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CONTENTS.

PAGE.

I. *The Genera Midotis, Ionomidotis and Cordierites.* By Elias J. Durand 1

II. *A Revision of the Atomic Weight of Boron. The Analysis of Boron Trichloride and Boron Tribromide.* By Gregory Paul Baxter and Arthur Ferdinand Scott 19

III. *Trajectory Surfaces and a Generalization of the Principal Directions in Any Space.* By Joseph Lipka 49

IV. *Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters.* By George W. Pierce 79

V. *The Compressibility and Pressure Coefficient of Resistance of Rhodium and Iridium.* By P. W. Bridgman 107

VI. *The Effect of Tension on the Thermal and Electrical Conductivity of Metals.* By P. W. Bridgman 117

VII. *The Thermal Conductivity of Liquids under Pressure.* By P. W. Bridgman 139

VIII. *The Compressibility of Five Gases to High Pressures.* By P. W. Bridgman 171

IX. *Summary of a Study of Stellar Distribution.* By Harlow Shapley and Annie J. Cannon 213

X. *A Revision of the Atomic Weight of Germanium. I. The Analysis of Germanium Tetrachloride.* By Gregory Paul Baxter and William Charles Cooper, Jr. 233

XI. *Marine Borers in Bermuda.* By E. L. Mark 257

XII. *New Species of Shipworms in Bermuda.* By W. F. Clapp . . . 277

XIII. *Some Properties of Three-Terminal Electrical Conducting Networks.* By A. E. Kennelly 295

XIV. *The Granites and Pegmatites of Cape Ann, Massachusetts.* By Charles H. Warren and Hugh E. McKinstry 313

XV. *Observations on Animal Life in the Thermal Waters of Yellowstone Park, With a Consideration of the Thermal Environment.* By Charles T. Brues 396

PAGE.

XVI. *The Bodily Proportions of Women in the United States; Based upon Measurements Taken from One Hundred Smith College Students.* By HARRIS HAWTHORNE WILDER AND MARGARET WASHINGTON PFEIFFER 439

XVII. RECORDS OF MEETINGS 605

BIOGRAPHICAL NOTICES 629

OFFICERS AND COMMITTEES FOR 1924–25 661

LIST OF FELLOWS AND FOREIGN HONORARY MEMBERS 663

STATUTES AND STANDING VOTES 683

RUMFORD PREMIUM 699

INDEX 701

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THE GENERA MIDOTIS, IONOMIDOTIS AND CORDIERITES.

BY ELIAS J. DURAND.

WITH TWO PLATES.

THE GENERA MIDOTIS, IONOMIDOTIS AND CORDIERITES.

By Elias J. Durand.

Received May 24, 1923. Presented by Roland Thaxter.

Note. The following paper, concerning which I had had some correspondence with Professor Durand shortly before his untimely death, is here published exactly as he left it except for the addition of a few Notes, as indicated, and of two plates which have been prepared to illustrate it. *Roland Thaxter.*

To the long established genera *Midotis* Fr. and *Cordierites* Mont. have been referred, from time to time, several little understood species of discomycetous fungi, mostly from the American tropics. While both genera as originally described were monotypic, either the type species were so rare, or so imperfectly described, or the type specimens are either missing entirely, or have been so inaccessible to those interested, that their nature and relationships have been involved in obscurity. The more recently described species are no better known, having been described from single collections in each case, and most inadequately characterized. Writers have referred them sometimes to one genus, sometimes to another, as their judgment, based always on too little information, seemed to dictate.

During the past twenty-five years several specimens have been sent to the writer by their collectors, or have been consulted in herbaria, which were evidently related to the genera under consideration, and which seemed to throw some light on their structure and relationship. That additional material needs to be consulted is only too evident to the writer. However, it seems to be worth while to put in form the information already at hand in the hope of adding something to our knowledge of these little-known plants.

Midotis Fr.

The genus *Midotis* was first published by Fries, in 1825,[1] as a genus of *Auricularini*, in the following terms:

"Hymenium laevissimum (exsiccatione rimosum) a pileo discretum & secedens. Asci immersi, sporidiis uniserialibus.

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[1] Syst. Orb. Veg. 363. 1825.

Ab Auricularia exacte differt ut Boletus a Polyporo. Species unica *Pleuropus*."

No species was mentioned, the terms *Mesopus*, *Pleuropus*, *Apus* etc., having been used by the author as descriptive, tribal headings rather than as the names of species, and having reference solely to the position of the stem, which in *Pleuropus* was lateral. Neither was there anything about the description to furnish more than the most general clue to the nature of the fungus. About all that was suggested was that the plant was an ascomycete somewhat resembling *Auricularia* in form.

A little additional light was thrown on the subject in 1828,[2] when the same author declared the hymenium to be inferior and *Peziza*-like. The only known species was described as *Midotis Lingua*, from material collected in Switzerland by Schleicher. It seems to have been a large plant (5 cm. high), attached by a short, lateral stipe, and was compared to a resupinate *Otidea leporina*, whatever that may be. It is hard to conceive of a resupinate *Otidea* with an inferior hymenium. *Midotis Lingua* is entirely unknown to Rehm and Boudier, nor has any other student of discomycetes collected or recognized the plant. The present writer saw no specimen in the Friesian Herbarium.

In 1849, Fries [3] again commented on the genus as follows:

"Excipulum coriaceo-suberosum, dimidiatum, verticale, disco atro unilaterali ceraceo ascigero, persistente."

"This paradoxical genus appears widely remote from others, but an exotic species (*M. heteromera* Mont. * * *) approaches so closely to the Dermateas, that it might be referred to the Encoelias, from which it differs as *Peziza leporina* from *P. cupularis*."

These notes convey the definite idea of an ear-shaped discomycete, much elongated on one side so as to be vertical, bearing the persistent hymenium on one surface, and differing from *Otidea* in being coriaceous rather than fleshy in texture. Of special importance is Fries's reference to a second species of *Midotis*, the *Peziza heteromera* Mont., from tropical America, from which, in the absence of specimens of *M. Lingua*, any modern conception of the genus must be derived.

Saccardo [4] included the genus *Midotis*, but unfortunately combined with it the little known genus *Wynnea*, of Berkeley, which Dr. Roland Thaxter [5] has more recently elucidated from study of fresh and authentic material, and shown to be a very distinct and well marked generic type.

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[2] Elench. Fung. 2: 29. 1828.
[3] Summa Veg. Scand. 362. 1849.
[4] Syll. Fung. 8: 547. 1889.
[5] Bot. Gaz. 39: 241–247, pl. 4. 1905.

The type material of *Peziza heteromera* Mont., collected in Guiana, by Leprieur, has not been available to the writer. However, there is a specimen from Cuba, in Cooke's herbarium, at Kew, marked "ex herb. Montagne," which may reasonably be regarded as authentic. A study of this, and certain other West Indian collections agreeing exactly with it, has convinced the writer that they represent a good generic type to which the name *Midotis* should be applied. Two other West Indian collections examined seem to be undescribed species, congeneric with *M. heteromera*. The present writer's conception of *Midotis* may be set forth as follows:

Ascomata superficial, solitary, or clustered, or several arising from a common base, spherically closed when young, then opening out, becoming vertically elongated on one side when mature, light colored; excipulum suberous or subcoriaceous, composed of interwoven hyphae which either project from the surface as very short, stout, septate pili, or are replaced by a thin cortical pseudo-parenchyma the cells of which project in groups, not becoming violet with KOH; asci cylindric clavate, opening by a pore, not blue with iodine; spores small, hyaline, continuous.

A. Plants large, nearly sessile, externally furfuraceous, rhubarb-color, spores 8–10 μ long..........................1. *M. heteromera*.
A. Plants small, slenderly stipitate, spores 4–6 μ long.
  B. Excipulum of hyphae projecting to form short, moniliform, brown pili; spores ovoid..............2. *M. occidentalis*.
  B. Cortical layer of excipulum parenchymatous, brown; spores elliptic.........................3. *M. infundibuliformis*.

1. MIDOTIS HETEROMERA (Mont.) Fr., Summa Veg. Scan. 362. 1849.

Peziza heteromera Mont., Ann. Sci. Nat. II. **13**: 206. 1840.
Midotis verruculosa B. & C., Journ. Linn. Soc. Bot. **10**: 370. 1868.

Ascomata solitary or several arising from a common stem-like base, irregularly elongated on one side, *Otidea*-like, tough, corky to leathery when dry, 2–3 cm. high and broad, externally verrucose, a beautiful rhubarb-color, disk reddish brown. *Excipulum* composed of closely interwoven, hyaline hyphae, remotely septate within, the external ones projecting from the surface are stouter and closely septate into cells as long as broad, with yellow walls, and aggregated in groups to form the external warts, not becoming violet with KOH. *Hymenium* about

150 μ thick. *Asci* clavate-cylindric, apex rounded, not blue with iodine, 80–90 × 5 μ. *Spores* 8, uniseriate, hyaline, smooth, 2-guttulate, oblong to oblong-cymbiform, 8–10 × 3.5–4 μ; *Paraphyses* cylindric, hyaline, slightly longer than the asci.

On rotten wood (apparently), tropical America.

MATERIAL EXAMINED.

Cuba: ex herb. Montagne (K.); Wright n. 580 (K); Wright n. 663 (type of *M. verruculosa* B. & C.) (K.).

Montserrat: Blake's Mt., Jan. 8, 1907, J. A. Shafer n. 908 (NY).

In general appearance and texture this plant has much in common with the genera *Cookeina* and *Phillipsia*, but the hymenium is quite different. Unfortunately the writer has seen the plant only in the dried state in which the colors probably have changed, although the substance seems to shrink but little in drying.

M. verruculosa B. & C. was said by its describers to differ from *M. heteromera* "in its verrucose, not simply mealy surface, and the broader rather shorter sporidia." After careful comparison of authentic specimens the writer is unable to detect any difference even in degree between them. No "large concatenate cells" such as were noted by Massee [6] have been observed in the excipulum.

NOTE. In the Curtis Herbarium are two specimens; the first, labeled "Midotis verruculosa B. & C., Fung. Cub.; 705. Ad lign. mort. Cuba, C. Wright (663)," the second "Cordierites lateritia B. & C., on rotten logs, May, 1857, Cuba, coll. Wright," with a note, apparently a memorandum of Wright, "clustered, cinnamon or light brown. Has the odor of Wintergreen or Black Birch." Also a note by Farlow "Was the name Cordierites lateritia ever published? See Midotis verruculosa B. & C. in Herb. Curtis." These two seem identical, and the habit herewith reproduced on Plate I, as well as figure 1 of Plate II, are taken from the second specimen. *R. T.*

Midotis occidentalis Durand n. sp.

Ascomata small, solitary or closely gregarious, stipitate, coriaceous; *stipe* slender, terete, longitudinally striate, blackish brown, 3–7 mm. long, 1 mm. thick, expanding to form the *cup* which is vertically elongated on one side and split down to the stem on the other, 3–5 mm. long and wide, externally blackish brown, vertically striate, hymenium brownish cervinous, margin irregular. *Excipulum* composed of slender hyphae densely interwoven, black at the surface and immediately beneath the hymenium, the intermediate zone nearly hyaline, the superficial hyphae projecting very slightly from the surface are slender, closely septate and moniliform, not violet with KOH. *Hymenium* about 150 μ thick, hyaline; *Asci* clavate-cylindric, apex rounded, not blue with iodine, about 80 μ long. *Spores* 8, uniseriate, hyaline, smooth, ovate-elliptic, 5–6 × 3 μ. *Paraphyses* slender, hyaline, not thickened above, longer than the asci.

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[6] Journ. Linn. Soc. Bot. 31: 467. 1896.

On a decaying log, Mabess River, Jamaica, Apr. 23, 1903, W. R. Maxon n. 1549 (D).

This material was communicated to the writer some years ago by Dr. P. L. Ricker. It so closely resembles *Cordierites Sprucei* in form, size, spores etc., that it was referred provisionally to that imperfectly described species. Subsequent comparison with authentic material, however, shows a difference in color, structure of excipulum, and reaction to KOH. It is congeneric with *M. heteromera.*

Midotis infundibuliformis Durand n. sp.

Ascomata solitary or gregarious, stipitate, 3–5 mm. high, the base of each usually surrounded by a group of minute conical or columnar young fruit bodies; *stipe* slender, 2–3 mm. long, 1 mm. thick, dark brown, furfuraceous, gradually expanding into the *cup* which is at first closed, then expanding to become cupulate, finally much elongated vertically on one side, externally cinnamon-brown, furfuraceous, usually more or less concentrically zonate near the margin, the disk griseus or brownish griseus, 2–3 mm. high and wide, often hood-shaped. *Excipulum* suberous, composed of densely interwoven hyaline hyphae, replaced by a brown cortical layer of pseudoparenchyma, the polygonal cells 10–12 μ in diameter, with brown walls, projecting from the surface in groups, not yielding a violet solution with KOH. *Hymenium* hyaline, about 80 μ thick. *Asci* clavate-cylindric, apex rounded, not blue with iodine, 50–55 μ long. *Spores* 8, uniseriate, hyaline, smooth, continuous, oblong-elliptic, 4–6 × 2–3 μ. *Paraphyses* cylindric, hyaline, somewhat longer than the asci.

On rotting logs, Emperor Valley, Port of Spain, Trinidad, December, 1912, R. Thaxter. Type in the Farlow Herbarium, Harvard University.

This species resembles *M. occidentalis* in size and form, but differs in color, the zonate surface, structure of excipulum, thinner hymenium, and shape of the spores. The writer is greatly indebted to Dr. Roland Thaxter for placing at his disposal this and several other unique *Midotis*-like forms described in the present paper.

Note. The cup in this species is not always "greatly elongated" on one side, although this asymmetry is usually characteristic. The "conical columnar young fruit bodies" referred to in the description are pycnidia, (Plate II, figure 4), which, when young, are filled with a mass of very minute, irregularly angular, brownish spores, which are subisodiametric, 1.5–2 μ in diameter and are discharged through a terminal pore. They are often crowded about the bases of the apothecial stalks, and become quite black; resembling the long-necked ostioles of some pyrenomycete, tapering from a somewhat swollen base to a subulate extremity, or nearly isodiametric throughout. The material was found growing gregariously on a hard decorticated log in the Emperor Valley, a part of the Botanic Garden at Port of Spain; a rich locality of native vegetation which has since been felled, burned over and effectually destroyed. *R. T.*

In addition to the four species already mentioned several other discomycetes have at one time or another been referred to the genus *Midotis*. Of these, *M. gigantea* (B. & C.) Sacc. and *M. macrotis* (Berk.) Sacc. are species of *Wynnea*. *M. crispata* B. & C. is a *Phillipsia*. *M. regularis* C. & P.,[7] from Brazil, if really erumpent as described, is not a true *Midotis*. *M. Patella* Fr.,[8] from Mexico, may be a *Wynnea*. Neither of the two last mentioned forms have been seen by the writer, and the descriptions are too inadequate to give more than a hint as to their relationship. *M. brasiliensis* Rick.,[9] with its large asci and spores is probably truly Pezizaceous. No specimen has been seen. Of special interest to students of North American fungi are *M. irregularis* (L. v. S.) Cke., of the Eastern States, and *M. plicata* Phil. & Hark., of California. A careful study of authentic material of these species has convinced the writer that they represent a distinct and undescribed generic type which, with its known species, may be characterized as follows:

Ionomidotis Durand, Nomen nov.[10]

Ascomata superficial, solitary, or several arising from a common base, irregular and much elongated on one side when mature, usually violet or olive; substance thin and fragile, exuding a deep violet solution with KOH; excipulum composed of interwoven hyphae, with a cortical layer of pseudoparenchyma; asci clavate-cylindric, opening by a pore, not blue with iodine; spores small, hyaline, continuous.

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[7] Grev. 9: 101. 1881.
[8] Nov. Symb. Mant. 231.
[9] Ann. Myc. 2: 246. 1904.
[10] 'ιόν (violet) + Midotis, referring to the violet solution with KOH.

7 species are known as follows:

A. Paraphyses with deciduous lanceolate tips; plants large. 1. *I. irregularis.*
A. Paraphyses without deciduous tips.
  B. Hymenium blackish brown or blackish violet.
    C. Blackish brown.
      D. Spores 5–6 μ, plants large..................2. *I. Chilensis.*
      D. Spores 15–18 μ........................3. *I. Nicaraguensis.*
    C. Blackish violet, spores minute.
      E. Ascomata irregularly goblet-shaped, nearly sessile....................................4. *I. plicata.*
      E. Ascomata vertical, stipe slender........5. *I. Sprucei.*
  B. Hymenium olivaceous.
    F. Ascomata urceolate, externally furfuraceous; spores 10–12 μ......................................6. *I. urceolata.*
    F. Ascomata unsymmetrically discoid, externally granular; spores 7–8 μ...............................7. *I. olivascens.*

Ionomidotis irregularis (L. v. S.) Durand.

Peziza irregularis L. v. S., Proc. Phil. Acad. Sci., **4**: 171. 1834.
Cordierites irregularis (L. v. S.) Cke., Bull. Buf. Acad. Sci., **3**: 26. 1875.
Peziza doratophora E. & E., Journ. Myc., **1**: 90. 1885.
Otidea doratophora (E. & E.), Sacc. Syll. Fung., **8**: 96. 1889.
Midotis irregularis (L. v. S.) Cke. in Sacc. Syll. Fung., **11**: 422. 1895.

Ascomata solitary or generally clustered, several arising from a common, stem-like base, up to 2 cm. long, which penetrates the substratum, at first closed then opening and expanding, becoming much elongated on one side and irregularly lobed or lacerated, up to 3 cm. long and wide, the clusters 5–7 cm. wide; externally scurfy, dark chestnut-brown, the hymenium darker or almost black. *Excipulum* thin (300 μ) and fragile, slightly gelatinous, dark violet-brown, opaque, yielding a deep violet solution with KOH, composed of slender interwoven hyphae, replaced at the surface by a thin parenchymatous layer of polygonal cells 8–10 μ in diameter, projecting from the surface in groups. *Hymenium* about 100 μ thick. *Asci* cylindric-clavate, the apex truncate-rounded, not blue with iodine, 50–70 × 4–5 μ; *Spores* 8, obliquely uniseriate or subbiseriate, clavate-oblong, hyaline with a tinge of violet, continuous, smooth, 2–3-guttulate, 8–10 × 3–4 μ. *Paraphyses* cylindric, hyaline with a violet tint, each terminated by an abrupt, lanceolate, acute head, 18–30 × 3–4 μ, becoming 1–3-septate, which projects about its length beyond the hymenium and easily separates from its base.

On rotten wood and branches lying on the ground. Eastern United States.

MATERIAL EXAMINED.

New Hampshire: White Mts., Miss Minns (type of *Peziza doratophora*) (NY).
Connecticut: West Haven, R. Thaxter (D.)
New York: Sandlake, C. H. Peck (A); Lake Placid, G. F. Atkinson (D).
Pennsylvania: Bethlehem, Schweinitz (S type) (K).
Ohio: A. P. Morgan (D) (K) (NY).

A remarkable discomycete both in habit and microscopical characters. The flesh is exceptionally thin and so saturated with the violet pigment as to be quite opaque even in thin sections, yielding a profuse, deep violet solution on treatment with dilute KOH. The lanceolate tips of the paraphyses readily separate from their slender bases at a septum, suggesting a conidial nature. They have not been seen germinating, and it is doubtful whether they so function. Somewhat similar conidium-like tips exist in *Lachnella diplocarpa* Curr., which Massee has made the type of a new genus, *Diplocarpa*, on this account. Morgan's Ohio specimens, previously reported as *Midotis plicata*, were given the manuscript name of *Diplocarpa tinctoria* by Massee. However, this species is quite different from Currey's plant.

NOTE. This form, which I have found at Burbank, E. Tennessee as well as at West Haven, Connecticut, may reach a length of more than seven centimeters, and varies from an almost simple to a copiously branching fasciculate habit, the thin apothecia spreading outward horizontally from the sides of fallen logs, with the hymenium directed downwards; so that it might easily be mistaken for some member of the Thelephoraceae. The peculiar deciduous tips of the paraphyses become tinged with brown, the protoplasm collecting along the inner margin of the wall. The separation of these tips, which project beyond the asci, is not an abjunction; but results from breakage, and seems certainly to have no significance such as is above suggested. The spores show a small clump of protoplasm at either pole and a broad equatorial band which, in alcoholic material, has the appearance of a pseudoseptum. *R. T.*

Ionomidotis chilensis Durand n. sp.

Ascomata solitary or cespitose, several arising from a common stem-like base, deeply cleft or lacerated to varying depths so as to resemble a broad-branched *Clavaria*, the lobes much elongated on one side, more or less vertical, ascigerous on one surface only, the cluster 2 cm. or more high and broad, very irregular and the margin fluted, at least when dry; disk black, the sterile surface brownish black, tissue thin and fragile; *Excipulum* yellowish brown when dry, blackish when wet, opaque, composed of interwoven brown hyphae, replaced at the surface by a thin cortical parenchyma of rounded or polygonal cells, 6–8 μ in diameter, yielding a violet solution with KOH. *Hymenium* brown, opaque, about 65 μ thick. *Asci* cylindric-clavate, apex rounded, not blue with iodine, 40–45 μ long. *Spores* 8, biseriate, minute, hyaline, continuous, cylindric, strongly curved, ends obtuse, 5–6 × 1.5 μ. *Paraphyses* cylindric, the tips variably thickened, nodulose, brown, slightly longer than the asci.

On rotting wood, Corral, Chile, December, 1905, R. Thaxter. Type in the Farlow Herbarium, Harvard University.

This species looks somewhat like *I. irregularis* externally, but the paraphyses lack the deciduous, lanceolate tips so characteristic of that plant, and the spores and method of branching are different.

It at first seemed possible that this might be an undescribed species of *Peltigeromyces* A. Möll., on account of the peculiar branching and lobing, but so far as one can judge from the description the single known species of that genus is a much more regular cupulate plant, attached by the center, and not *Otidea*-like.

NOTE. Two gatherings of this peculiar form were made at Corral which differ slightly in habit. That communicated to Dr. Durand differs in the more deeply and copiously cleft or lacerated character of the margin, but in other respects the two seem to be identical. The hymenium in both is at least partly inferior. The species is distinguished microscopically from the others by the paraphyses, the tips of which may be unmodified; but are usually, sometimes very considerably and abruptly, enlarged. *R. T.*

Ionomidotis Nicaraguensis Durand n. sp.

Ascomata closely clustered or coalesced at the base, at first closed, then opening at the apex and expanding so as to become applanate (when dry), sessile, attached at one side of the base, 1–2 cm. in diameter; externally dark ferruginous brown, verrucose, the disk blackish brown becoming dark reddish purple when wet. *Excipulum* tough and corky when dry, corky-gelatinous when moist, composed of slender hyphae, 3–4 μ thick, closely interwoven, passing at the surface into a pseudoparenchyma of cells 10–12 μ in diameter which project in groups from the surface, yielding a violet solution with KOH. *Hymenium* about 200 μ thick. *Asci* cylindric-clavate, apex rounded, not blue with iodine, 135–150 × 8 μ. *Spores* 4–6 in each ascus, obliquely uniseriate, hyaline, smooth, continuous, oblong, 15–18 × 5–7 μ. *Paraphyses* slender, hyaline, slightly thickened and brownish at the tips.

On rotten logs, Volcan Mombacho, Department of Grenada, Nicaragua, 20 February, 1903, C. F. Baker no. 2500 (D).

The specimens on which the above description is based were sent to the writer many years ago, by Prof. F. S. Earle, in a batch of tropical American discomycetes. They evidently belonged to a species related to *Peziza irregularis* L. v. S., but never could be placed satisfactorily. The plants evidently do not change form much in drying. No more than six spores have been detected in any ascus.

Ionomidotis plicata (Phil. & Hark.) Durand.

Midotis plicata Phil. & Hark., Bull. Cal. Ac. Sci., **1**: 24. 1884.

Ascomata small, clustered, or several arising from a common base which is seated on the wood, at first closed, then opening by a pore at the apex, becoming unsymmetrically goblet-shaped, resembling a miniature *Urnula craterium*, 1–2 mm. high and wide, entirely black when moist, brownish black when dry, externally granular-furfuraceous, flesh thin and dark violet-brown and opaque under the microscope. *Excipulum* composed of interwoven hyphae, passing at the surface into a thin parenchymatous cortex the cells of which project in groups to form the external granules, yielding a deep violet solution with KOH. *Hymenium* about 75–80 μ thick. *Asci* cylindric-clavate, apex rounded, not blue with iodine, 65–70 μ long. *Spores* 8, irregularly biseriate, hyaline, smooth, continuous, narrowly oblong, straight or strongly curved, 5–8 × 2–3 μ. *Paraphyses* numerous, cylindric, septate, often forked, the contents pale violet-brown, the tips slightly swollen and darker.

On dead *Umbellularia californica*, Sausalito, California, Aug. 1881, no. 2743.

This species is known only from the original collection as above. While the collector's name is not given it was probably H. W. Harkness.

The description given is drawn from the material preserved in the herbarium of the California Academy of Science, for access to which the writer is indebted to Miss Alice Eastwood. The somewhat plicate hymenium mentioned in the original description is not evident in the specimens seen. How the paraphyses could have been overlooked by the describers is a mystery as they are the most conspicuous thing about the hymenium.

Ionomidotis olivascens Durand n. sp.

Ascomata solitary or gregarious, rarely cespitose, at first closed, then opening by a pore, finally becoming discoid, nearly plain with a slightly elevated margin, 1.5–3 mm. across, entirely blackish olive, the disk with a yellowish tint, externally granular or nearly smooth, abruptly narrowed to a slender stem, 1–2 mm. high, 0.5 mm. thick, attached to one side of the cup below, so that the latter varies from merely unsymmetrical to *Otidea*-like, scarcely changing in form or color on drying. *Excipulum* composed of slender, interwoven, brown hyphae replaced by a cortical layer of brown, polygonal cells, 12–14 μ in diameter, a few of the superficial ones projecting to form the granules, yielding a deep violet solution with KOH. Hymenium 60–65 μ thick. *Asci* clavate, apices rounded, not blue with iodine, 45–50 μ long. *Spores* 8, biseriate, hyaline, smooth, continuous, straight or slightly curved, narrowly oblong, 7–8 × 3 μ. *Paraphyses* cylindric-filiform, scarcely thickened above, brownish.

On rotten wood, Cocoanut Grove, Florida, November, 1897, R. Thaxter no. 111. Type in the Farlow Herbarium, Harvard University.

The olive hymenium of this species suggests the genus *Chlorosplenium*, but its other characters are different. The irregular, unsymmetrical ascomata call to mind *C. versiforme*, but they are smaller, tougher, less contorted than in that species, and the structure and reaction of the excipulum to KOH are quite distinct. Nevertheless, *I. olivascens* may have been described somewhere as a *Chlorosplenium* although attempts to refer it to any published species have failed.

Ionomidotis Sprucei (Berk.) Durand.

Cordierites Sprucei Berk., in Hook. Journ. Bot., **8**: 280. pl. 10, f. 5. 1856.

Ascomata small, solitary or closely gregarious, stipitate, entirely blackish vinaceous; stipe terete, 5 mm. long, 1 mm. thick, minutely furfuraceous, expanding above to form the cup which is oblique, much elongated vertically on one side, split down the other to the summit of the stipe, 3–5 mm. long and wide, externally minutely furfuraceous. *Excipulum* composed of vinous-brown hyphae, closely interwoven, replaced by a cortical parenchymatous layer of polygonal cells with blackish-brown walls, 8–12 μ in diameter, the superficial ones projecting in groups making the surface rough, the whole yielding a violet solution with KOH. *Hymenium* about 125 μ thick, brown. *Asci* clavate-cylindric, apex rounded, not blue with iodine, 65 μ long. *Spores* 8, uniseriate, hyaline, smooth, continuous, ovate-elliptic, 5–6 × 3 μ. Paraphyses slender, pale brown, longer than the asci, often nodulose above.

On decaying trunks, Panuré, Brazil, Spruce (NY).

The above description is drawn from two specimens in the herbarium of the New York Botanical Garden marked "*Cordierites Sprucei* Berk. Brazil (Spruce)," and which are undoubtedly parts of the original collection. In the original description the asci were reported as not seen. In the material examined the plants appear to be old, and only an occasional ascus is to be found among the numerous paraphyses. This species is clearly congeneric with the other members of the genus *Ionomidotis*, and appears to have little in common with Montagne's *Cordierites guienensis*, as Saccardo has already pointed out. However, Massee, in his redescriptions of Berkeley's types,[11] retained it in that genus, and figured the stipe as forked in a dendroid manner.

Ionomidotis urceolata Durand n. sp.

Ascomata solitary or cespitose, several arising from a common, stroma-like base seated on the bare wood or occupying cracks in the bark, at first closed, then opening by a pore at the apex, becoming urceolate, the margin incurved, either regular, or in some cases, split down one side and *Otidea*-like, when wet entirely dark violet-brown, the exterior brown and the disk olive when dry, 1–2 mm. high and wide. *Excipulum* composed of slender, interwoven, pale brown hyphae, replaced by a cortical layer of pseudoparenchyma of polygonal cells, 8–10 μ in diameter, with dark brown walls, which project in groups to form the lumps on the surface, the whole yielding a deep violet solution with KOH. *Hymenium* 100–120 μ thick. *Asci* narrowly clavate, apices rounded, not blue with iodine, 85–90 × 5–7 μ. *Spores* 8, obliquely uniseriate, hyaline, smooth, continuous, 1–2-guttulate, narrowly oblong, ends rounded, straight or curved, 10–12 × 3–4 μ. *Paraphyses* very slender, hyaline, cylindric, longer than the asci, scarcely thickened at the tips.

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[11] Journ. Linn. Soc. 31: 466, pl. 18, f. 4–5. 1896.

On rotten wood, Palm Beach, Florida, December, 1897, R. Thaxter no. 110. Type in the Farlow Herbarium, Harvard University.

The ascomata are about the size of those of *I. plicata*, but differ in their more regular form, their color, more furfuraceous exterior and smaller spores. The species is evidently related to the imperfectly described *Midotis regularis* Cke. & Phil., but differs in color of disk, at least.

DESCRIPTION OF PLATE I.

NOTE. These figures have been photographed at Cornell University through the kindness of Professor Fitzpatrick, and are about the natural size, as reproduced, of the dried material. Unfortunately all the apothecia of *Midotis infundibuliformis* were broken off in transit before the photograph was made, so that only a few pycnidia and broken bases of apothecia are visible. For the general habit of the fungus, reference should be made to figure 2, of Plate II.

Midotis heteromera (Mont.) Fries. Young and mature apothecia ex Herb. Curtis sub "*Cordierites lateritia* B. & C."

Midotis infundibuliformis Durand. The apothecia broken, only their bases and pycnidia remaining. See Plate II, figure 2.

Ionomidotis irregularis (L. v. S.) Durand. Left hand figure seen from below, showing surface of hymenium: right hand figure showing upper surface. Material collected at West Haven, Connecticut.

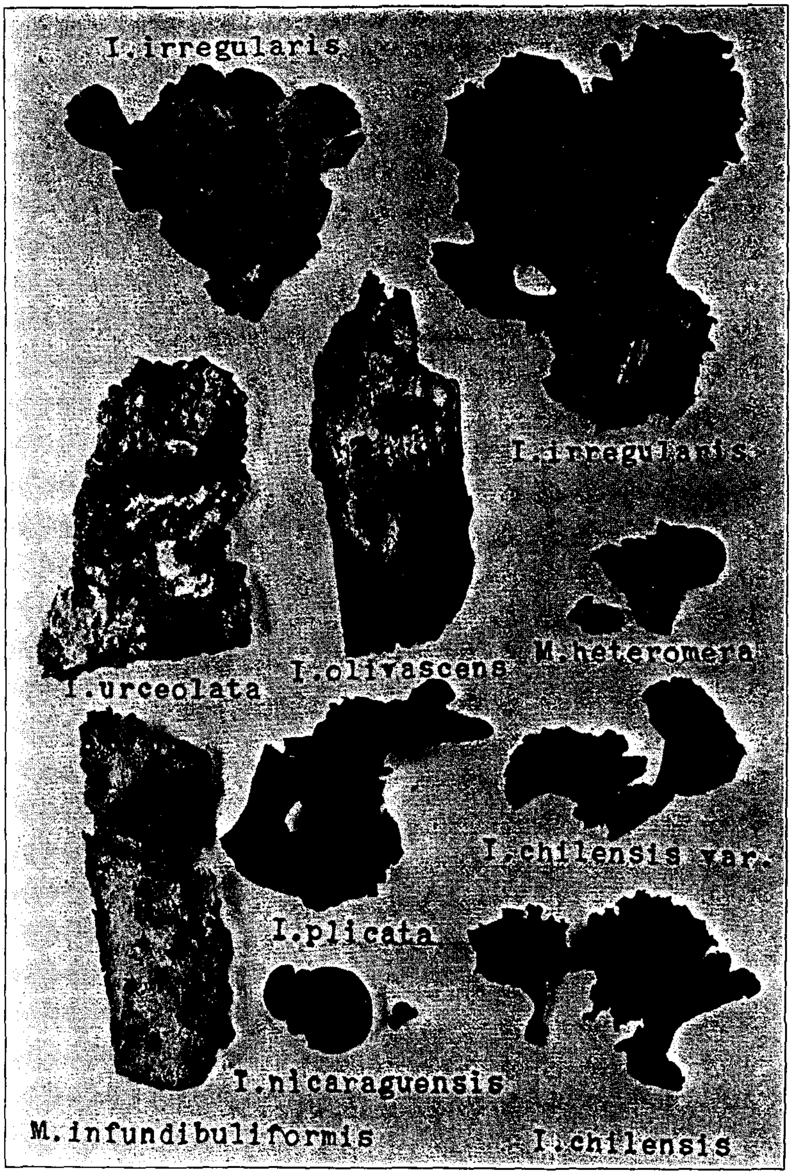
Ionomidotis urceolata Durand. Type specimen from Palm Beach, Florida.

Ionomidotis olivascens Durand. Type material from Cocoanut Grove, Florida.

Ionomidotis plicata (Phil. & Hark.) Durand. Part of original gathering from California in Herb. Durand.

Ionomidotis nicaraguensis Durand. Type material in Herb. Durand.

Ionomidotis chilensis Durand. Lower figure typical form from original gathering at Corral, Chile. Upper figure a less lacerate and fluted form from same locality.


I. irregularis
I. irregularis
I. urceolata
I. olivascens
M. heteromera
I. chilensis var.
I. plicata
I. nicaraguensis
M. infundibuliformis
I. chilensis

DESCRIPTION OF PLATE II.

NOTE. The figures of spores, asci and paraphyses were drawn with Leitz water immersion and No. 4 ocular in all cases. Figure 5 was drawn natural size from fresh material. Figures 2 and 4 were outlined with Zeiss a * obj. and oc. 4. The linear dimensions of the original drawings have been reduced somewhat less than a third in reproduction.

Midotis heteromera (Mont.) Fres.

Fig. 1. Ascus from specimen in Herb. Curtis labeled *Cordierites lateritia* B. & C.

Midotis infundibuliformis Durand.

Fig. 2. General habit, side and front views of dried material; the zonation, which is often hardly visible, somewhat exaggerated.

Fig. 3. Asci and paraphysis.

Fig. 4. Two pycnidia, one discharging spores shown at right enlarged, and between them the broken stalk of an apothecium. Type material from Trinidad, B. W. I., dry.

Ionomidotis irregularis (L. v. S.) Durand.

Fig. 5. Habit drawn natural size from fresh material collected at New Haven, Connecticut; here reduced about one third.

Fig. 6. Ascus and paraphyses with one of the separable tips of the latter.

Ionomidotis chilensis Durand.

Fig. 7. Ascus, spores and paraphyses, showing variations in tips of latter.

Ionomidotis nicaraguensis Durand.

Fig. 8. Ascus and paraphyses from type.

Ionomidotis plicata (Phil. & Hark.) Durand.

Fig. 9. Ascus, spores and paraphyses from original material.

Ionomidotis olivascens Durand.

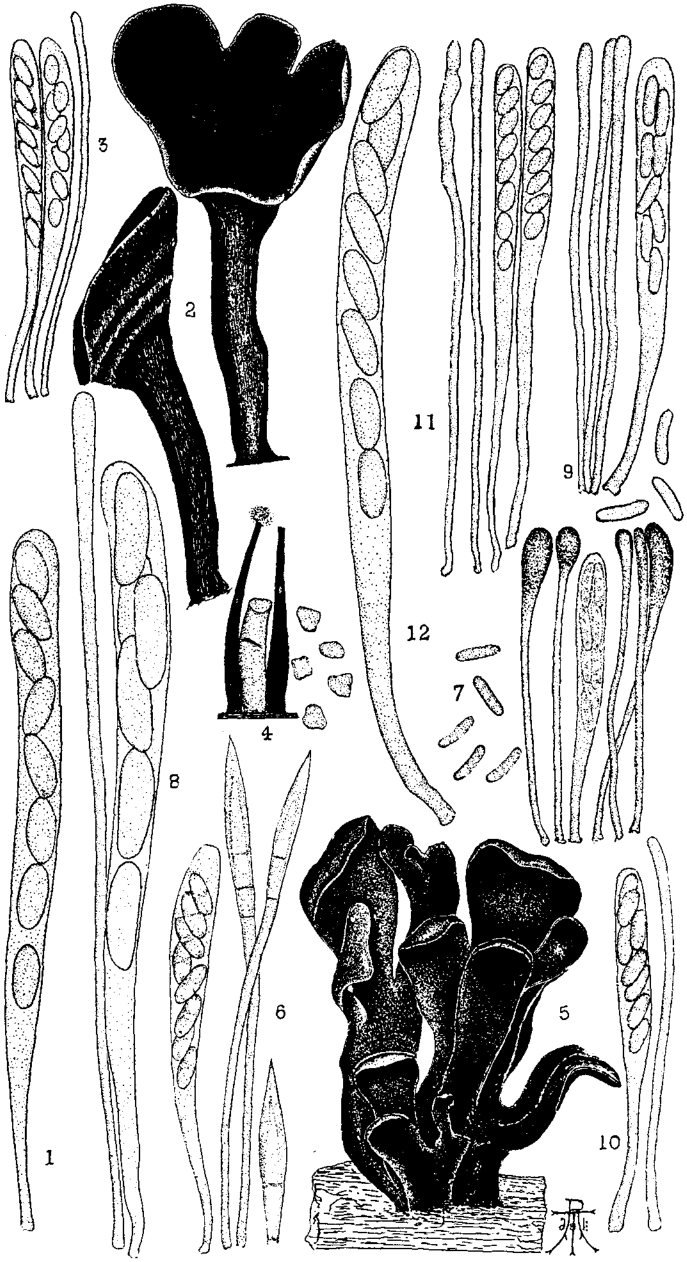
Fig. 10. Ascus and paraphyses from type.

Ionomidotis Sprucei (Berk.) Durand.

Fig. 11. Asci and paraphyses. Original material from Herb. N. Y. Bot. Garden, in Herb. Durand.

Ionomidotis urceolatus Durand.

Fig. 12. Ascus and paraphysis from type.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(CONTRIBUTION FROM THE T. JEFFERSON COOLIDGE, JR. MEMORIAL LABORATORY OF HARVARD UNIVERSITY.)

A REVISION OF THE ATOMIC WEIGHT OF BORON.

THE ANALYSIS OF BORON TRICHLORIDE AND BORON TRIBROMIDE.

BY GREGORY PAUL BAXTER AND ARTHUR FERDINAND SCOTT.

(CONTRIBUTION FROM THE T. JEFFERSON COOLIDGE JR. MEMORIAL LABORATORY OF HARVARD UNIVERSITY.)

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CONTENTS.

Introduction 21
Purification of Materials 22
Preparation of Boron Trichloride 25
Preparation of Boron Tribromide 31
Analysis of Boron Trichloride 33
Analysis of Boron Tribromide 37
Results 39
Discussion of Results 42
Summary 47

Introduction.

During recent years until 1918 the value 11.0 for the atomic weight of boron was in common use.[1] In that year, however, Smith and Van Haagen [2] published the results of a strikingly consistent series of experiments in which weighed quantities of anhydrous borax were converted into sodium chloride, sulfate, fluoride, nitrate and carbonate, either directly or with the intervention of acetic acid. While the number of experiments of each sort was in general small, the remarkable concordance in the values for the atomic weight of boron obtained from the varied ratios fills the reader with admiration for the extraordinarily accurate analytical technique of the experiments, and leaves no question but that in this respect nothing more could be desired. The value for the atomic weight of boron was found to be 10.900, a value subsequently adopted by the International Committee on Atomic Weights. One of the chief obstacles encountered by Smith and Van Haagen was found to be the complete dehydration of borax, and they impute to this difficulty the divergence from theirs of most of the earlier values based on the analysis of borax.

Only one sample of borax was prepared by Smith and Van Haagen, by neutralizing recrystallized sodium carbonate with a slight excess of boric acid. Although this specimen was crystallized, fused and recrystallized before the final dehydration, the question may well be raised as to whether its composition could be represented exactly by the conventional formula. Smith and Van Haagen state:

> "Reliable tests indicating the presence, in borax, of traces of other borates (either more basic or more acid in character) are lacking. However, such borates are prepared only with difficulty and as, in the present instance, the proportion of the components and the conditions of formation were those corresponding to the tetraborate with ten molecules of water, it may safely be assumed that the preparation was pure also from this point of view."

It is nevertheless true that borates in which the ratio of sodium to boron varies from 1 to 1, even to 1 to 5 have been prepared.[3] Therefore it is conceivable, owing to the complicated nature of borax solutions and to the presence of the slight excess of boric acid used by Smith and Van Haagen in the preparation of their borax, that their *material occluded small amounts of salts more acid than borax.* The effect of such occlusion would be to raise the apparent atomic weight of boron.

In view of this possibility it is unfortunate that specimens of borax crystallized initially from solutions of somewhat different composition were not compared, in order to make certain that the composition of the recrystallized salt is really independent of small variations in the conditions of preparation.

Because of the low atomic number and isotopic character of boron, its atomic weight is a particularly interesting one. Therefore in order to obtain further evidence in the matter we have purified and *analyzed both the trichloride and the tribromide. In a brief note the* preliminary result of these analyses has already been stated [4] to be 10.83 ± 0.01. The final result, 10.82, agrees exactly with the result of the analysis of boron trichloride recently published by Hönigschmid and Birckenbach.[5] Still more recently, in a preliminary communication, Stock and Kuss,[6] by measuring the hydrogen evolved in the decomposition of diboran with water, have obtained the value 10.81.

PURIFICATION OF MATERIALS.

Water.— Ordinary distilled water was twice redistilled, once from alkaline permanganate and once from very dilute sulfuric acid. The condensers were block tin tubes, fitted to Pyrex flasks with constricted necks, which served as stills. The connection was made by a water seal, no cork or rubber being used. The water was collected in Jena flasks, generally just previous to use.

Nitric Acid.— Concentrated C. P. nitric acid was distilled through a quartz condenser, the first two-thirds as well as the last tenth being rejected.

Bromine.— This material was purified according to the method which has been frequently used at Harvard.[7] Crude bromine was first distilled from aqueous potassium bromide in order to remove chlorine. Next one-fourth of the product was converted to potassium bromide by means of recrystallized potassium oxalate, and the remaining three-fourths of the bromine was distilled from solution in this potassium bromide. All the product was then converted to potassium bromide by means of potassium oxalate, and the potassium bromide was fused in a platinum dish with enough recrystallized potassium permanganate to oxidize all organic matter. In order to obtain bromine the purified potassium bromide was dissolved in a solution of pure sulfuric acid. The excess of potassium permanganate contained by the salt caused a small quantity of bromine to be liberated, which was removed by distillation since it might have contained a trace of iodine. Enough pure permanganate to liberate the greater portion of the bromine was then added, and the bromine was distilled into a receiver cooled with ice. In this last step the bromine received a third distillation from a bromide. The product was separated from the water, dried with resublimed phosphorus pentoxide and once redistilled. Small portions of similar bromine were furnished by Dr. P. F. Weatherill and Mr. M. J. Dorcas.

Silver.— This substance was prepared by standard methods. These consisted in brief of the following processes: double precipitation as chloride, followed by reduction with alkaline sugar solution, fusion of the metal on charcoal, solution and reprecipitation with ammonium formate, fusion on pure lime, electrolytic transport, fusion on pure lime in hydrogen, etching, drying in a vacuum at 500°C. Recent work by Baxter and Parsons [8] and Baxter [9] as well as earlier work have shown that these processes yield a product of adequate purity. Four other samples of silver which had been prepared in essentially the same way by Dr. Weatherill,[10] and Messrs. Tani,[11] Dorcas and Hilton,[12] were used in certain analyses as indicated later.

Chlorine.— This substance was drawn directly from a large tank of liquid material, a considerable portion of which had already been used. It was purified and dried by passing through two towers filled with beads wet with water, then three similar towers containing concentrated sulfuric acid, and finally through a tube containing resublimed phosphorus pentoxide. The purifying apparatus contained no rubber except one very short connection with the tank.

Helium.— This gas was very kindly provided by the United States Navy Department. In order to remove the greater part of the nitrogen and hydrocarbons the helium, after passing through a gas washing bottle containing concentrated sulfuric acid, was conducted through a hot hard glass tube containing metallic calcium to remove nitrogen, and copper oxide to oxidize hydrogen and hydrocarbons. The product then passed over aqueous silver nitrate, concentrated aqueous sodium hydroxide, concentrated sulfuric acid and resublimed phosphorus pentoxide in sequence. Rubber connections were avoided beyond the hard glass tube.

Boron.— Since the halides were prepared by synthesis from the elements, it was necessary to prepare elementary boron. This was done by the reduction process of Moissan,[13] which consists in combining magnesium with boron trioxide and extracting the product with acid. In one preparation an excess of boric oxide was used, so that the so-called boron suboxide resulted. This material proved unsatisfactory from the standpoint of reactivity with halogens, so that nearly all our material was prepared with an excess of magnesium. The greater portion of this excess was ultimately extracted with acid, but residual magnesium or magnesium boride could do no harm owing to the low volatility of magnesium halides.

To prepare the suboxide the trioxide was mixed with powdered magnesium in the proportions of 3 to 1, and the mixture, contained in a sand crucible, was placed in a muffle which already had been heated to dull redness with a blast flame. At this temperature the reaction soon started and progressed rapidly throughout the mass. After cooling, the contents of the crucible were extracted with an excess of hydrobromic acid and the residue was washed by decantation until the washings were neutral. Centrifugal settling in glass receptacles materially hastened the progress of the washing. The product was first dried in an oven, and finally at 200° in a vacuum. This material was originally intended for the preparation of boron tribromide and on this account was extracted with hydrobromic acid. It was actually used in preparing the first specimen of the trichloride.

For all the other work boron was made from a mixture of the trioxide with magnesium in the proportions of 1 to 1.5 by a process similar to that outlined above. The bulk of the magnesia resulting from the reaction together with a large part of the excess of magnesium was first extracted with acid and the residue washed. Then the boron was twice boiled for some time with concentrated acid with intermediate washing. After a final thorough washing with water the material was dried in the air at 110° to 120° for some time, and then at 200° in a high vacuum. Since in spite of the careful extraction the material was found to retain halide, for preparing the trichloride and tribromide separate specimens were made in exactly the same way, except that each was extracted with the appropriate acid.

The Preparation of Boron Trichloride.

Two specimens of boron trichloride were prepared. The first was made by heating boron suboxide in a current of dry chlorine. The suboxide was contained in a hard glass tube connected by ground glass joints at one end to the chlorine purifying train (see page 23), at the other to two receiving bulbs also connected by a ground glass joint. The hard glass tube was electrically heated to dull redness. The first receiving bulb was cooled with ice, the second with alcohol-carbon dioxide mixture. At first the chlorine was almost completely used up, but after several hours operation the gases produced showed by their color that they contained much uncombined chlorine. The process was therefore stopped at this point. After a second similar preparation had been made, the two portions of trichloride were combined in one bulb, and, after the addition of mercury, the bulb was exhausted by means of an efficient water pump through a capillary tube and sealed. Owing to the relatively high vapor pressure of boron trichloride the greater part of the air contained by the bulb was swept out by the tri-chloride vapor during the exhaustion. At the same time the low boiling impurities such as hydrogen chloride were partially eliminated.

At this point the trichloride was known or suspected to contain the following impurities: free chlorine, the products of hydrolysis owing to unavoidable slight exposure to moist air, i.e., boric acid and hydrogen chloride, boron tribromide from impurity in the boron which had been extracted with hydrobromic acid, and silicon tetrachloride. Wöhler and Deville [14] noted the contamination of boron trichloride with silicon tetrachloride in the case of material prepared in glass. As suggested by Stock and Priess,[15] who prepared the trichloride in a similar fashion, this difficulty may have resulted from impurity of silicon in the boron which was made in sand crucibles or from reaction of the boron and chlorine together on the hard glass tube. It was unlikely though possible that a small amount of carbon tetrachloride was present. Stock and Priess [15] and Stock and Kuss [16] found that the boron trihalides do not add halogen, and Stock, Kuss and Priess [17] found that boron hydrides are converted to trihalide by an excess of halogen so that higher halides and substituted hydrides were not to be feared.

The free chlorine was readily removed by shaking with mercury in the first bulb. The original yellow color of the liquid nearly disappeared and the liquid after distillation into a second bulb, also containing mercury, scarcely tarnished the latter substance. The remaining substances were eliminated by fractional distillation in vacuum. From the following table it can be seen that the boiling points of all the impurities are so far removed from that of boron trichloride that, unless constant boiling mixtures with boron trichloride exist, sufficiently complete separation could be hoped for.

	Bpt.
BCl_3	12.5°
BBr_3	90.5°
HCl	−83.1°
CCl_4	76.7°
$SiCl_4$	58.0°

The apparatus employed in the early stages of the fractionation is illustrated by Figure 1. The original bulb *A* containing the trichloride was provided at the outset with a special joint *B*, by means of which it could be connected with apparatus sealed to it at *C*. The system terminates in a second large bulb *F* also attached to a special joint *G*. The special joint is shown on a large scale in the figure. The sealed-in capillary *K* is closed at the end and is scratched with a file at several points to facilitate breaking. To break the joint the closed tube of glass weighted with mercury *L* is allowed to strike the capillary with some force. This device was apparently first used by Bruner and Bekier [18] and has recently been employed in several investigations.[19] After the system to the right of *B* has been exhausted

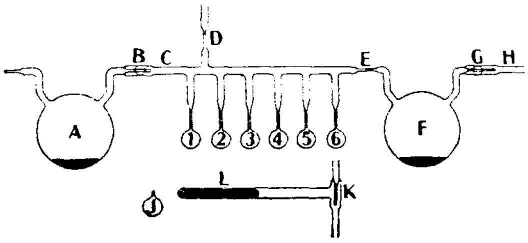

FIGURE 1.

with an efficient Gaede mercury pump and sealed at *D*, the joint at *B* is opened by breaking the capillary. The more volatile fractions of material are condensed in some of the small bulbs, 1, 2, 3, by means of liquid air or carbon dioxide-alcohol mixture and are sealed off as soon as filled. Then the main portion of material is distilled into *F*, which is sealed off at *E*, leaving the least volatile fraction in *A*. Subsequently, if desired, the residue in *A* can be distilled into 4, 5 and 6 for examination. A new system can now be sealed to *F* at *H* and the process repeated.

In this way the boron trichloride was distilled twenty times. In the early distillations several small, more volatile fractions were rejected as well as several small, less volatile fractions. Later, since the silicon tetrachloride was removed rather slowly, none of the more volatile material was rejected, and the less volatile residue was collected as a single fraction. After the twentieth distillation we found that in spite of the low boiling point of the trichloride, a Hempel column could be employed without difficulty. Figure 2 shows the arrangement of a system for this purpose. The trichloride is at room temperature in bulb *A*. The Hempel column *E* filled with large glass pearls is cooled by filling the jacket *F* with alcohol-carbon dioxide until refluxing is apparent in the column. A temperature of −15° to −20° was found satisfactory. By chilling the bulb 1 to a still lower temperature a light fraction may be condensed, and after the joint *L* has been opened the main distillation into *B* may be carried

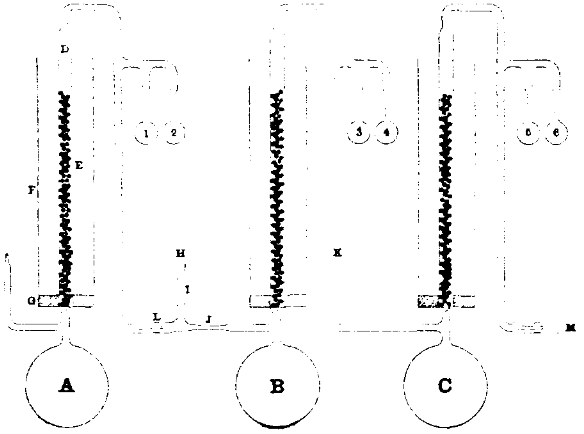

FIGURE 2.

out. The bulb *A* is then disconnected by sealing *J* and the residue in *A* is condensed in the small bulb 2. With suitable adjustment of temperatures in similar apparatus many substances have recently been successfully and very efficiently fractionated in this laboratory.

The complete fractionation of Sample I of boron trichloride is indicated in Figure 3, in which each large circle represents a distillation. More volatile fractions removed are shown as small circles to the right, less volatile fractions to the left. These fractions were usually condensed in small bulbs, but sometimes merely were left in the distilling flask. The special sealed-in joints are indicated as at *a*, Hempel columns as at *b*.

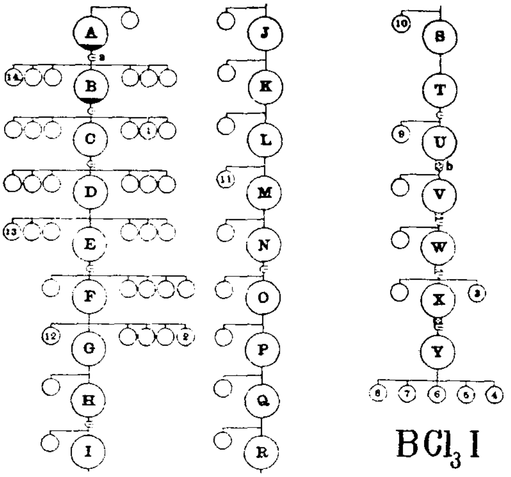

FIGURE 3.

The least volatile fraction from the second distillation, 14, was decomposed with water in a platinum dish and the silica content was found to be 0.07 gram. Fraction 13 in a similar way yielded 0.10 gram, but since this fraction was much larger than 14, the result really indicates a smaller proportion. Repeated fractionation slowly increased the purity of the trichloride so that Fraction 10, about 3 gm., yielded only 2.0 mg. of silica. At this point the use of a Hempel fractionating column was begun. Fraction 9, weighing approximately 2.5 gram was found to yield 0.6 mg. silica. This proportion would raise the apparent atomic weight of boron 0.007 unit. After three more distillations with Hempel columns, the remaining material was collected in five fractions for analysis (Fractions 4 to 8).

The second sample of boron trichloride, as stated before, was made from boron which had been completely reduced with magnesium and extracted with hydrochloric acid. Otherwise its preparation did not differ from that of the first sample except that it was secured in a single operation. The bulb *A* (Figure 4), containing the trichloride and mercury, was cooled and exhausted by means of an efficient water aspirator. After the trichloride had been allowed to boil gently for a short time the system was sealed. In order to remove the free chlorine by combination with the mercury the bulb was frequently shaken for a week. Since the yellow color still persisted, the material was distilled through a fractionating column into another bulb *B*, also containing mercury, with which it was left in contact for some time. Upon distillation into *C*, it was apparent that the chlorine had not been completely removed. Therefore a large part of the material was distilled back again into *B*, leaving a colorless liquid in *C*. The bulb *C* was then sealed off. After some time the material in both *B* and *C* was distilled into *D* also containing mercury. Prolonged agitation with mercury in *D* finally removed all trace of color.

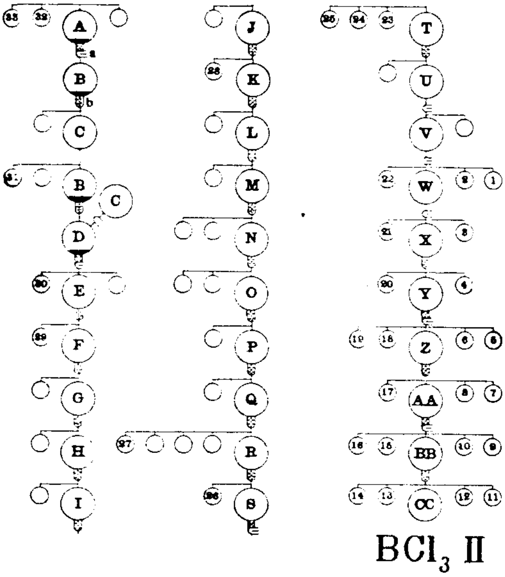

FIGURE 4.

In order to avoid the necessity of continually attaching and exhausting new distilling apparatus, a system was constructed in which the same fractionating columns could be repeatedly employed.[15] The construction of this apparatus is shown in Figure 5. After sealing to the bulb *A* containing the trichloride, the system was exhausted and sealed off at *N*. By chilling *B* with carbon dioxide-alcohol mixture and suitable cooling of the column *E* the trichloride, was distilled into *B* and bulb *A* was sealed off at *P*. With *B* at room temperature, 13 was next chilled and a small fraction was removed through the column *Q*. The greater portion of the remainder was distilled into *C*, and when the residue in *B* was small, connection between *B* and *C* was closed by forcing pure mercury from *G* into the U-tube *O*. This mercury valve is a modification of one used by Stock and Priess.[15] The residue in *B* was finally collected in the bulb 1 while *B* and *Q* were both surrounded with hot water. The process was then repeated, *C* serving as still and *R* as the fractionating column. No volatile fraction was rejected but the least volatile was collected in 14. In all, thirteen such distillations were made and the remaining material was collected in *D* (Figure 5).

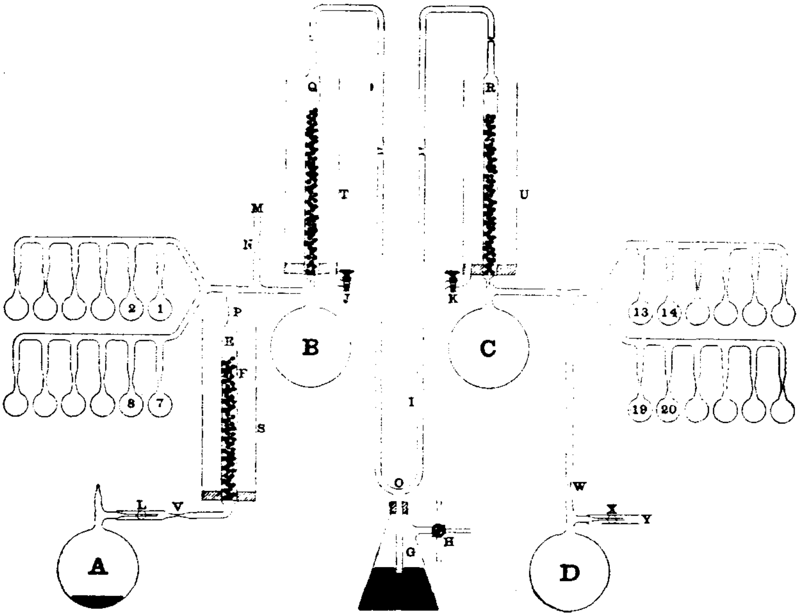

FIGURE 5.

In Figure 4 the fractionation with the above apparatus includes the steps *E* to *S*. From *S* the distillation was continued by the earlier method. Since a crack in the fractionating column of *S* was suspected to have admitted a small amount of air, after distilling from *U* to *V*, a portion was distilled back to *U* which was immediately sealed off. In this way permanent gases were effectually flushed out. After distillation into *W*, the material was set aside for nine months. No change in appearance could be detected during this time. Eventually the distillation was continued until all the material had been collected in fractions.

In Figure 4 the fractions ultimately examined are numbered in the order of decreasing volatility, i.e., Fraction 1 is the most volatile of these.

Fractions 30 and 29 were found to contain considerable silicon tetrachloride, but since the proportion in 28 was much smaller than in 29, no other direct tests for silicon were made.

THE PREPARATION OF BORON TRIBROMIDE.

The boron tribromide was synthesized from the elements by saturating an inert gas with pure bromine and passing the mixture over boron which had been extracted with hydrobromic acid (see page 24). Since nitrogen reacts readily with boron at elevated temperatures, the gas used as a carrier for the bromine was helium. The purification of this gas is described on page 24. After passing through the purifying train the helium was bubbled through bromine in a glass bulb connected by a ground joint with the hard glass reaction tube containing the boron. This tube, as in the preparation of the trichloride, was connected by a second ground joint with two condensing bulbs the second of which was cooled in an ice bath, since the comparatively high boiling point of the tribromide, 90.5°, prevents serious loss from evaporation at this temperature. The whole system was swept out with helium before bromine was introduced into the saturating bulb. Then when the reaction tube had been filled with helium saturated with bromine it was heated to redness. As high a concentration of bromine as possible was maintained by warming the saturating bulb and connection with the reaction tube. After twelve hours operation it was evident that considerable unchanged bromine was passing through the apparatus. The process was therefore discontinued.

Besides boric acid and hydrogen bromide formed by hydrolysis, silicon tetrabromide was certainly present, possibly carbon tetrabromide. The relative boiling points of the impurities lead to the expectation of comparatively rapid purification by fractional distillation. In fact Stock and Kuss [20] have already found that, although owing to the higher temperature at which the synthesis is effected, the tribromide is likely to be more highly contaminated with the corresponding silicon compound than the trichloride, the speed of removal by fractionation is distinctly greater.

	Bpt.
BBr_3	90.5°
HBr	−69.0°
CBr_4	190. °
$SiBr_4$	153. °

Purification by fractional distillation was then carried out as illustrated in Figure 6. After the material had been poured into bulb *A*, attached to a special joint, the bulb was exhausted as far as possible by an efficient water-pump and sealed. Distillation into *B*, containing mercury, freed the material from boric acid formed by hydrolysis during the unavoidable exposure to air during the preparation. Since even vigorous shaking with the mercury failed to remove all the excess of bromine, distillation into *C*, also containing mercury, followed. In *C* the color due to free bromine completely disappeared. From

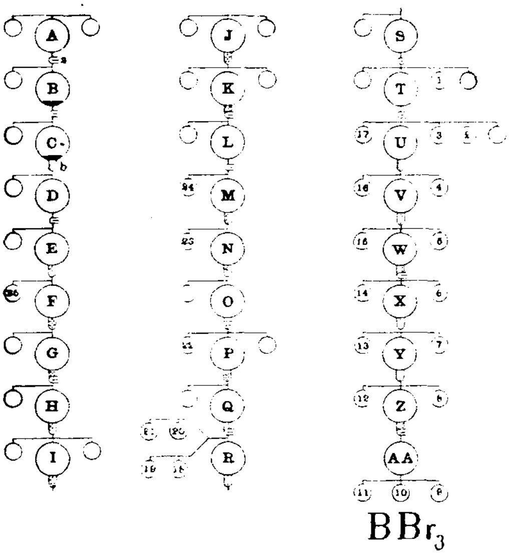

FIGURE 6.

this point on distillation with fractionating columns in apparatus somewhat similar to that shown in Figure 2 was continued for twenty-four distillations. Usually the still was warmed and the receiver cooled with ice. Under these conditions the fractionating column required no cooling beyond that of the air. In the distillation from *K* to *L* the receiver was cooled with carbon dioxide refrigerant. Although Fraction 25 contained considerable silicon tetrachloride, Fraction 23, consisting of about 7.5 grams was analyzed quantitatively for silicon, and none could be discovered. More and less volatile fractions were then collected for analysis, the more volatile fractions with the use of liquid air, until finally the tribromide was completely fractionated.

The Analysis of Boron Trichloride.

After being weighed the bulb containing boron trichloride was broken under an excess of ammonia and the glass was collected and weighed. The solution was diluted to considerable volume and made acid with nitric acid, and then was precipitated with a weighed very nearly equivalent amount of pure silver. The point of exact equivalence between chloride and silver was then found with the assistance of a nephelometer.

The bulb was soaked first in cleaning solution and then for at least a day in pure water. Generally the capillary was slightly scratched with a file to facilitate breaking. Next the bulb, suspended in a platinum wire basket, was weighed under water of known temperature, and the weight of the basket under water was determined at the same time. Sometimes these operations were carried out before the cleaning. After being dried with a clean nearly lintless cloth, the bulb was allowed to remain at least over night in a desiccator containing fused potassium hydroxide. Weighing by substitution followed, and the temperature, pressure and humidity of the balance case were observed at the same time. From the loss in weight under water the volume of the bulb was computed, and then from the atmospheric conditions the buoyant effect of the air on the bulb. In the later experiments the atmospheric density was found from the weight of a sealed, standardized globe.[21]

For breaking the bulb and hydrolyzing the trichloride a 2-liter heavy walled Jena or Pyrex flask, fitted with a carefully ground glass stopper, was employed. About one hundred and twenty-five cubic centimeters of pure water were placed in the flask and ammonia, generated by boiling a concentrated solution, was introduced in slight excess of the amount necessary to neutralize the acids formed in hydrolysis. After the bulb had been carefully lowered into the solution, the flask was stoppered and cooled with ice. With the walls of the flask wet with ammonia solution the bulb was broken by shaking the flask. No difficulty was experienced in this step in the process with the trichloride, but, as stated later, the tribromide is likely to react with explosive violence.

Although the fog produced in the decomposition disappeared within a half hour, the flask was never opened for two hours. Occasional shaking was employed to hasten the absorption of the fog. In every case, except for glass fragments, the resulting solution was perfectly clear.

The glass was now washed thoroughly by decantation and was finally collected on a filter paper, while the solution was received in a large glass-stoppered precipitating flask or bottle.

After the filter paper with the glass had been dried in a weighed platinum crucible, the paper was charred and the residue burned at the lowest possible temperature. Following a preliminary weighing the glass was fused in the crucible and the system reweighed. In nearly every case the change in weight on fusion was within the limit of error of weighing, and the average weight of glass was used in the final computations.

In order to determine the efficiency of the washing of the filter, empty glass bulbs which had been weighed open to the air were broken under a solution prepared by decomposing boron trichloride with ammonia, and the glass fragments were collected and weighed as above. Small differences between the original weight of the bulb and the final weight of the glass fragments were found as follows:

	gram
	−0.00007
	+0.00011
	+0.00025
	+0.00022
Average	+0.00013

This difference, though small, was traced to slightly faulty technique in washing, for two similar experiments where the method of washing was slightly improved, the corresponding difference was only 0.02 mg. in each case.

If the excess in weight in the earlier experiments is boric oxide, a corresponding amount of chloride must have been lost. These errors partially compensate so that with a five gram sample of material the observed atomic weight would be raised only 0.0004 unit. In the later analyses this difficulty was, of course, avoided.

From the weight of the bulb corrected for the buoyancy of the air on bulb and weights was subtracted the weight of glass, also corrected to vacuum, in order to find the correct weight of boron trichloride.

In all the analyses of Series I, after the ammoniacal solution of ammonium borate and chloride had been diluted to a volume of between one and one-half liters, it was made slightly acid by the addition of dilute nitric acid, and then, after precipitation, the acidity was increased by the addition of concentrated nitric acid to 0.1 normal concentration. In Series II and IV immediately before precipitation it was made still more strongly acid by the addition of 3 or 6 normal nitric acid through a funnel tube extending to the bottom of the solution, until in general the final solution was about normal in acidity. Since Hönigschmid [22] has found that even three normal nitric acid liberates no chloride from a similar quantity of potassium chloride the acidity was not dangerous, especially as the condition was temporary.

From the corrected weight of boron trichloride the weight of silver necessary to precipitate the chloride was computed. This quantity was weighed out, chiefly in the form of a very few large buttons, the final adjustment being made with small electrolytic crystals. After careful solution of the silver in chloride-free nitric acid, and elimination of the nitrous acid and oxides of nitrogen, in a flask provided with a spray trap in the form of a column of bulbs ground into the neck, the solution was diluted to less than tenth normal concentration and then was added slowly with constant agitation to the chloride solution. The mixture was allowed to stand for several days with occasional shaking, before testing for excess of chloride or silver in a nephelometer. If a deficiency of either was found, it was made up by adding hundredth normal solution until the end point had been reached. Even then the solutions were allowed to stand for some weeks longer with occasional shaking in order to allow included or occluded material to be extracted from the precipitate. Only slight changes were ever produced by thus standing.

In Analyses 12, 14 and 15 of Series II and in Analysis 13 of Series IV, fifty cubic centimeters additional of concentrated nitric acid were added to the silver nitrate solution before precipitation. Variations in the acidity of the solutions before and after precipitation produced no apparent effect.

The manipulation of precipitation and testing of the solutions were carried out in ruby light. In using the nephelometer all the precautions noted by Richards and Wells [23] were observed, such as preparing the comparison tubes under as nearly as possible identical conditions of temperature, concentration and time, allowing the tubes to come to constant ratio by standing for an hour or more, and taking the average of several readings of each ratio.

In several analyses the weight of the silver chloride was determined. The first three experiments were completed in the usual way. In order to lower the solubility of the precipitate 0.05 gram of silver nitrate per liter of solution was added, and after vigorous shaking the analysis was allowed to stand for twenty-four hours.

Filtration of the supernatant liquid through a weighed platinum sponge crucible was followed by thorough washing of the precipitate by decantation with a solution of silver nitrate also containing 0.05 gram per liter. Pure water was used for the final washing and transference of the precipitate to the crucible. The precipitate was dried at about 260° in an electrically heated porcelain oven for sixteen hours before being weighed. Moisture retained by the precipitate was determined by fusing the main bulk and finding the loss in weight on fusion. The solubility of the silver chloride in the mother liquor and silver nitrate washings was assumed to be 0.05 mg. per liter.[24] The silver chloride dissolved in the aqueous washings together with that obtained by rinsing the precipitation flask with ammonia was estimated by nephelometric comparison with standards. The concentration of silver chloride in the aqueous washings did not vary much from 1.2 mg. per liter in the different experiments.

Since in these first three experiments the weight of silver chloride was perceptibly less than that corresponding to the silver used in the comparison, the suspicion was raised that owing to the rather high concentration of ammonium salts and nitric acid in the mother liquor the solubility of the silver chloride might be larger than the assumed value. In the next analysis (Series III, Analysis 4) no silver nitrate was added to either mother liquor or washing liquid. These solutions were combined and evaporated to dryness. The residue was dissolved and filtered and the vessel in which the evaporation occurred was rinsed with ammonia and the rinsing was poured through the filter. A small quantity of potassium iodide was added to this solution and then an excess of nitric acid. The resulting silver iodide was collected on a small platinum sponge crucible, dried and weighed. The iodide thus obtained indicates a solubility of silver chloride in the mother liquor of 2.40 mg. per liter, a value nearly twice as large as that in pure water. In the next experiment (Series III, Analysis 5) the mother liquor alone was evaporated, and by a similar treatment was found to contain 2.37 mg. per liter. This result is in accord with earlier observations of the increase in solubility of silver chloride in increasing concentrations of nitric acid.[25]

In another analysis (Series III, Analysis 6) an excess of 0.5 gram of silver nitrate was added to the mother liquor, which was then cooled with ice. The solubility under these conditions was assumed to be negligible. Ice cold water was used in rinsing the precipitate, and the dissolved silver chloride was determined by nephelometric comparison.

In the seventh analysis (Series III), no silver nitrate was added either to mother liquor or to washing liquid, but the mother liquor and washing liquid were ice cold during filtration. After evaporation the dissolved silver was determined as silver iodide. In this experiment, after the silver chloride had been fused in a quartz crucible and weighed, it was again fused, in a current of dry chlorine. A gain of 0.29 mg. resulted. A second fusion in air then caused a loss in weight of 0.02 mg. and fusion in chlorine for over four hours resulted in a loss of 0.08 mg. more. The weight after the first fusion in chlorine is used in the calculation.

THE ANALYSIS OF BORON TRIBROMIDE.

The weight of boron tribromide was found in very nearly the same way as in the experiments with trichloride. Considerable difficulty was experienced in hydrolyzing the tribromide, owing to the extraordinary violence of the reaction. In four cases the pressure developed was high enough to burst the heavy walled reaction flask. This effect has already been noted by Abrahall[26] and by Gautier.[27] We were unable to determine the exact cause of the effect, nor did variations in the temperature, volume and concentration of the ammonia solution, nor substitution of pure water for ammonia (Series IV, Analyses 5 and 6) eliminate the difficulty. Furthermore the decomposition flasks were apparently weakened by use, for in several cases a flask which had been successfully used several times succumbed to the effect of a sample of tribromide smaller than some already hydrolyzed in the same flask. As soon as the danger was discovered, the breaking flask was surrounded with cloth and a stout wooden box during the decomposition, so that no untoward results occurred on those occasions when the flask burst.

The comparison of the solution of tribromide with silver was conducted as described in the case of the chloride. Freedom from chloride was indicated by the fact that at the end of the comparison with silver the mother liquor gave almost no opalescence with either silver or bromide solutions.

In Series IV, Analysis 8, the solutions were both diluted to over three liters before precipitation.

The tribromide used in Analyses 6, 7 and 13 had a slight yellow tint, possibly owing to decomposition when the capillary of the bulb was scaled with a blast flame.

In two experiments the silver bromide was collected and weighed. Owing to the lower solubility of silver bromide, the process offers fewer difficulties than that of determining silver chloride. The precipitate was washed only with pure water in the first experiment and a correction of 0.1 mg. per liter was applied to correct for solubility.[28] In the second experiment 0.02 gram of silver nitrate was added for each liter of mother liquor and after shaking and standing the latter was poured through a weighed platinum sponge crucible. Washing with a solution containing 0.01 gram silver nitrate per liter was followed by rinsing and transferring to the crucible with chilled water. No correction for solubility was applied in this experiment. Moisture retained by the bromide was determined as described in the case of the silver chloride.

Weighings were made on a No. 10 Troemner balance, sensitive to 0.02 mg. with a load of fifty grams. The beam was graduated to 0.05 milligrams, and the 5 milligram rider was used to determine all quantities larger than this, interpolation from the zero points being employed for amounts less than 0.05 milligram.

The weights were of gold plated brass, except the fractional weights, and were compared by the Richards substitution method.[29]

All weighings were by substitution. In the case of the bulb and the silver, the weights were substituted for the object weighed. In the case of the glass and the silver halides, the crucibles were substituted for similar counterpoises. A small quantity of impure radium bromide was kept in the balance case to prevent electrostatic effects.

Vacuum corrections were applied as follows:

TABLE 1.

	Density	Vacuum Correction per Gram
Weights	8.3	
Air	0.001293 at 0° and 760 mm.	
Glass	2.5	+0.000335
Ag	10.49	−0.000031
AgCl	5.56	+0.000071
AgBr	6.47	+0.000041

Humidity was read on an accurate hair hygrometer.

RESULTS.

TABLE 2.

$BCl_3 : 3\ Ag$

SERIES I. SAMPLE I.

Ag = 107.880 Cl = 35.458

Number of Analysis	Fraction of BCl_3	Weight of BCl_3 in vacuum	Weight of Ag in vacuum	Weight of Ag added or subtracted in solution	Corrected Weight of Ag in Vacuum	Ratio $BCl_3 : 3\ Ag$	Atomic Weight of Boron
		gm.	gm.	gm.	gm.		
1	6	4.84825	13.39534	−0.00474	13.39060	0.363063	10.804
2	4	3.80944	10.52174	−0.00225	10.51949	0.362132	10.826
3	8	5.67738	15.67705	+0.00368	15.68073	0.362060	10.803
4	5	5.28752	14.60628	−0.00250	14.60378	0.362065	10.805
5	7	2.86895	7.92386	−0.00021	7.92365	0.362074	10.808
					Average	0.362079	10.809
			SERIES II.	SAMPLE II.			
1	25	6.63574	18.32700	−0.00315	18.32385	0.362136	10.828
2	1	6.36923	17.58436	+0.00030	17.58466	0.362204	10.850
3	24	6.37226	17.59566	+0.00100	17.59666	0.362129	10.825
4	2	5.87852	16.23532	−0.00403	16.23129	0.362172	10.839
5	3	3.73634	10.31892	−0.00094	10.31798	0.362119	10.822
6	21	3.95311	10.91956	−0.00205	10.91751	0.362089	10.812
7	4	3.53556	9.76477	−0.00241	9.76236	0.362162	10.836
8	5	3.55078	9.80597	−0.00041	9.80556	0.362119	10.822
9	19	3.45120	9.53065	+0.00123	9.53188	0.362069	10.806
10	7	3.59682	9.93334	+0.00005	9.93339	0.362094	10.814
11	9	4.33672	11.97673	+0.00005	11.97678	0.362094	10.814
12	16	3.07091	8.48141	−0.00010	8.48131	0.362080	10.810
13	11	3.70594	10.23471	+0.00076	10.23547	0.362068	10.806
14	14	3.17865	8.77846	+0.00016	8.77862	0.362090	10.813
15	6	4.35457	12.02684	−0.00055	12.02629	0.362088	10.912
16	18	4.10982	11.34969	+0.00060	11.35029	0.362089	10.812
17	12	3.62520	10.01112	+0.00060	10.01172	0.362096	10.815
					Average	0.362112	10.820

TABLE 3.

SERIES III.

$BCl_3 : 3 AgCl$

Number of Analysis	Fraction of BCl_3	Weight of BCl_3 in vacuum	Weight of AgCl in vacuum	Loss on Fusion	Dissolved AgCl	Corrected Weight of AgCl in vacuum	Ratio $BCl_3 : 3 AgCl$	Atomic Weight of Boron
		gm.	gm.	gm.	gm.	gm.		
1	21	3.95311	14.50419	0.00023	0.00156	14.50552	0.272525	10.816
2	5	3.55078	13.02480	0.00027	0.00168	13.02621	0.272587	10.842
3	19	3.45120	12.66314	0.00020	0.00186	12.66480	0.272503	10.806
4	16	3.07091	11.26100	0.00027	0.00746	11.26819	0.272529	10.817
5	14	3.17865	11.65561	0.00020	0.00741	11.66282	0.272546	10.825
6	7	3.59682	13.19721	0.00022	0.00090	13.19789	0.272530	10.818
7	4	3.53556	12.96603	0.00020	0.00462	12.97045	0.272586	10.842
						Average	0.272544	10.824

TABLE 4.
SERIES IV.
$BBr_3 : 3 Ag$

Ag = 107.880 Br = 79.916

Number of Analysis	Fraction of BBr_3	Weight of BBr_3 in vacuum	Weight of Ag in vacuum	Weight of Ag added or subtracted in solution	Corrected Weight of Ag in vacuum	Ratio $BBr_3 : 3 Ag$	Atomic Weight of Boron
		gm.	gm.	gm.	gm.		
1	24	11.94682	15.42985	−0.00254	15.42731	(0.774394)	(10.877)
2	22	16.70949	21.58073	+0.00200	21.58273	0.774207	10.816
3	18	10.62202	13.71837	+0.00120	13.71957	0.774224	10.822
4	20	5.72663	7.39441	+0.00230	7.39671	0.774213	10.818
5	4	8.72710	11.27176	−0.00003	11.27173	0.774247	10.829
6	3	8.01796	10.35691	−0.00178	10.35513	0.774298	10.846
7	5	7.99444	10.32534	+0.00049	10.32583	0.774218	10.820
8	6	7.05884	9.11711	−0.00049	9.11662	0.774283	10.841
9	15	10.56234	13.64286	−0.00015	13.64271	0.774211	10.818
10	14	7.92601	10.23772	−0.00046	10.23726	0.774232	10.824
11	13	9.11150	11.76854	+0.00079	11.76933	0.774173	10.805
12	8	10.45731	13.50635	+0.00020	13.50655	0.774240	10.827
13	9	9.20250	11.88626	−0.00067	11.88559	0.774257	10.833
14	10	8.85374	11.43584	+0.00071	11.43655	0.774162	10.802
15	11	4.75503	6.14174	+0.00037	6.14211	0.774169	10.804
				Average, omitting Analysis 1		0.774224	10.822

TABLE 5.
SERIES V.
$BBr_3 : 3 AgBr$

Number of Analysis	Fraction of BBr_3	Weight of BBr_3 in vacuum	Weight of AgBr in vacuum	Loss on Fusion	Dissolved AgBr	Corrected Weight of AgBr in vacuum	Ratio $BBr_3 : 3 AgBr$	Atomic Weight of Boron
		gm.	gm.	gm.	gm.	gm.		
1	10	8.85374	19.90872	0.00028	0.00039	19.90883	0.444714	10.799
2	8	10.45731	23.51239	0.00020	0.00000	23.51219	0.444761	10.825
						Average	0.444738	10.812

DISCUSSION OF RESULTS.

It has already been pointed out that the first sample of boron trichloride was made from boron suboxide which had been extracted with hydrobromic acid, and therefore may have contained a trace of tribromide, in spite of the fact that the material at first contained an excess of chlorine. The difference in the boiling points of the trichloride and tribromide is so large, however, that it seems reasonable to expect that if the tribromide was actually present as impurity in the trichloride it was eliminated in the fractional distillation together with the silicon halides. Furthermore, the effect of tribromide as impurity would be to raise the apparent atomic weight of boron, whereas the experiments of Series I give a result somewhat lower than those of the later series. In Series I also the uncertainty exists as to the washing of the filter paper in the determination of the glass, but as already pointed out on page 34 this uncertainty is scarcely large enough to affect the third decimal place in the atomic weight of boron. The earlier analyses of Series II and III are subject to the latter uncertainty, but there seems to be insufficient reason to reject analyses on this account.

The question of the ultimate elimination of silicon halides has been answered in part by direct evidence both of the progressive separation by the fractional distillation and of the absence of appreciable proportions of silicon compounds in fractions less pure than those analyzed.

The only evidence as to the complete elimination of hydrogen halides must necessarily come from the results of the analyses. In Table 6 the analyses are arranged in the order of decreasing volatility of the fractions. No systematic trend is apparent except possibly a tendency toward high results with the most volatile fractions. Since hydrogen chloride would lower, not raise the apparent atomic weight of boron the tendency can not be ascribed to this impurity. Nor can it be ascribed to silicon halides since these would be contained in the less volatile fractions. On the whole the uniformity of the material over such a wide range of fractions is satisfactory.

TABLE 6.

Series I.

Fraction	Atomic Weight
4	10.826
5	10.805
6	10.804
7	10.808
8	10.803

Series II and III.

Fraction	Atomic Weight	
1	10.850	
2	10.839	
3	10.822	
4	10.836	10.842
5	10.822	10.842
6	10.812	
7	10.814	10.818
9	10.814	
11	10.806	
12	10.815	
14	10.813	10.825
16	10.810	10.817
18	10.812	
19	10.806	10.806
21	10.812	10.816
24	10.825	
25	10.828	

Series IV and V.

Fraction	Atomic Weight	
3	10.846	
4	10.829	
5	10.820	
6	10.841	
8	10.827	10.825
9	10.833	
10	10.802	10.799
11	10.804	
13	10.805	
14	10.824	
15	10.818	
18	10.822	
20	10.818	
22	10.816	

The most convincing evidence that both halides were ultimately pure and that no serious analytical error exists is the concordance between the results of the chloride and bromide analyses. It is difficult to believe that two substances differing so widely in physical properties both contained after fractionation proportions of impurity which produced identical effects.

Further comparison of the results may be made by grouping the experiments according to the specimen of pure silver used. This has been done in Table 7. Here again the differences are no larger than the experimental error.

TABLE 7.

Sample of Silver	Analyses	Average Atomic Weight of Boron
Scott	Series II, Analyses 5–12 Series IV, Analyses 7–14	10.818
Tani	Series I, Analyses 1–5 Series II, Analyses 1–4	10.822
Weatherill	Series IV, Analyses 1–6	10.824
Dorcas and Hilton	Series II, Analyses 13–17	10.813

The mean of the averages of Series I to V is 10.817. If each average is weighted according to the number of analyses, the mean is 10.820. This latter value is almost identical with the average of Series II and IV, the most reliable of the four. It is interesting to note that Abrahall [26] in his final series of analyses of boron tribromide by comparison with silver obtained a result identical with that of our analyses of the same substance. Furthermore Hönigschmid and Birckenbach believe that the value 10.82 best represents the outcome of their work. It is especially reassuring that material prepared and analyzed under such widely differing conditions should yield so concordant results as theirs and ours.

Direct confirmation of the existence of isotopes of boron was obtained by Aston [31] by the use of the mass spectrograph. These isotopes were found to have the masses 10 and 11, the latter being more abundant. Aston states, "The relative photographic intensity of the lines 5 and 5.5[32] does not agree well with an atomic weight as high as 10.9, ..." Aston later [33] estimates the mean atomic weight from the relative intensities of the mass spectrum lines to be 10.75 ± 0.07. The value 10.82 is in accord with this estimate and corresponds to relative proportions of 1 to 4.5 of the isotopes 10 and 11 respectively.

With two isotopes of boron, chlorine and bromine, there are evidently possible eight different trichlorides of boron as well as eight different tribromides, with molecular weights ranging from 115, at intervals of one unit, to 122 in the case of the chloride, and from 247 to 254 in the case of the bromide. Of these, the chloride $B^{11}(Cl^{35})_3$, 116, and $B^{11}(Br^{79})_3$, 248, are presumably the most abundant, and $B^{10}(Cl^{37})_3$, 121, and $B^{10}(Br^{81})_3$ the least abundant. The question therefore may be raised as to whether partial separation may have occurred during the fractionation of the two liquids.

As pointed out by Baxter, Weatherill and Scripture [9] in the case of the silicon halides, separation, if it occurs at all, should continue throughout the fractionation. Since no systematic difference in the composition of fractions covering a large proportion of the material is apparent, it is reasonable to conclude that no appreciable separation of isotopic substances was taking place. Furthermore perceptible separation is to be expected only during distillation at extremely low pressures, as has already been pointed out several times by others, whereas in the distillation of the boron trihalides this was far from being the case.

Experimental evidence that isotopic separation was inappreciable is furnished by the ratio between the silver used in a comparison with the boron halide and the silver halide obtained in the second step in the same analysis, for separation of the halogen isotopes is as probable as the separation of boron isotopes. The following tables give the values of the silver-halide ratios.

TABLE 8.

Series II and III.

Fraction of BCl_3	Ratio Ag : AgCl
4	0.752662
5	0.752756
7	0.752650
14	0.752701
16	0.752677
19	0.752628
21	0.752645
Average	0.752674
Richard and Wells found[34]	0.752627

Series IV and V.

Fraction of BBr_3	Ratio Ag : AgBr
10	0.574449
8	0.574446
Average	0.574448
Baxter[35] found	0.574453

The differences between the values of the chloride ratio and that found by Richards and Wells is not too great to explain by the difficulty experienced from the increased solubility of silver chloride in the mother liquors of an analysis. Certainly there is no systematic trend with continued fractionation. The bromide ratio is essentially identical with that found directly by Baxter.

In order to test the matter further several syntheses of silver chloride were made from hydrochloric acid obtained from fractionated silicon and boron halides. The silver chloride produced in the analyses of the above halides was reduced with clean zinc in slightly acid solution. After adding an excess of sulfuric acid to the resulting solution of zinc halide, the hydrochloric acid was distilled and collected and was further purified by two distillations through a quartz condenser. Very small extreme fractions were rejected. The silicon tetrachloride material was obtained in the analyses of Series IV of the work of Baxter, Weatherill and Scripture.[36] The boron trichloride material consisted of the silver chloride obtained in the analyses of Series III of this paper plus that of two analyses in which the silver chloride was not weighed. One analysis (Analysis 3) was made with ordinary redistilled hydrochloric acid.

Weighed portions of pure silver were dissolved as previously de-

TABLE 9.

Ag : AgCl

No. of Synthesis	Sample of Ag	Source of HCl	Weight of Ag in vacuum	Weight of AgCl in vacuum	Loss on Fusion	Change in Cl_2	Weight of Asbestos	Dissolved AgCl*	Corrected Weight of AgCl	Ratio Ag : AgCl
			gm.	gm.	gm.	gm.	gm.	gm.		
1	Weatherill	$SiCl_4$	9.07788	12.06067	0.00020	−0.00001	0.00006	0.00082	12.06134	0.752643
2	Weatherill	$SiCl_4$	8.74097	11.61309	0.00010	−0.00008	0.00005	0.00046	11.61342	0.752661
3		ordinary	7.89493	10.48919	0.00025	+0.00012	lost	0.00027	10.48933	0.752663
4	Dorcas	BCl_3	9.83087	13.06167	0.00031	−0.00002		0.00050	13.06184	0.752641
5	Dorcas	BCl_3	10.32779	13.72293	0.00068	+0.00004		0.00035	13.72264	0.752610

Average of Syntheses 1 and 2........................0.752652
Average of Syntheses 4 and 5........................0.752626

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* Determined as AgI.

scribed and after dilution were precipitated with a slight excess of the hydrochloric acid solutions in question. The precipitate, after coagulation and standing for several days, was collected on a Gooch crucible with an asbestos mat in the first three analyses of Table 9. The precipitate was dried as already described (page 36) and, after being weighed, was fused in a quartz crucible and the loss due to moisture was found. A second fusion in dry chlorine followed, in order to convert occluded silver nitrate into chloride. Changes in weight in the latter process were inappreciable. Asbestos shreds in the filtrate were collected on a filter paper and determined. In the last two analyses a platinum sponge crucible was employed, after it had been shown that a solution similar to the filtrate obtained in the analyses when passed through the crucible did not alter the weight of the crucible. The filtrates and washings of each experiment were evaporated separately and the dissolved silver salt was determined as silver iodide, in the fashion previously described (page 36).

The differences between the results of these syntheses and the value to be expected for the silver-silver chloride ratio, 0.752627, is not large enough to indicate any real difference between the chlorine of the distilled halides and normal chlorine.

SUMMARY.

The results of this investigation may be summarized as follows:

1. Improved methods for the fractional distillation in vacuum of boron halides are described.
2. The analysis of pure boron trichloride and tribromide has yielded the value 10.820 for the atomic weight of boron (Ag = 107.880).

We are very greatly indebted to the Wolcott Gibbs and Bache Funds of the National Academy of Sciences for generous assistance in providing the necessary apparatus and materials, and to the United States Navy Department for furnishing helium.

CAMBRIDGE 38, MASSACHUSETTS.

Notes.

[1] The earlier determinations are discussed by F. W. Clarke in Memoirs of the Nat. Acad. Sci., Vol. XVI, No. 3. The Constants of Nature. Part V. A Recalculation of the Atomic Weights. 4th Ed., p. 205 (1920); by Brauner in Abegg's Handb. d. anorg. Chem., Vol. III, Pt. 1, 5, (1906); and by Smith and Van Haagen, Reference 2.

[2] Smith and Van Haagen, Carnegie Inst. Pub., No. 267 (1918).

[3] For a resumé of the subject see Abegg, Handb. anorg. Chem., III, 1, 31, (1906).

[4] Baxter and Scott, Science, N. S., **54**, 524 (1921).

[5] Hönigschmid and Birckenbach, Anales Soc. Españ. Fís. Quím., **20**, 167 (1922).

[6] Stock and Kuss, Ber. d. d. chem. Gesell., **56B**, 314 (1923).

[7] Baxter, Moore and Boylston, Proc. Am. Acad., **47**, 585 (1912); Jour. Amer. Chem. Soc., **34**, 1644 (1912); Baxter and Grover, *Ibid.*, **37**, 1029 (1915).

[8] Baxter and Parsons, Jour. Amer. Chem. Soc., **44**, 577 (1922).

[9] Baxter, *Ibid.*, **44**, 591 (1922).

[10] Baxter, Weatherill and Scripture, These Proceedings, **58**, 247 (1923).

[11] Baxter, Tani and Chapin, Jour. Amer. Chem. Soc., **43**, 1080 (1921).

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[13] Moissan, Ann. chim. phys., (7) **6**, 296 (1895).

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[15] Stock and Priess, Ber. d. d. chem. Gesell., **47**, 3109 (1914).

[16] Stock and Kuss, ibid., **47**, 3113 (1914).

[17] Stock, Kuss and Priess, *ibid.*, **47**, 3115 (1914).

[18] Bruner and Bekier, Z. Elektrochem., **18**, 369 (1912).

[19] *See especially Reference 10, p. 250.*

[20] Reference 16, page 3115.

[21] Baxter, Jour. Amer. Chem. Soc., **43**, 1317 (1921).

[22] Hönigschmid, Z. Elektrochem., **26**, 403 (1920); Hönigschmid and Birckenbach Ber. d. d. chem. Gesell., **54 B**, 1873 (1921).

[23] Richards and Wells, Amer. Chem. Jour., **31**, 235 (1904); Richards, **35**, 510 (1906).

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[25] Glówczyński, Koll. chem. Beihefte, **6**, 147 (1914).

[26] Abrahall, Jour. Chem. Soc., **61**, 650 (1892).

[27] Gautier, Ann. chim. phys., (7) **18**, 352 (1899).

[28] Böttger, Z. physik. Chem., **46**, 602 (1903), 0.08 mg. per liter at 20°. Kohlrausch, Ibid., **50**, 356 (1905), 0.1 mg. per liter at 21°.

[29] Richards, Jour. Amer. Chem. Soc., **22**, 144 (1900).

[31] Aston, Phil. Mag. (6) **40**, 628 (1920).

[32] Second order.

[33] Aston, "Isotopes," 1922, p. 114.

[34] Richards and Wells, Carnegie Inst. Pub. No. 28, 1905.

[35] Baxter, These Proceedings, **42**, 201 (1906).

[36] See Reference 10, page 264.

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TRAJECTORY SURFACES AND A GENERALIZATION OF THE PRINCIPAL DIRECTIONS IN ANY SPACE.

By Joseph Lipka.

TRAJECTORY SURFACES AND A GENERALIZATION OF THE PRINCIPAL DIRECTIONS IN ANY SPACE.

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1. **Introduction.** The differential geometry of a surface (spread of two dimensions) owes much of its development upon the notion of the curvature of the surface introduced by Gauss; similarly, the differential geometry of a spread of n dimensions bases much of its subject matter on the notion of the curvature of such a spread as introduced by Riemann. This important concept of the curvature of a space according to Riemann may be summarized as follows: In a space V_n defined by the totality of points given by the values of n real independent variables $x_1, x_2, \ldots, x_n$, we have for the element of length or first fundamental form, the quadratic differential form

$$ds^2 = \sum_{ik} a_{ik}\, dx_i\, dx_k.$$

Consider, now, any two directions issuing from a point P of this space; these determine a plane pencil of directions or an *orientation* V_2 in V_n, and the geodesics of V_n passing through P in these directions determine a spread of two dimensions, called a *geodesic surface.* The Gaussian curvature of this geodesic surface is defined as the Riemannian curvature of the space V_n *relative to the above orientation* V_2. Thus, in general, the Riemannian curvature of a space V_n varies with the assigned orientation through a point P.

Now in defining the above concept, we have used the geodesics of the space, defined as usual as the curves which render

$$\delta \int ds = 0,$$

(where δ is the ordinary symbol of variation). We may generalize the concept of the geodesics of V_n by defining a system of curves in V_n which render

$$\delta \int F(x_1, x_2, \ldots, x_n)\, ds = \delta \int e^{\phi(x_1, x_2, \ldots, x_n)}\, ds = 0,$$

where, as indicated, F or ϕ is an arbitrary function of the coördinates only, (e being the Napierian base of logarithms). We shall call these curves the *trajectories* of the space.[1] A point and a direction of V_n determine uniquely one *trajectory*. As above, *any two directions through a point P determine a plane pencil of directions or an orientation* V_2 *in* V_n, *and the trajectories of* V_n *passing through P in these directions determine a surface or spread of two dimensions, which we shall call a trajectory surface.* The geodesic surfaces are the trajectory surfaces for which F or ϕ reduces to a constant. The function ϕ determines uniquely a vector at each point of space, which we shall call the *trajectory vector*.

We shall first study the Gaussian curvatures of the trajectory surfaces through a point P, and the variation of these curvatures with the variation of the assigned orientation through P(§2). Defining as *corresponding surfaces*, the geodesic and the trajectory surfaces determined by the same orientation through a point P in V_n, we find that for two corresponding surfaces to have the same curvature, it is necessary and sufficient that the orientation contain the direction of the trajectory vector (§3). The curvature of a trajectory surface enters naturally into a study of the motion of a direction around a closed infinitesimal cycle by *conformal parallelism*, and thus leads to a geometric interpretation of certain invariants or differential parameters of an arbitrary function ϕ of the coördinates (§4).

A second part of the paper is concerned with a generalization of some of the work of Ricci on the *principal directions* of a Riemannian space. The generalization consists in replacing geodesics and geodesic surfaces by trajectories and trajectory surfaces. We begin by considering a space of *three* dimensions, and we find that through each point of such a space there are, in general, three mutual orthogonal directions in which the curvature of the trajectory surfaces normal to these directions, is a maximum or a minimum. We thus determine the *principal trajectory curvatures* of the space in terms of which the curvature of any other trajectory surface is expressed (§5).

Proceeding to a space of n dimensions, we define the *median trajectory curvature*, and we find that at each point of the space there are, in general, n mutually orthogonal directions in which the median trajectory curvature is a maximum or a minimum. These directions determine *n principal trajectory congruences and n principal median trajectory curvatures* or *invariants* in V_n (§6). These directions and invariants are then characterized in terms of the Ricci coefficients of rotation and the function ϕ (§7). Returning to the case where $n = 3$, we show that here the directions of principal trajectory curvature and the directions of principal median trajectory curvature coincide, and we find the equations which express the invariants of one type in terms of the invariants of the other type (§8). Finally our results are shown to be valid if we replace the trajectories by the more general system of curves called *velocity curves* (§9).

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[1] These curves are sometimes called "*a natural family of curves*." We have chosen the brief description "*trajectories*," since any such system of curves may be considered as the dynamical trajectories by the principle of least action in a conservative field of force and for a given constant of energy. The geodesics of V_n are the trajectories under a constant force.

It is the purpose of the author to continue the developments here begun in papers which are to follow.

2. **The Curvature of a Trajectory Surface.** If the first fundamental form of V_n is given by[2]

$$ds^2 = \sum_{ik} a_{ik}\, dx_i\, dx_k, \tag{1}$$

and the trajectories are defined by

$$\delta \int e^{\phi} ds = 0, \tag{2}$$

where ϕ is a function of the coördinates only, the differential equations of the trajectories, found by an application of the ordinary methods of the calculus of variation, are[3]

$$\frac{d\xi^{(i)}}{ds} + \sum_{\lambda\mu} \left\{ \begin{matrix} \lambda\mu \\ i \end{matrix} \right\} \xi^{(\lambda)} \xi^{(\mu)} = \phi^{(i)} - \xi^{(i)} \sum_k \phi_k \xi^{(k)}, \qquad (i = 1, 2, \ldots, n); \tag{3}$$

where the $\xi^{(i)} = \dfrac{dx_i}{ds}$ are the *parameters* of any direction, i.e.

$$\sum_{ik} a_{ik}\, \xi^{(i)} \xi^{(k)} = 1, \tag{4}$$

and where the $\phi_i = \dfrac{\partial \phi}{\partial x_i}$ form a covariant system determining the direction of the trajectory vector ϕ, and the $\phi^{(i)}$ are the corresponding contravariant system; these systems are related by

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[2] All the summations are to extend from 1 to n for the indicated subscripts unless otherwise specified.

[3] See the author's paper, *Some geometric investigations on the general problem of dynamics*, Proc. Am. Acad. of Arts and Sciences, Vol. 55 (1920), pp. 283-322.

$$(5) \qquad \phi^{(i)} = \sum_k a^{(ik)} \phi_k; \qquad \phi_i = \sum_k a_{ik} \phi^{(k)},$$

the $a^{(ik)}$ being the coefficients of the fundamental form reciprocal to (1). Further, the $\left\{ {\lambda\mu \atop i} \right\}$ are the Christoffel symbols of the second kind and between these and the $\left[{\lambda\mu \atop i} \right]$, the symbols of the first kind, we have the relations

$$(6) \qquad \left\{ {\lambda\mu \atop i} \right\} = \sum_k a^{(ik)} \left[{\lambda\mu \atop k} \right]; \qquad \left[{\lambda\mu \atop i} \right] = \sum_k a_{ik} \left\{ {\lambda\mu \atop k} \right\}.$$

Consider a point P_0 with coördinates x_i^0 and two directions with parameters $\xi_1^{(i)}$ and $\xi_2^{(i)}$ issuing from P_0. These determine a plane pencil of directions or orientation V_2, and the trajectories of V_n passing out from P_0 in these directions form a trajectory surface σ with pole P_0. Any direction in the pencil has parameters

$$(7) \qquad \zeta^{(i)} = \alpha\, \xi_1^{(i)} + \beta\, \xi_2^{(i)}.$$

Following the work of Bianchi for geodesics,[4] let us develop the coördinates x_i of any point of the trajectory (3) in a direction $\zeta^{(i)}$ in powers of the arc s measured from P_0, i.e.

$$x_i = x_i^0 + \zeta^{(i)} s + \tfrac{1}{2} \frac{d\zeta^{(i)}}{ds} s^2 + \cdots\cdot,$$

so that along a trajectory,

$$x_i = x_i^0 + \zeta^{(i)} s + \tfrac{1}{2} \Bigg[- \sum_{\lambda\mu} \left\{ {\lambda\mu \atop i} \right\} \zeta^{(\lambda)} \zeta^{(\mu)} + \phi^{(i)} \sum_{\lambda\mu} a_{\lambda\mu} \zeta^{(\lambda)} \zeta^{(\mu)}$$
$$- \zeta^{(i)} \sum_{\lambda} \phi_\lambda \zeta^{(\lambda)} \Bigg] s^2 + \cdots\cdot.$$

If we substitute here the value of $\zeta^{(i)}$ from (7) and choose $u_1 = s\alpha$, $u_2 = s\beta$ as variables on the trajectory surface σ, we get

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[4] Bianchi, *Geometria differenziale*, 2d edition, vol. 1, p. 339.

$$(8)\qquad x_i = x_i^0 + \xi_1^{(i)} u_1 + \xi_2^{(i)} u_2 - \tfrac{1}{2} \sum_{\lambda\mu} \left\{ \begin{matrix} \lambda\mu \\ i \end{matrix} \right\} (\xi_1^{(\lambda)} u_1 + \xi_2^{(\lambda)} u_2)\, (\xi_1^{(\mu)} u_1$$
$$+ \xi_2^{(\mu)} u_2) + \tfrac{1}{2} \phi^{(i)} \sum_{\lambda\mu} a_{\lambda\mu} (\xi_1^{(\lambda)} u_1 + \xi_2^{(\lambda)} u_2)\, (\xi_1^{(\mu)} u_1 + \xi_2^{(\mu)} u_2)$$
$$- \tfrac{1}{2} \sum_{\lambda} \phi_\lambda (\xi_1^{(i)} u_1 + \xi_2^{(i)} u_2)\, (\xi_1^{(\lambda)} u_1 + \xi_2^{(\lambda)} u_2) + \cdots\cdot.$$

From these we immediately have the values of the derivatives at P_0,

$$(9)\qquad \left(\frac{\partial x_i}{\partial u_k}\right)^0 = \xi_k^{(i)};\ \left(\frac{\partial^2 x_i}{\partial u_h \partial u_k}\right)^0 = - \sum_{\lambda\mu} \left\{ \begin{matrix} \lambda\mu \\ i \end{matrix} \right\} \xi_h^{(\lambda)} \xi_k^{(\mu)} + \psi_i (h, k),$$

where h and k have the values 1 and 2, and where we have employed the abbreviation

$$(9')\qquad \psi_i (h, k) = \phi^{(i)} \sum_{\lambda\mu} a_{\lambda\mu} \xi_h^{(\lambda)} \xi_k^{(\mu)} - \tfrac{1}{2} \sum_{\lambda} \phi_\lambda (\xi_h^{(i)} \xi_k^{(\lambda)} + \xi_k^{(i)} \xi_h^{(\lambda)}).$$

Let the fundamental form for the trajectory surface σ be given by

$$(10)\qquad ds^2 = b_{11}\, du_1^2 + 2\, b_{12}\, du_1\, du_2 + b_{22}\, du_2^2,$$

where

$$(10')\qquad b_{ik} = \sum_{\lambda\mu} a_{\lambda\mu} \frac{\partial x_\lambda}{\partial u_i} \frac{\partial x_\mu}{\partial u_k} \qquad (i, k = 1, 2).$$

Now the Gaussian curvature of the surface σ at P_0 or of the binary form (10) is given by

$$(11)\qquad \mathrm{T} = \frac{b_{12.12}^{\ 0}}{(b_{11} b_{22} - b_{12}^2)^0},$$

where the Riemannian four-indices symbol $b_{12.12}$ is defined by [5]

$$(12)\qquad b_{12,12} = \frac{\partial}{\partial u_2} \begin{bmatrix} 11 \\ 2 \end{bmatrix}_b - \frac{\partial}{\partial u_1} \begin{bmatrix} 12 \\ 2 \end{bmatrix}_b$$
$$+ \sum_{lm}^{1,2} b^{(lm)} \left(\begin{bmatrix} 12 \\ m \end{bmatrix}_b \begin{bmatrix} 12 \\ l \end{bmatrix}_b - \begin{bmatrix} 11 \\ m \end{bmatrix}_b \begin{bmatrix} 22 \\ l \end{bmatrix}_b \right).$$

We shall first compute the value of this symbol at P_0.

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[5] Bianchi, ibid., p. 73.

We have[6]

$$(13)\qquad \begin{bmatrix} ik \\ l \end{bmatrix}_b = \sum_{\lambda\mu\nu} \begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a \frac{\partial x_\lambda}{\partial u_i}\frac{\partial x_\mu}{\partial u_k}\frac{\partial x_\nu}{\partial u_l} + \sum_{\lambda\mu} a_{\lambda\mu}\frac{\partial^2 x_\lambda}{\partial u_i \partial u_k}\frac{\partial x_\mu}{\partial u_l},$$

and hence,

$$(14)\qquad \left\{\begin{aligned} \begin{bmatrix} 11 \\ 2 \end{bmatrix}_b &= \sum_{\lambda\mu\nu} \begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a \frac{\partial x_\lambda}{\partial u_1}\frac{\partial x_\mu}{\partial u_1}\frac{\partial x_\nu}{\partial u_2} + \sum_{\lambda\mu} a_{\lambda\mu}\frac{\partial^2 x_\lambda}{\partial {u_1}^2}\frac{\partial x_\mu}{\partial u_2}, \\ \begin{bmatrix} 12 \\ 2 \end{bmatrix}_b &= \sum_{\lambda\tau\nu} \begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix}_a \frac{\partial x_\lambda}{\partial u_1}\frac{\partial x_\tau}{\partial u_2}\frac{\partial x_\nu}{\partial u_2} + \sum_{\lambda\mu} a_{\lambda\mu}\frac{\partial^2 x_\lambda}{\partial u_1 \partial u_2}\frac{\partial x_\mu}{\partial u_2}. \end{aligned}\right.$$

Thus,

$$\begin{aligned} \frac{\partial}{\partial u_2}\begin{bmatrix} 11 \\ 2 \end{bmatrix}_b - \frac{\partial}{\partial u_1}\begin{bmatrix} 12 \\ 2 \end{bmatrix}_b &= \sum_{\lambda\mu\nu\tau}\left(\frac{\partial}{\partial x_\tau}\begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a - \frac{\partial}{\partial x_\mu}\begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix}_a\right)\frac{\partial x_\lambda}{\partial u_1}\frac{\partial x_\mu}{\partial u_1}\frac{\partial x_\nu}{\partial u_2}\frac{\partial x_\tau}{\partial u_2} \\ &+ \sum_{\lambda\mu\nu}\begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a \frac{\partial x_\mu}{\partial u_1}\frac{\partial x_\nu}{\partial u_2}\frac{\partial^2 x_\lambda}{\partial u_1 \partial u_2} - \sum_{\lambda\tau\nu}\begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix}_a \frac{\partial x_\lambda}{\partial u_1}\frac{\partial x_\tau}{\partial u_2}\frac{\partial^2 x_\nu}{\partial u_1 \partial u_2} \\ &+ \sum_{\lambda\mu} a_{\lambda\mu}\frac{\partial^2 x_\lambda}{\partial {u_1}^2}\frac{\partial^2 x_\mu}{\partial {u_2}^2} - \sum_{\lambda\mu} a_{\lambda\mu}\frac{\partial^2 x_\lambda}{\partial u_1 \partial u_2}\frac{\partial^2 x_\mu}{\partial u_1 \partial u_2} + \sum_{\lambda\mu\nu}\begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a \frac{\partial x_\lambda}{\partial u_1}\frac{\partial x_\mu}{\partial u_1}\frac{\partial^2 x_\nu}{\partial {u_2}^2} \\ &- \sum_{\lambda\tau\nu}\begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix}_a \frac{\partial x_\tau}{\partial u_2}\frac{\partial x_\nu}{\partial u_2}\frac{\partial^2 x_\lambda}{\partial {u_1}^2} + \sum_{\lambda\mu\nu}\frac{\partial a_{\lambda\mu}}{\partial x_\nu}\frac{\partial x_\mu}{\partial u_2}\frac{\partial x_\nu}{\partial u_2}\frac{\partial^2 x_\lambda}{\partial {u_1}^2} - \sum_{\lambda\mu\nu}\frac{\partial a_{\lambda\mu}}{\partial x_\nu}\frac{\partial x_\mu}{\partial u_2}\frac{\partial x_\nu}{\partial u_1}\frac{\partial^2 x_\lambda}{\partial u_1 d u_2} \end{aligned}$$

Substituting here the values of the derivatives at P_0 as given by (9), we find after lengthy reductions

$$\begin{aligned} (15)\qquad \frac{\partial}{\partial u_2}\begin{bmatrix} 11 \\ 2 \end{bmatrix}_b - \frac{\partial}{\partial u_1}\begin{bmatrix} 12 \\ 2 \end{bmatrix}_b &= \sum_{l\tau} a_{l\tau}\left\{\psi_\tau(2,2)\psi_l(1,1) - \psi_\tau(1,2)\psi_l(1,2)\right\} \\ &+ \sum_{\lambda\mu\nu\tau}\left\{\frac{\partial}{\partial x_\tau}\begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix}_a - \frac{\partial}{\partial x_\mu}\begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix}_a\right. \\ &\left. + \sum_l\left(\begin{bmatrix} \mu\nu \\ l \end{bmatrix}_a \begin{Bmatrix} \lambda\tau \\ l \end{Bmatrix}_a - \begin{bmatrix} \lambda\mu \\ l \end{bmatrix}_a \begin{Bmatrix} \nu\tau \\ l \end{Bmatrix}_a\right)\right\} \xi_1^{(\lambda)}\xi_1^{(\mu)}\xi_2^{(\nu)}\xi_2^{(\tau)}. \end{aligned}$$

The first term in the right member may be expanded as follows. From (9′) we have

$$\psi_\nu(i, k) = \phi^{(\nu)}\sum_{\lambda\mu} a_{\lambda\mu}\xi_i^{(\lambda)}\xi_k^{(\mu)} - \tfrac{1}{2}\sum_\lambda \phi_\lambda\left(\xi_i^{(\nu)}\xi_k^{(\lambda)} + \xi_k^{(\nu)}\xi_i^{(\lambda)}\right),$$

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[6] Bianchi, ibid., p. 335.

and since

$$(16)\qquad \sum_{\lambda\mu} a_{\lambda\mu}\, \xi_1^{(\lambda)}\, \xi_1^{(\mu)} = \sum_{\lambda\mu} a_{\lambda\mu}\, \xi_2^{(\lambda)}\, \xi_2^{(\mu)} = 1;\ \ \sum_{\lambda\mu} a_{\lambda\mu}\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)} = \cos\theta,$$

where θ is the angle between the directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$, we may write

$$\begin{cases} \psi_l(1,\ 1) = \phi^{(l)} - \sum\limits_{\lambda} \phi_\lambda\, \xi_1^{(l)}\, \xi_1^{(\lambda)}, \\ \psi_\tau(2,\ 2) = \phi^{(\tau)} - \sum\limits_{\mu} \phi_\mu\, \xi_2^{(\tau)}\, \xi_2^{(\mu)}, \\ \psi_l(1,\ 2) = \phi^{(l)} \cos\theta - \frac{1}{2} \sum\limits_{\lambda} \phi_\lambda \left(\xi_1^{(l)}\, \xi_2^{(\lambda)} + \xi_2^{(l)}\, \xi_1^{(\lambda)}\right) \\ \psi_\tau(1,\ 2) = \phi^{(\tau)} \cos\theta - \frac{1}{2} \sum\limits_{\mu} \phi_\mu \left(\xi_1^{(\tau)}\, \xi_2^{(\mu)} + \xi_2^{(\tau)}\, \xi_1^{(\mu)}\right) \end{cases}$$

We find, after simple reductions,

$$(17)\qquad \sum_{l\tau} a_{l\tau} \left\{\psi_\tau(2,2)\, \psi_l(1,1) - \psi_\tau(1,2)\, \psi_l(1,2)\right\}$$
$$= \sin^2\theta \sum_{\lambda\mu} a_{\lambda\mu} \phi^{(\lambda)} \phi^{(\mu)} - \tfrac{5}{4} \sum_{\lambda\mu} \phi_\lambda \phi_\mu \left(\xi_1^{(\lambda)} \xi_1^{(\mu)} + \xi_2^{(\lambda)} \xi_2^{(\mu)} - 2\cos\theta\, \xi_1^{(\lambda)} \xi_2^{(\mu)}\right)$$

To complete the calculation of $\overset{0}{b}_{12,12}$, we still have to find the value of the second term in the right member of (12). From (13) and (9), we find

$$\begin{bmatrix} i\,k \\ l \end{bmatrix}_b^0 = \sum_{\lambda\mu} a_{\lambda\mu}\, \psi_\lambda(i, k)\, \xi_l^{(\mu)},$$

and hence

$$\begin{bmatrix} 11 \\ 1 \end{bmatrix}_b^0 = 0;\ \begin{bmatrix} 22 \\ 2 \end{bmatrix}_b^0 = 0;\ \begin{bmatrix} 12 \\ 2 \end{bmatrix}_b^0 = -\tfrac{1}{2} \begin{bmatrix} 22 \\ 1 \end{bmatrix}_b^0;\ \begin{bmatrix} 12 \\ 1 \end{bmatrix}_b^0 = -\tfrac{1}{2} \begin{bmatrix} 11 \\ 2 \end{bmatrix}_b^0;$$
$$\begin{bmatrix} 11 \\ 2 \end{bmatrix}_b^0 = \sum_{\mu} \phi_\mu\, \xi_2^{(\mu)} - \cos\theta \sum_{\mu} \phi_\mu\, \xi_1^{(\mu)};$$
$$\begin{bmatrix} 22 \\ 1 \end{bmatrix}_b^0 = \sum_{\mu} \phi_\mu\, \xi_1^{(\mu)} - \cos\theta \sum_{\mu} \phi_\mu\, \xi_2^{(\mu)}.$$

Further, from (10′) and (9),

$$b_{ik}^0 = \sum_{\lambda\mu} a_{\lambda\mu}\, \xi_i^{(\lambda)}\, \xi_k^{(\mu)},$$

and from (16), we have

$$b_{11}^0 = b_{22}^0 = 1,\ b_{12}^0 = \cos\theta;\ (b_{11}\,b_{22} - b_{12}^2)^0 = \sin^2\theta.$$

We finally have for the second term of the right member of (12),

$$\sum_{lm}^{1,2} b^{(lm)} \left(\begin{bmatrix} 12 \\ m \end{bmatrix}_b \begin{bmatrix} 12 \\ l \end{bmatrix}_b - \begin{bmatrix} 11 \\ m \end{bmatrix}_b \begin{bmatrix} 22 \\ l \end{bmatrix}_b \right)^0 \tag{18}$$

$$= \tfrac{1}{4} \sum_{\lambda\mu} \phi_\lambda\,\phi_\mu\,(\xi_1^{(\lambda)}\,\xi_1^{(\mu)} + \xi_2^{(\lambda)}\,\xi_2^{(\mu)} - 2\cos\theta\,\xi_1^{(\lambda)}\,\xi_2^{(\mu)})\,.$$

Hence, combining (12), (15), (17), and (18), we finally obtain

$$\begin{aligned} (19)\quad b_{12.12}^0 = \sum_{\lambda\mu\nu\tau} \Bigg\{ & \frac{\partial}{\partial x_\tau} \begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix} - \frac{\partial}{\partial x_\mu} \begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix} \\ & + \sum_l \left(\begin{bmatrix} \mu\nu \\ l \end{bmatrix} \left\{ \begin{matrix} \lambda\tau \\ l \end{matrix} \right\} - \begin{bmatrix} \lambda\mu \\ l \end{bmatrix} \left\{ \begin{matrix} \nu\tau \\ l \end{matrix} \right\} \right) \Bigg\} \xi_1^{(\lambda)}\,\xi_2^{(\mu)}\,\xi_1^{(\nu)}\,\xi_2^{(\tau)} \\ + \sin^2\theta \sum_{\lambda\mu} a_{\lambda\mu}\,\phi^{(\lambda)}\,\phi^{(\mu)} - & \sum_{\lambda\mu} \phi_\lambda\,\phi_\mu \left(\xi_1^{(\lambda)}\,\xi_1^{(\mu)} + \xi_2^{(\lambda)}\,\xi_2^{(\mu)} - 2\cos\theta\,\xi_1^{(\lambda)}\,\xi_2^{(\mu)} \right). \end{aligned}$$

Introducing the Riemann symbol in V_n[7]

$$a_{\lambda\mu.\nu\tau} = \frac{\partial}{\partial x_\tau} \begin{bmatrix} \lambda\mu \\ \nu \end{bmatrix} - \frac{\partial}{\partial x_\mu} \begin{bmatrix} \lambda\tau \\ \nu \end{bmatrix} + \sum_l \left(\begin{bmatrix} \mu\nu \\ l \end{bmatrix} \left\{ \begin{matrix} \lambda\tau \\ l \end{matrix} \right\} - \begin{bmatrix} \lambda\mu \\ l \end{bmatrix} \left\{ \begin{matrix} \nu\tau \\ l \end{matrix} \right\} \right),$$

the first differential invariant of ϕ (or the square of the magnitude of the trajectory vector)

$$\Delta\phi = \sum_{\lambda\mu} a_{\lambda\mu}\,\phi^{(\lambda)}\,\phi^{(\mu)} = \sum_\lambda \phi^{(\lambda)}\,\phi_\lambda = R^2\,, \tag{20}$$

and the value

$$(b_{11}\,b_{22} - b_{12}^2)^0 = \sin^2\theta\,,$$

we obtain from (11) for *the curvature of the trajectory surface at* P_0 *determined by the directions* $\xi_1^{(i)}$ *and* $\xi_2^{(i)}$ *through* P_0,

$$\begin{aligned} (21)\quad T = \frac{1}{\sin^2\theta} & \sum_{\lambda\mu\nu\tau} a_{\lambda\mu,\nu\tau}\,\xi_1^{(\lambda)}\,\xi_2^{(\mu)}\,\xi_1^{(\nu)}\,\xi_2^{(\tau)} + \Delta\phi \\ & - \frac{1}{\sin^2\theta} \sum_{\lambda\mu} \phi_\lambda\,\phi_\mu\,(\xi_1^{(\lambda)}\,\xi_1^{(\mu)} + \xi_2^{(\lambda)}\,\xi_2^{(\mu)} - 2\cos\theta\,\xi_1^{(\lambda)}\,\xi_2^{(\mu)}) \end{aligned}$$

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[7] Bianchi, ibid., p. 335.

The last term may be written more symmetrically by introducing the relations (16)

$$\sum_{\nu\tau} a_{\nu\tau}\, \xi_1^{(\nu)}\, \xi_1^{(\tau)} = 1; \;\; \sum_{\nu\tau} a_{\nu\tau}\, \xi_2^{(\nu)}\, \xi_2^{(\tau)} = 1; \;\; \sum_{\nu\tau} a_{\nu\tau}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)} = \cos\theta;$$

thus we have

$$\sum_{\lambda\mu} \phi_\lambda\, \phi_\mu\, (\xi_1^{(\lambda)}\, \xi_1^{(\mu)} + \xi_2^{(\lambda)}\, \xi_2^{(\mu)} - 2 \cos\theta\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)})$$

$$= \sum_{\lambda\mu\nu\tau} (a_{\mu\tau}\, \phi_\lambda\, \phi_\nu - a_{\lambda\tau}\, \phi_\mu\, \phi_\nu + a_{\lambda\nu}\, \phi_\mu\, \phi_\tau - a_{\mu\nu}\, \phi_\lambda\, \phi_\tau)\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)}\,.$$

Let us write

$$(22) \qquad \phi_{\lambda\mu,\nu\tau} = a_{\mu\tau}\, \phi_\lambda\, \phi_\nu - a_{\lambda\tau}\, \phi_\mu\, \phi_\nu + a_{\lambda\nu}\, \phi_\mu\, \phi_\tau - a_{\mu\nu}\, \phi_\lambda\, \phi_\tau.$$

The quantity $\phi_{\lambda\mu,\nu\tau}$ is evidently a covariant having properties similar to those of the Riemannian covariant $a_{\lambda\mu,\nu\tau}$, viz.,

$$(22') \qquad \begin{cases} \phi_{\lambda\mu,\nu\tau} = \phi_{\nu\tau,\lambda\mu} = -\,\phi_{\lambda\mu,\tau\nu} = -\,\phi_{\mu\lambda,\nu\tau}; \\ \phi_{\lambda\mu,\nu\nu} = \phi_{\lambda\lambda,\nu\tau} = 0; \\ \phi_{\lambda\mu,\nu\tau} + \phi_{\lambda\nu,\tau\mu} + \phi_{\lambda\tau,\mu\nu} = 0. \end{cases}$$

Let us also express $\sin^2\theta$ in terms of the fundamental coefficients a_{ik} and the fundamental directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$, by means of (9) and (10); we find

$$\sin^2\theta = \sum_{\lambda\mu\nu\tau} (a_{\lambda\nu}\, a_{\mu\tau} - a_{\lambda\tau}\, a_{\mu\nu})\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)}.$$

We may finally write

$$(21') \quad \mathrm{T} = \frac{\sum\limits_{\lambda\mu\nu\tau} (a_{\lambda\mu,\nu\tau} - \phi_{\lambda\mu,\nu\tau})\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)}}{\sum\limits_{\lambda\mu\nu\tau} (a_{\lambda\nu}\, a_{\mu\tau} - a_{\lambda\tau}\, a_{\mu\nu})\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)}} + \sum_{\lambda\mu} a_{\lambda\mu}\, \phi^{(\lambda)} \phi^{(\mu)},$$

for *the curvature of the trajectory surface at* P_0 *determined by two directions* $\xi_1^{(i)}$ *and* $\xi_2^{(i)}$ *through* P_0.

3. **Corresponding Geodesic and Trajectory Surfaces.** The plane pencil of directions or orientation determined by a point P and by any two directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$ through P determine a geodesic surface σ_g and a trajectory surface σ_t; we shall say that σ_g and σ_t are *corresponding surfaces*.

Now, the curvature of σ_g at P is given by [8]

$$(23) \qquad G = \frac{1}{\sin^2\theta} \sum_{\lambda\mu\nu\tau} a_{\lambda\mu,\nu\tau}\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)},$$

while the curvature of σ_t is given by (21). We deduce immediately

$$(24)\ T = G + \Delta\phi - \frac{1}{\sin^2\theta} \sum_{\lambda\mu} \phi_\lambda\, \phi_\mu\, (\xi_1^{(\lambda)}\, \xi_1^{(\mu)} + \xi_2^{(\lambda)}\, \xi_2^{(\mu)} - 2\cos\theta\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}).$$

Equation (24) indicates the relation between the curvatures of corresponding surfaces. By introducing the angles α and β between the trajectory vector ϕ and the fundamental directions ξ_1, ξ_2,

$$\sum_\lambda \phi_\lambda\, \xi_1^{(\lambda)} = R\cos\alpha; \quad \sum_\lambda \phi_\lambda\, \xi_2^{(\lambda)} = R\cos\beta; \quad R^2 = \Delta\phi;$$

(24) may be written in the alternate form

$$(24') \qquad T = G + R^2 - \frac{R^2}{\sin^2\theta}\,[\cos^2\alpha + \cos^2\beta - 2\cos\theta\cos\alpha\cos\beta].$$

We may, without loss of generality, assume the two fundamental directions ξ_1 and ξ_2 at right angles, and (24) and (24′) take the forms

$$(25) \qquad T = G + \Delta\phi - \sum_{\lambda\mu} \phi_\lambda\, \phi_\mu\, (\xi_1^{(\lambda)}\, \xi_1^{(\mu)} + \xi_2^{(\lambda)}\, \xi_2^{(\mu)});$$

$$(25') \qquad T = G + R^2 - R^2\,(\cos^2\alpha + \cos^2\beta).$$

For the special case $n = 2$, the corresponding surfaces coincide with the space; hence $T = G$. We may verify this from (25′), for since, here, ϕ, ξ_1, and ξ_2 are directions in a same plane pencil, $\cos^2\beta = \sin^2\alpha$.

In general, for any V_n, we may ask: Under what conditions will the curvatures of corresponding geodesic and trajectory surfaces be the same? From (25′), we note that if $T = G$, we must have

$$\cos^2\alpha + \cos^2\beta = 1 \quad \text{or} \quad \cos^2\beta = \sin^2\alpha, \quad \text{or} \quad \cos\beta = \pm \sin\alpha,$$

hence, ϕ, ξ_1, and ξ_2 must lie in the same plane pencil; i.e., the trajectory vector ϕ must lie in the orientation determined by the fundamental directions. We thus have the theorem:

Corresponding geodesic and trajectory surfaces at a point P which

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[8] Bianchi, ibid., p. 343.

contain the direction of the trajectory vector have the same curvatures at P; and conversely, if corresponding geodesic and trajectory surfaces are to have the same curvature at a point P, they must contain the direction of the trajectory vector at P.

Now, the geodesic surfaces which contain the direction of the trajectory vector are the osculating geodesic surfaces of the trajectories through P.[9] Hence, we may state the above theorem thus:

The osculating geodesic surface to a trajectory at a point P and its corresponding trajectory surface have the same curvature at P.

4. **Conformal Parallelism.** In a previous paper,[10] the author defined the motion of a direction by conformal parallelism as follows: Given in a space V_n two infinitely near points P and Q and a direction $\xi^{(i)}$ issuing from P; a direction $\eta^{(i)}$ in V_n through Q is said to be conformally parallel to $\xi^{(i)}$, (1) if $\eta^{(i)}$ is a direction in the trajectory surface determined by the directions $\xi^{(i)}$ and PQ, and (2) if $\eta^{(i)}$ makes at Q the same angle with the trajectory uniquely determined by P and the direction PQ as $\xi^{(i)}$ makes with it at P. If in this definition we replace trajectory and trajectory surface by geodesic and geodesic surface, we get the definition of parallelism as given by Levi-Civita and Severi.[11] The motion by conformal parallelism in V_n is evidently the conformal representation of the motion by parallelism in a space V'_n which is conformally represented on V_n, i.e. such that their first fundamental forms are related by

$$ds' = e^{\phi}\, ds,$$

the geodesics and geodesic surfaces of V'_n being represented by the corresponding trajectories and trajectory surfaces in V_n. Now, let P' be any point in V'_n and let $\eta_1^{(i)}$ and $\eta_2^{(i)}$ be any directions through P' which determine a geodesic surface containing an infinitesimal cycle of area $\delta\sigma'$ through P'. If any direction through P' and lying in the geodesic surface moves by parallelism in V'_n completely around the cycle, and if $\delta\epsilon'$ is the angle between its initial and final positions, then[12]

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[9] See the author's paper: *Natural families of curves in a general curved space of n dimensions.* Trans. Am. Math. Soc., Vol. 13 (1912), pp. 77–95.

[10] *On conformal parallelism*, Journ. of Math. and Phys., Vol. 2 (1923), No. 3.

[11] T. Levi-Civita: *Nozione di parallelismo in una varietà qualunque e conseguente specificazione geometrica della curvatura Riemanniana.* Rend. del Circ. Mat. di Palermo, t. XLII (1917).

F. Severi: *Sulla curvatura delle superficie e varietà.* Rend. del Circ. Mat. di Palermo, t. XLII (1917).

$$(26)\qquad \frac{\delta\epsilon'}{\delta\sigma'} = \frac{1}{\sin^2\theta'} \sum_{\lambda\mu\nu\tau} a'_{\lambda\mu,\nu\tau}\, \eta_1^{(\lambda)}\, \eta_2^{(\mu)}\, \eta_1^{(\nu)}\, \eta_2^{(\tau)} = G',$$

where G' is the curvature of the geodesic surface.[13] If we transform to the conformal space V'_n, the corresponding conformal direction moves by conformal parallelism around an infinitesimal cycle of area $\delta\sigma$ through the corresponding point P and lying on a trajectory surface with P as pole determined by the directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$, where

$$\xi_1^{(i)} = e^{\phi}\, \eta_1^{(i)},\ \xi_2^{(i)} = e^{\phi}\, \eta_2^{(i)},\ \delta\sigma = e^{-2\phi}\, \delta\sigma'.$$

If $\delta\epsilon$ is the angle between the initial and final positions of the moving direction, then $\delta\epsilon = \delta\epsilon'$, and (26) becomes

$$\frac{\delta\epsilon}{\delta\sigma} = \frac{e^{-2\phi}}{\sin^2\theta} \sum_{\lambda\mu\nu\tau} a'_{\lambda\mu,\nu\tau}\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)}.$$

We still have to express $a'_{\lambda\mu,\nu\tau}$ in terms of the elements of V_n. For this purpose we may show that[14]

$$(27)\quad \begin{aligned} e^{-2\phi} a'_{\lambda\mu,\nu\tau} = a_{\lambda\mu,\nu\tau} &+ a_{\lambda\tau}(\phi_{\mu\nu} - \phi_\mu\phi_\nu) - a_{\lambda\nu}(\phi_{\mu\tau} - \phi_\mu\phi_\tau) \\ &+ a_{\mu\nu}(\phi_{\lambda\tau} - \phi_\lambda\phi_\tau) - a_{\mu\tau}(\phi_{\lambda\nu} - \phi_\lambda\phi_\nu) + (a_{\lambda\tau}a_{\mu\nu} - a_{\lambda\nu}a_{\mu\tau})\Delta\phi, \end{aligned}$$

where, as usual, we designate by ϕ_{ik} the covariant derivative of ϕ_i, i.e.,

$$\phi_{ik} = \frac{\partial\phi_i}{\partial x_k} - \sum_l \begin{Bmatrix} ik \\ l \end{Bmatrix} \phi_l.$$

Introducing (27), using the relations (16), and collecting terms, we may write

$$(28)\quad \begin{aligned} \frac{\delta\epsilon}{\delta\sigma} = \frac{1}{\sin^2\theta} &\sum_{\lambda\mu\nu\tau} a_{\lambda\mu,\nu\tau}\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}\, \xi_1^{(\nu)}\, \xi_2^{(\tau)} - \Delta\phi \\ &- \frac{1}{\sin^2\theta} \sum_{\lambda\mu} (\phi_{\lambda\mu} - \phi_\lambda\, \phi_\mu)\, (\xi_1^{(\lambda)}\, \xi_1^{(\mu)} + \xi_2^{(\lambda)}\, \xi_2^{(\mu)} - 2\cos\theta\, \xi_1^{(\lambda)}\, \xi_2^{(\mu)}). \end{aligned}$$

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[12] See T. Levi-Civita, *Qüestions de Mecànica clàssica i relativista.* Institute D'Estudis Catalans. Secció de Ciències, Barcelona, 1922.

[13] Here $\delta\epsilon'$ is considered positive or negative according as the infinitesimal rotation necessary to bring the initial direction into the final position is in the same sense as the description of the cycle or in the opposite sense.

[14] Cf. Levi-Civita, *ds^2 einsteiniani in campi newtoniani.* III, Rendic. della R. Acc. dei Lincei, t. XXVII, p. 183.

We may further introduce the curvature T and G of the corresponding trajectory and geodesic surfaces determined by ξ_1 and ξ_2, and given by (21) and (23); then (28) becomes

$$(29)\quad \frac{\delta\epsilon}{\delta\sigma} = 2G - T - \frac{1}{\sin^2\theta} \sum_{\lambda\mu} \phi_{\lambda\mu} \left(\xi_1^{(\lambda)} \xi_1^{(\mu)} + \xi_2^{(\lambda)} \xi_2^{(\mu)} - 2 \cos\theta\, \xi_1^{(\lambda)} \xi_2^{(\mu)}\right).$$

Finally, let us consider the plane pencil of directions determined by two directions ξ_1 and ξ_2 through a point P. This orientation determines two corresponding surfaces, one geodesic and the other trajectory. If a direction of the pencil moves by parallelism completely around an infinitesimal cycle of area $\delta\sigma$ on the geodesic surface, it will return to a final position in the pencil and make with the initial position an angle $\delta\bar{\epsilon}$ where $\delta\bar{\epsilon}/\delta\sigma = G$. On the other hand, if this same initial direction moves by conformal parallelism completely around an infinitesimal cycle of area $\delta\sigma$ in the trajectory surface, it will return to a final position in the pencil and make with the initial position an infinitesimal angle $\delta\epsilon$, where $\delta\epsilon/\delta\sigma$ is given by (29). If we designate by $\delta\omega$ the angle between the two final positions, i.e. $\delta\omega = \delta\bar{\epsilon} - \delta\epsilon$, and by K the difference of the curvatures, i.e. $K = G - T$, we may write (29) in the form

$$(30)\quad \frac{\delta\omega}{\delta\sigma} + K = \frac{1}{\sin^2\theta} \sum_{\lambda\mu} \phi_{\lambda\mu} \left(\xi_1^{(\lambda)} \xi_1^{(\mu)} + \xi_2^{(\lambda)} \xi_2^{(\mu)} - 2 \cos\theta\, \xi_1^{(\lambda)} \xi_2^{(\mu)}\right)$$

Let us define the covariant system of the fourth order

$$(31)\qquad \bar{\phi}_{\lambda\mu,\nu\tau} = a_{\mu\tau}\, \phi_{\lambda\nu} - a_{\lambda\tau}\, \phi_{\mu\nu} + a_{\lambda\nu}\, \phi_{\mu\tau} - a_{\mu\nu}\, \phi_{\lambda\tau},$$

which has properties similar to the covariant $\phi_{\lambda\mu,\nu\tau}$ defined by (22) and to the Riemannian co-variant $a_{\lambda\mu,\nu\tau}$. Using the relations (16), we may write (30) in the form

$$(30')\qquad \frac{1}{\sin^2\theta} \sum_{\lambda\mu\nu\tau} \bar{\phi}_{\lambda\mu,\nu\tau}\, \xi_1^{(\lambda)} \xi_2^{(\mu)} \xi_1^{(\nu)} \xi_2^{(\tau)} = \frac{\delta\omega}{\delta\sigma} + K,$$

which gives us *a geometric interpretation of the invariant in the left member.*

When $n = 2$, the left member of (30′) reduces to

$$\Delta_2\phi = \sum_{\lambda\mu}^{1,2} a^{(\lambda\mu)}\, \phi_{\lambda\mu},$$

the *second differential parameter of* ϕ in the two-spread, and since here $K = 0$, (30′) becomes

$$\Delta_2 \phi = \frac{\delta\omega}{\delta\sigma},$$

a *geometric interpretation of the second differential parameter of a function,* (as already shown in a previous paper [15]).

5. **Maximum and Minimum Trajectory Curvatures in $\mathbf{V}_3$.** For the special case $n = 3$, the expression

$$\text{(21′)} \qquad \mathrm{T} = \frac{\sum\limits_{\lambda\mu\nu\tau} (a_{\lambda\mu,\nu\tau} - \phi_{\lambda\mu,\nu\tau})\, \xi_1^{(\lambda)}\xi_2^{(\mu)}\xi_1^{(\nu)}\xi_2^{(\tau)}}{\sum\limits_{\lambda\mu\nu\tau} (a_{\lambda\nu}\, a_{\mu\tau} - a_{\lambda\tau}\, a_{\mu\nu})\, \xi_1^{(\lambda)}\xi_2^{(\mu)}\xi_1^{(\nu)}\xi_2^{(\tau)}} + \Delta\phi$$

(all summations being carried from 1 to 3 for the indicated subscripts) for the curvature of the trajectory surface determined by two directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$ at a point P may be given a much simpler form by the following considerations. We shall follow the method of Ricci in his "*Lezioni sulla teoria delle superficie,*" and consider as equivalent two indices whose difference is divisible by 3. Using the properties of the covariants $a_{\lambda\mu,\nu\tau}$ and $\phi_{\lambda\mu,\nu\tau}$, viz.,

$$a_{\lambda\mu,\nu\tau} = a_{\nu\tau,\lambda\mu} = -\, a_{\mu\lambda,\nu\tau} = -\, a_{\lambda\mu,\tau\nu}; \quad a_{\lambda\lambda,\nu\tau} = 0;$$

$$\phi_{\lambda\mu,\nu\tau} = \phi_{\nu\tau,\lambda\mu} = -\, \phi_{\mu\lambda,\nu\tau} = -\, \phi_{\lambda\mu,\tau\nu}; \quad \phi_{\lambda\lambda,\nu\tau} = 0;$$

and letting in turn

$$\lambda = r + 1,\ \nu = r + 2 \quad (r = 1, 2, 3, \text{ modulo } 3)$$
$$\lambda = r + 2,\ \nu = r + 1 \quad (r = 1, 2, 3, \text{ modulo } 3),$$

and then

$$\mu = s + 1,\ \tau = s + 2 \quad (s = 1, 2, 3, \text{ modulo } 3)$$
$$\mu = s + 2,\ \tau = s + 1 \quad (s = 1, 2, 3, \text{ modulo } 3),$$

we easily find

$$\text{(32)} \qquad \sum_{\lambda\mu\nu\tau} a_{\lambda\mu,\nu\tau}\, \xi_1^{(\lambda)}\xi_2^{(\mu)}\xi_1^{(\nu)}\xi_2^{(\tau)} = \sum_{rs} a_{r+1\, r+2,\, s+1\, s+2} \big(\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)}\big)\big(\xi_1^{(s+1)}\xi_2^{(s+2)} - \xi_1^{(s+2)}\xi_2^{(s+1)}\big),$$

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[15] Cf. reference in footnote 10.

and

$$(33)\quad \sum_{\lambda\mu\nu\tau} \phi_{\lambda\mu,\nu\tau}\,\xi_1^{(\lambda)}\xi_2^{(\mu)}\xi_1^{(\nu)}\xi_2^{(\tau)} = \sum_{rs} (a_{r+2\,s+2}\,\phi_{r+1}\,\phi_{s+1} - a_{r+1\,s+2}\,\phi_{r+2}\,\phi_{s+1}$$
$$+ a_{r+1\,s+1}\,\phi_{r+2}\,\phi_{s+2} - a_{r+2\,s+1}\,\phi_{r+1}\,\phi_{s+2})\,(\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)})$$
$$(\xi_1^{(s+1)}\xi_2^{(s+2)} - \xi_1^{(s+2)}\xi_2^{(s+1)}).$$

Similarly,

$$(34)\qquad \sum_{\lambda\mu\nu\tau} (a_{\lambda\nu}\,a_{\mu\tau} - a_{\lambda\tau}\,a_{\mu\nu})\,\xi_1^{(\lambda)}\xi_2^{(\mu)}\xi_1^{(\nu)}\xi_2^{(\tau)}$$
$$= \sum_{rs} (a_{r+1\,s+1}\,a_{r+2\,s+2} - a_{r+1\,s+2}\,a_{r+2}\,a_{s+1})\,(\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)})$$
$$(\xi_1^{(s+1)}\xi_2^{(s+2)}\xi_1^{(s+2)}\xi_2^{(s+1)})\,.$$

We now write

$$(32')\qquad \frac{a_{r+1\,r+2,\,s+1\,s+2}}{a} = \beta^{(rs)},$$

where, as Ricci has shown, $\beta^{(rs)}$ is a symmetric contravariant of the second order, a being the value of the determinant of the fundamental coefficients a_{ik}. Further, from the properties of the determinant $|\,a_{ik}\,|$ and the reciprocal determinant $|\,a^{(ik)}\,|$, we have

$$\frac{a_{r+1\,s+1}}{a} = \begin{vmatrix} a^{rs} & a^{r\,s+2} \\ a^{r+2\,s} & a^{r+2\,s+2} \end{vmatrix}, \text{ etc.,}$$

so that

$$(33')\quad a_{r+2\,s+2}\,\phi_{r+1}\,\phi_{s+1} - a_{r+1\,s+2}\,\phi_{r+2}\,\phi_{s+1} + a_{r+1\,s+1}\,\phi_{r+2}\,\phi_{s+2}$$
$$- a_{r+2\,s+1}\,\phi_{r+1}\,\phi_{s+2}$$
$$= a \sum_{pq} \phi_p\,\phi_q\,(a^{(rs)}\,a^{(pq)} - a^{(ps)}\,a^{(qr)})$$
$$= a\left[a^{(rs)} \sum_{pq} a^{(pq)}\,\phi_p\,\phi_q - \sum_{pq} a^{(ps)}\,a^{(qr)}\,\phi_p\,\phi_q\right];$$
$$\text{where}\quad \left\{\begin{matrix} p = r+1,\ r+2, \text{ mod. } 3 \\ q = s+1,\ s+2, \text{ mod. } 3 \end{matrix}\right\}.$$
$$= a\,[a^{(rs)}\,\Delta\phi - \phi^{(r)}\,\phi^{(s)}].$$

Also,

$$(34')\qquad \frac{a_{r+1\,s+1}\,a_{r+2\,s+2} - a_{r+1\,s+2}\,a_{r+2\,s+1}}{a} = a^{(rs)}.$$

Introducing (32′), (33′), (34′) into (32), (33), (34) and these into (21′), we obtain

$$(35)\quad T = \frac{\sum_{rs} (\beta^{(rs)} + \phi^{(r)}\phi^{(s)}) (\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)}) (\xi_1^{(s+1)}\xi_2^{(s+2)} - \xi_1^{(s+2)}\xi_2^{(s+1)})}{\sum_{rs} a^{(rs)} (\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)}) (\xi_1^{(s+1)}\xi_2^{(s+2)} - \xi_1^{(s+2)}\xi_2^{(s+1)})}.$$

Let us finally introduce the direction $\zeta^{(i)}$ of the normal to the surface determined by $\xi_1^{(i)}$ and $\xi_2^{(i)}$; we have

$$(36)\qquad \zeta_r = (\xi_1^{(r+1)}\xi_2^{(r+2)} - \xi_1^{(r+2)}\xi_2^{(r+1)}) \sqrt{a},$$

and we thus find

$$(37)\qquad T = \frac{\sum_{rs} (\beta^{(rs)} + \phi^{(r)}\phi^{(s)}) \zeta_r \zeta_s}{\sum_{rs} a^{(rs)} \zeta_r \zeta_s} = \frac{\sum_{rs} (\beta_{rs} + \phi_r \phi_s) \zeta^{(r)}\zeta^{(s)}}{\sum_{rs} a_{rs} \zeta^{(r)} \zeta^{(s)}},$$

(where β_{rs} is the covariant reciprocal of $\beta^{(rs)}$) for *the curvature of the trajectory surface at P normal to the direction $\zeta^{(i)}$ through P.* We shall say that the trajectory surface through P is determined by the direction of its normal $\zeta^{(i)}$.

Let us now ask for the directions $\zeta^{(i)}$ for which the normal trajectory surfaces have a maximum or minimum curvature. We must evidently have

$$\frac{\partial T}{\partial \zeta^{(r)}} = 0, \quad (r = 1, 2, 3),$$

and indicating the value of T in the required direction by ω, and applying this condition to (37), we obtain the linear system

$$(38)\qquad \sum_{s} (\beta_{rs} + \phi_r \phi_s - \omega\, a_{rs}) \zeta^{(s)} = 0, \quad (r = 1, 2, 3).$$

Hence ω must satisfy the determinantal equation

$$(39)\qquad | \beta_{rs} + \phi_r \phi_s - \omega\, a_{rs} | = 0.$$

It is well known that because the quantities β_{rs}, $\phi_r \phi_s$, and a_{rs} are symmetric functions with respect to r and s, equation (39) has 3 real roots, which we shall designate by ω_1, ω_2, ω_3. If these roots are distinct each will give a single system of values of the ratios of the ζ's when substituted into (38), and thus three directions are determined. If one of the ω's is a double root, there will correspond to this value a plane pencil of directions. We may then state:

There are, in general, three directions through each point in V_3 *for which the corresponding trajectory surfaces have a maximum or minimum curvature.*

We shall call these directions the *principal trajectory directions* and the corresponding curvatures ω_1, ω_2, ω_3, the *principal trajectory curvatures* at a point P.

If ω_1 and ω_2 are two distinct roots of (39), and $\zeta_1^{(i)}$ and $\zeta_2^{(i)}$ are the corresponding directions, then, from (38), we have

$$\sum_s (\beta_{rs} + \phi_r \phi_s)\, \zeta_1^{(s)} = \omega_1 \sum_s a_{rs}\, \zeta_1^{(s)}$$

$$\sum_r (\beta_{rs} + \phi_r \phi_s)\, \zeta_2^{(r)} = \omega_2 \sum_r a_{rs}\, \zeta_2^{(r)}$$

Multiplying the first of these by $\zeta_2^{(r)}$ and summing with respect to r, and the second by $\zeta_1^{(s)}$ and summing with respect to s, and subtracting, we find

$$(\omega_1 - \omega_2) \sum_{rs} a_{rs}\, \zeta_1^{(r)}\, \zeta_2^{(s)} = 0,$$

and hence, since $\omega_1 \neq \omega_2$,

$$\sum_{rs} a_{rs}\, \zeta_1^{(r)}\, \zeta_2^{(s)} = 0,$$

i.e. the corresponding directions are orthogonal. Hence

The 3 *principal trajectory directions at a point P in* V_3 *are mutually orthogonal.*

Let us indicate the three principal trajectory directions by the parameters $\zeta_h^{(r)}$ $(h = 1, 2, 3)$ or by the corresponding moments $\zeta_{h/r}$ $(h = 1, 2, 3)$. Equations (38) may be written

$$\sum_s (\beta_{rs} + \phi_r \phi_s)\, \zeta_h^{(s)} = \omega_h \sum_s a_{rs}\, \zeta_h^{(s)} = \omega_h\, \zeta_{h/r}\,.$$

Multiplying by $\zeta_{h/t}$ and summing on h, we have

$$\sum_{sh} (\beta_{rs} + \phi_r \phi_s)\, \zeta_h^{(s)}\, \zeta_{h/t} = \sum \omega_h\, \zeta_{h/r}\, \zeta_{h/t}\,,$$

and since

$$\sum_h \zeta_h^{(s)} \zeta_{h/t} = \epsilon_{st} = \begin{cases} 0, \text{ if } s \neq t \\ 1, \text{ if } s = t \end{cases},$$

we obtain

$$(40) \qquad \beta_{rs} + \phi_r \phi_s = \sum_h \omega_h \zeta_{h/r} \zeta_{h/s},$$

a canonical form for the covariant $\beta_{rs} + \phi_r \phi_s$.[16]

Let $\zeta^{(i)}$ be any direction through the point P making angles $\gamma_1, \gamma_2, \gamma_3$ with the principal trajectory directions $\zeta_1^{(i)}, \zeta_2^{(i)}, \zeta_3^{(i)}$, respectively, i.e.,

$$\cos \gamma_h = \sum_r \zeta^{(r)} \zeta_{h/r}, \ (h = 1, 2, 3).$$

The curvature of the trajectory surface normal to this arbitrary direction $\zeta^{(i)}$ is given by (37). If we introduce into this the expression (40), we have

$$T = \sum_{rsh} \omega_h \zeta^{(r)} \zeta^{(s)} \zeta_{h/r} \zeta_{h/s} = \sum_h \omega_h \cos^2 \gamma_h.$$

But $\omega_1, \omega_2, \omega_3$ are the 3 principal trajectory curvatures T_1, T_2, T_3, hence

$$(41) \qquad T = \sum_h T_h \cos^2 \gamma_h,$$

which expresses the curvature of any trajectory surface at a point P in terms of the 3 principal trajectory curvatures at P.

If we consider any 3 mutually perpendicular directions through P making angles $\gamma_{h/1}, \gamma_{h/2}, \gamma_{h/3}$ with the principal direction $\zeta_h^{(i)}$, we shall have for the curvatures of the corresponding trajectory surfaces

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[16] Ricci and Levi-Civita have shown that a symmetric covariant α_{rs} of the second order in any number of dimensions can be expressed in one and only one way in the form $\alpha_{rs} = \sum_h \rho_h \lambda_{h/r} \lambda_{h/s}$, where the directions $\lambda_{h/r}$ $(h = 1, 2, \ldots n)$ form an orthogonal ennuple at each point of V_n, and they have designated this form by the name *canonical form.* These canonical forms play a leading rôle in the geometry of the absolute calculus. Cf., Ricci, *Sulla teoria delle linee geodetiche e dei sistemi isotermi di Liouville*, §2, Atti del R. Ist. Veneto di Science, Lettere ed Arti (1894), and also, Ricci et Levi-Civita, *Méthodes de calcul différentiel absolu et leurs applications*, Math. Annalen, Bd. LIV, (1901), Chapt. II.

$$T^k = \sum_h T_h \cos^2\gamma_{h/k} \qquad (k = 1, 2, 3),$$

and summing with respect to k, we get

$$\sum_k T^k = \sum_h T_h$$

Hence, *the sum of the curvatures of any three trajectory surfaces normal to three mutually perpendicular directions through a point P is constant, i. e. independent of the three directions chosen.*

All these theorems reduce to the corresponding theorems found by Ricci by a discussion of the curvatures of *geodesic surfaces* in V_3, and where he has introduced the terms *principal directions* and *principal curvatures* at a point in V_3.[17] Our expressions reduce to those found by Ricci, if we replace ϕ by a constant. We may here derive an interesting relation between the sum of the principal trajectory curvatures and the sum of the principal curvatures at a point P. From the canonical form (40), we have

$$\sum_{rs} a^{(rs)} (\beta_{rs} + \phi_r \phi_s) = \sum_{rsh} a^{(rs)} \omega_h \zeta_{h/r} \zeta_{h/s},$$

or

$$\sum_{rs} a^{(rs)} \beta_{rs} + \Delta\phi = \sum_h \omega_h. \tag{42}$$

On the other hand, from the corresponding canonical form for β_{rs} in terms of the principal curvatures and the principal directions, viz.,

$$\beta_{rs} = \sum_h \omega'_h \zeta'_{h/r} \zeta'_{h/s},$$

we get

$$\sum_{rs} a^{(rs)} \beta_{rs} = \sum_h \omega'_h, \tag{43}$$

By comparison of (42) and (43) we deduce

$$\sum_h \omega_h = \sum_h \omega'_h + \Delta\phi. \tag{44}$$

Hence, *the sum of the principal trajectory curvatures minus the sum of the principal curvatures at a point P is equal to the square of the magnitude of the trajectory vector at P.*

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[17] Ricci, *Sui gruppi continui di movimenti in una varietà qualunque a tre dimensioni.* Memorie della Società Italiana delle Science, detta dei XL, Serie III^a, t. XII.

6. **Median Trajectory Curvature and Principal Trajectory Directions in $\mathbf{V_n}$.** As usual, let us define a congruence of lines in V_n by the equations

$$\frac{dx_1}{\lambda^{(1)}} = \frac{dx_2}{\lambda^{(2)}} = \ldots\ldots = \frac{dx_n}{\lambda^{(n)}},$$

the λ's being functions of the coördinates $x_1, x_2, \ldots, x_n$. We say that the system $\lambda^{(r)}$ $(r = 1, 2, \ldots, n)$ is the *contravariant coördinate system of the congruence* and that the reciprocal λ_r, $(r = 1, 2, \ldots, n)$ is the *covariant coördinate system.* We represent by [1], [2], . . ., [n] n such congruences of lines, with $\lambda_1^{(r)}, \lambda_2^{(r)}, \ldots, \lambda_n^{(r)}$ $(r = 1, 2, \ldots, n)$ or with $\lambda_{1/r}, \lambda_{2/r}, \ldots, \lambda_{n/r}$ $(r = 1, 2, \ldots, n)$ their contravariant or covariant coördinate systems. If these congruences determine n mutual orthogonal directions at each point of V_n, they are said to form an *orthogonal ennuple.*[18] For such an ennuple, we have the identities

$$(45)\quad \begin{cases} \sum\limits_r \lambda_h^{(r)} \lambda_{k/r} = \epsilon_{hk}\,;\ \sum\limits_r \lambda_r^{(h)} \lambda_{r/k} = \epsilon_{hk}\,;\ \epsilon_{hk} = \begin{cases} 0, \text{ if } h \neq k \\ 1, \text{ if } h = k \end{cases} \\ a_{rs} = \sum\limits_h \lambda_{h/r} \lambda_{h/s}\,;\ a^{(rs)} = \sum\limits_h \lambda_h^{(r)} \lambda_h^{(s)}. \end{cases}$$

Let us now return to the expression (21) for the curvature of a trajectory surface determined by two directions $\xi_1^{(i)}$ and $\xi_2^{(i)}$. The curvature of the trajectory surface determined at the point P by the directions of the lines of two congruences [h] and [k] passing through P is evidently given by

$$(46)\quad T_{hk} = \sum_{rstu} a_{rs,tu} \lambda_h^{(r)} \lambda_k^{(s)} \lambda_h^{(t)} \lambda_k^{(u)} + \Delta\phi - \sum_{rs} \phi_r \phi_s (\lambda_h^{(r)} \lambda_h^{(s)} + \lambda_k^{(r)} \lambda_k^{(s)}),$$

since $\cos\theta = 0$, $\sin\theta = 1$.

If we allow k to take all values from 1 to n, except $k = h$, i. e. if we associate with the direction of the congruence [h] the directions of $n-1$ orthogonal congruences at P, and sum with respect to k the corresponding expressions (46), we obtain

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[18] Cf. Ricci et Levi-Civita, reference in footnote 16. Also, Ricci, *Dei sistemi di congruenze ortogonali in una varietà qualunque.* Memorie della R. Acc. dei Lincei, Classe dei Scienze, t. II (1896).

$$\sum_k{}' T_{hk} = \sum_{rstu} a_{rs,tu} \lambda_h^{(r)} \lambda_h^{(t)} \left(\sum_k{}' \lambda_k^{(s)} \lambda_k^{(u)} \right) + (n-1) \Delta\phi$$
$$- (n-1) \sum_{rs} \phi_r \phi_s \lambda_h^{(r)} \lambda_h^{(s)} - \sum_{rs} \phi_r \phi_s \left(\sum_k{}' \lambda_k^{(r)} \lambda_k^{(s)} \right),$$

where $\sum{}'$ indicates that in the summation k takes all values except the value h. If to the right member we add

$$\sum_{rstu} a_{rs,tu} \lambda_h^{(r)} \lambda_h^{(t)} \lambda_h^{(s)} \lambda_h^{(u)} + \sum_{rs} \phi_r \phi_s \lambda_h^{(r)} \lambda_h^{(s)} - \sum_{rs} \phi_r \phi_s \lambda_h^{(r)} \lambda_h^{(s)},$$

which vanishes identically, we may write

$$\sum_k{}' T_{hk} = \sum_{rstu} a_{rs,tu} \lambda_h^{(r)} \lambda_h^{(t)} \left(\sum_k \lambda_k^{(s)} \lambda_k^{(u)} \right) + (n-1) \Delta\phi$$
$$- (n-2) \sum_{rs} \phi_r \phi_s \lambda_h^{(r)} \lambda_h^{(s)} - \sum_{rs} \phi_r \phi_s \left(\sum_k \lambda_k^{(r)} \lambda_k^{(s)} \right).$$

Using the last identity (45) and introducing the covariant system

$$\alpha_{rs} = \sum_{pq} a^{(pq)} a_{pr,qs}, \tag{47}$$

we finally get, after a change of indices,

$$\sum_k{}' T_{hk} = \sum_{rs} [\alpha_{rs} - (n-2) \phi_r \phi_s] \lambda_h^{(r)} \lambda_h^{(s)} + (n-2) \Delta\phi, \tag{48}$$

which is independent of k. We thus have the result:

If at a point P and to a direction $[h]$ in V_n we associate any $(n-1)$ other directions which form an orthogonal ennuple with it, the sum of the curvatures of the trajectory surfaces determined by the direction $[h]$ and each of the $(n-1)$ associated directions, is constant, i.e., *independent of the particular $(n-1)$ associated directions considered. This sum we shall designate by the term median trajectory curvature of V_n at the point P and in the direction $[h]$.*

If we sum (48) on h, we get

$$\sum_{hk} T_{hk} = \sum_{rs} [\alpha_{rs} - (n-2) \phi_r \phi_s] \cdot \sum_h \lambda_h^{(r)} \lambda_h^{(s)} + n(n-2) \Delta\phi,$$

or

$$\sum_{hk} T_{hk} = \sum_{rs} a^{(rs)} \alpha_{rs} + (n-1)(n-2) \Delta\phi. \tag{49}$$

The right member is a function of the coördinates of P only, and we may state the result:

The sum of the median trajectory curvatures associated with an arbitrary orthogonal ennuple at a point P is constant, i.e. *independent of the particular ennuple chosen.*

It is interesting to note that since the sum of the median curvatures of the corresponding geodesic surfaces at a point P is given by

$$\sum_{hk} G_{hk} = \sum_{rs} a^{(rs)} \alpha_{rs},$$

we have the relation

$$\sum_{hk} T_{hk} - \sum_{hk} G_{hk} = (n-1)(n-2)\Delta\phi. \tag{50}$$

Let us now ask for the directions $[h]$ through a point P for which the corresponding trajectory median curvature is a maximum or a minimum. Writing (48) in the form

$$\sum_k T_{hk} = \frac{\sum_{rs} [\alpha_{rs} - (n-2)\phi_r \phi_s] \lambda_h^{(r)} \lambda_h^{s}}{\sum_{rs} a_{rs} \lambda_h^{(r)} \lambda_h^{(s)}} + (n-2)\Delta\phi,$$

we note that the last term $(n-2)\Delta\phi$ is independent of the direction. Indicating by ρ the value of $\sum_k T_{hk}$ in the required direction, we have for the maximum or minimum condition, (as in §5),

$$\sum_s [\alpha_{rs} + (n-2)(a_{rs}\Delta\phi - \phi_r\phi_s) - \rho a_{rs}] \lambda_h^{(s)} = 0, \quad (r = 1, 2, \ldots, n); \tag{51}$$

hence, the ρ's must satisfy the determinantal equation

$$|\alpha_{rs} + (n-2)(a_{rs}\Delta\phi - \phi_r\phi_s) - \rho a_{rs}| = 0. \tag{52}$$

An equation of this form has, as is well known, n real roots, in general distinct, which when substituted into (51) determine n directions through the point P and, as in §5, it is a simple matter to show that these n directions are mutually orthogonal. We thus have the theorem:

At each point of V_n, there exist, in general, n mutually orthogonal directions corresponding to which the median trajectory curvature is a maximum or a minimum.

Analogous to the work of Ricci,[19] we shall name the directions thus determined at each point P, the *principal trajectory directions*, corresponding ennuples the *principal trajectory ennuples* or *principal trajectory congruences*, and the invariants ρ_h $(h = 1, 2, \ldots, n)$, the *principal trajectory invariants* or *principal trajectory median curvatures* of the space V_n.

From (51), we immediately deduce

$$(52) \qquad \alpha_{rs} + (n-2)\,(a_{rs}\Delta\phi - \phi_r\phi_s) = \sum_h \rho_h \lambda_{h|r}\lambda_{h|s},$$

the *canonical form for the symmetric covariant* $\alpha_{rs} + (n-2)\,(a_{rs}\Delta\phi - \phi_r\phi_s)$.

The corresponding canonical form which determines the principal invariants of Ricci, ρ_h' $(h = 1, 2, \ldots, n)$ is

$$(53) \qquad \alpha_{rs} = \sum_h \rho_h' \lambda_{h|r}' \lambda_{h|s}'.$$

Multiplying (52) by $a^{(rs)}$ and summing with respect to r and s, we obtain

$$\sum_{rs} a^{(rs)}\alpha_{rs} + (n-1)\,(n-2)\,\Delta\phi = \sum_h \rho_h.$$

Similarly, from (53) we obtain

$$\sum_{rs} a^{(rs)}\alpha_{rs} = \sum_h \rho_h'.$$

By comparison,

$$(54) \qquad \sum_h \rho_h = \sum_h \rho_h' + (n-1)\,(n-2)\,\Delta\phi.$$

This gives *the relation between the sums of the principal invariants and the principal trajectory invariants at a point P in V_n.*

7. **Characterization of the Principal Trajectory Invariants and Directions.** We may characterize the principal trajectory invariants and directions as follows. Ricci has defined the *coefficients of rotation*[20] in terms of the directions of an orthogonal ennuple [1], [2], . . ., [n] as

$$\gamma_{hij} = \sum_{rs} \lambda_{h|rs}\lambda_i^{(r)}\lambda_j^{(s)},$$

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[19] *Direzione e invarianti principali in una varietà qualunque.* Atti del R. Ist. Veneto di Scienze, Lettere, ed Arti, t. LXIII, (1903–4).

[20] Cf. reference in footnote 19.

where $\lambda_{h/rs}$ is the covariant derivative of $\lambda_{h/r}$ with respect to s. From these we form the derived quantities

$$\gamma_{hi,kl} = \frac{\partial \gamma_{hik}}{\partial s_l} - \frac{\partial \gamma_{hil}}{\partial s_k} + \sum_j \{\gamma_{hij}\ (\gamma_{jkl} - \gamma_{jlk}) + \gamma_{jhl}\gamma_{jik} - \gamma_{jhk}\gamma_{jil}\}.$$

From these it may be shown that

$$\gamma_{hi,kl} = \sum_{rstu} a_{rs,tu}\, \lambda^{(r)}_h\, \lambda^{(s)}_i\, \lambda^{(t)}_k\, \lambda^{(u)}_l,$$

or, by solving for the Riemann symbols,

$$a_{pr,qs} = \sum_{hikl} \gamma_{hi,kl}\, \lambda_{h/p}\, \lambda_{i/r}\, \lambda_{k/q}\, \lambda_{l/s}.$$

Applying this to (47) we may write

$$(55) \qquad \alpha_{rs} = \sum_{pq} a^{(pq)}\, a_{pr,qs} = \sum_{hil} \gamma_{hi,hl}\, \lambda_{i/r}\, \lambda_{l/s}.$$

Now if β_i is the angle between the trajectory vector $\phi^{(r)}$ and the congruence $[i]$, then

$$\cos\beta_i = \frac{1}{\sqrt{\Delta\phi}} \sum_r \phi_r \lambda^{(r)}_i,$$

or

$$\phi_r = \sqrt{\Delta\phi} \sum_i \cos\beta_i\, \lambda_{i/r},$$

and hence,

$$(56) \qquad \phi_r\, \phi_s = \Delta\phi \sum_{il} \cos\beta_i \cos\beta_l\, \lambda_{i/r}\, \lambda_{l/s}.$$

Furthermore,

$$a_{rs} = \sum_i \lambda_{i/r}\, \lambda_{i/s}.$$

Hence,

$$(57) \quad \alpha_{rs} + (n-2)\,(a_{rs}\,\Delta\phi - \phi_r\,\phi_s) =$$

$$\sum_i \left[\sum_h \gamma_{hi,hi} + (n-2)\,\Delta\phi \sin^2\beta_i \right] \lambda_{i/r}\, \lambda_{i/s}$$

$$+ \sum_{il}^{i \neq l} \left[\sum_h \gamma_{hi,hl} - (n-2)\,\Delta\,\phi \cos\beta_i \cos\beta \right] \lambda_{i/r}\, \lambda_{l/s}.$$

Comparing this with the canonical form for the left member as given by (52), we find

$$(58)\qquad \sum_h \gamma_{hi,hi} + (n-2)\,\Delta\,\phi\,\sin^2\beta_i = \rho_i\,,$$

which define the principal trajectory invariants ρ_i, and

$$(59)\qquad \sum_h \gamma_{hi,hl} - (n-2)\,\Delta\phi\,\cos\beta_i\cos\beta_l = 0, \quad (i \neq l),$$

which serve to characterize the principal trajectory ennuples of any V_n.

8. **Principal Trajectory Directions when n = 3.** In § 5, we derived, for the case $n = 3$, certain relations by means of a direct discussion of the curvatures of trajectory surfaces at a point P, whereas in § 6, we derived similar relations by means of a discussion of the median trajectory curvature at a point P. We may ask how the directions and invariants found in these two discussions are related. For this purpose, let us study the canonical forms

$$(40)\qquad \beta_{rs} + \phi_r\,\phi_s = \sum_h \omega_h\,\zeta_{h/r}\,\zeta_{h,s}\,,$$

and

$$(52)\qquad \alpha_{rs} + (a_{rs}\,\Delta\,\phi - \phi_r\,\phi_s) = \sum_h \rho_h\,\lambda_{h/r}\,\lambda_{h/s}\,,$$

which serve to characterize the two sets of directions. Here the $\beta^{(rs)}$ are defined by

$$(32')\qquad \beta^{(rs)} = \frac{a_{r+1\,r+2,\,s+1\,s+2}}{a},$$

and the α_{rs} by

$$(47)\qquad \alpha_{rs} = \sum_{pq} a^{(pq)}\,a_{pr,qs}\,,$$

(all summations being carried from 1 to 3 for the indicated subscripts). Using the properties of the Riemann symbol, we have

$$\begin{aligned}\alpha_{rs} = {} & a^{(r+1\,s+1)}\,a_{r+1\,r,\,s+1\,s} + a^{(r+2\,s+1)}\,a_{r+2\,r,\,s+1\,s} + \\ & \qquad a^{(r+1\,s+2)}\,a_{r+1\,r,\,s+2\,s} + a^{(r+2\,s+2)}\,a_{r+2\,r,\,s+2\,s} \\ = {} & a[a^{(r+1\,s+1)}\,\beta^{(r+2\,s+1)} - a^{(r+2\,s+1)}\,\beta^{(r+1\,s+2)} - a^{(r+1\,s+2)}\,\beta^{(r+2\,s+1)} + \\ & \qquad a^{(r+2\,s+2)}\,\beta^{(r+1\,s+1)}].\end{aligned}$$

But

$$a\,.\,a^{(r+1\,s+1)} = \begin{vmatrix} a_{rs} & a_{r\,s+2} \\ a_{r+2\,s} & a_{r+2\,s+2} \end{vmatrix}, \text{ etc.}$$

Hence

$$\alpha_{rs} = \sum_{pq} \beta^{(pq)} (a_{rs} a_{pq} - a_{ps} a_{qr}), \text{ where } \left\{ \begin{array}{l} p = r+1, r+2, \text{ mod. } 3 \\ q = s+1, s+2, \text{ mod. } 3 \end{array} \right\}$$

$$= a_{rs} \sum_{pq} a_{pq} \beta^{pq} - \sum_{pq} a_{ps} a_{qr} \beta^{(pq)}$$

$$= a_{rs} \sum_{pq} a^{(pq)} \beta_{pq} - \beta_{rs}.$$

Now, we have shown, by (42), that

$$\sum_{pq} a^{(pq)} \beta_{pq} + \Delta \phi = \sum_h \omega_h = L,$$

which is independent of the directions through P; therefore

$$\alpha_{rs} = a_{rs} (L - \Delta\phi) - \beta_{rs}.$$

But

$$a_{rs} = \sum_h \zeta_{h|r} \zeta_{h|s},$$

hence,

$$\beta_{rs} = \sum_h (L - \Delta\phi) \zeta_{h|r} \zeta_{h|s} - \alpha_{rs};$$

substituting this into (40), we have the equivalent canonical form for determining the invariants ω,

(40′) $$\alpha_{rs} + (a_{rs} \Delta \phi - \phi_r \phi_s) = \sum_h (L - \omega_h) \zeta_{h|r} \zeta_{h|s}.$$

Finally, comparing the two canonical forms (40′) and (52) for the same expression $\alpha_{rs} + (a_{rs} \Delta\phi - \phi_r \phi_s)$, we deduce that the directions $\zeta_h^{(i)}$ coincide with the directions $\lambda_h^{(i)}$ and that the invariants ω_h and ρ_h are related by

(60) $$\rho_h = \omega_1 + \omega_2 + \omega_3 - \omega_h, \qquad (h = 1, 2, 3).$$

This may be written in the equivalent forms

(61) $$\left\{ \begin{array}{l} \rho_h = \omega_{h+1} + \omega_{h+2} \\ 2\omega_h = \rho_{h+1} + \rho_{h+2} - \rho_h \end{array} \right., \quad (h = 1, 2, 3)$$

We may then state the result:

In any space of 3 dimensions, the principal trajectory directions defined by means of the principal trajectory curvatures and by means of the principal trajectory median curvatures coincide for every point of the space; in other words, the principal trajectory congruences thus deter-

mined coincide. The relations between the two sets of invariants ω_h *(the principal trajectory curvatures) and* ρ_h *(the principal trajectory median curvatures) are given by* (61).

It is interesting to note that due to the absence of the function ϕ from equations (61), the relations between the principal trajectory invariants ρ and ω have exactly the same form as the relations between the principal invariants ρ' and ω'.

9. **A Generalization. Velocity Systems.** We note that if in equations (3), § 2,

$$\frac{d\xi^{(i)}}{ds} + \sum_{\lambda\mu} \left\{ {\lambda\mu \atop i} \right\} \xi^{(\lambda)} \xi^{(\mu)} = \phi^{(i)} - \xi^{(i)} \sum_k \phi_k \xi^{(k)}, \quad (i = 1, 2, \ldots, n), \tag{3}$$

the quantities ϕ_i are not taken as the partial derivatives of a function ϕ with respect to the coördinates, but as components of an arbitrary point vector, these equations no longer represent a system of curves defined by a variation problem of the form

$$\delta \int e^{\phi}\, ds = 0. \tag{2}$$

In this case the curves are said to form a *velocity system*, and a dynamical interpretation of these is found in the following statement:— A curve is a velocity curve corresponding to a given constant speed c, if a particle starting from any point of such a curve in the direction of the curve and with that speed, describes a trajectory, in a field under any positional forces, osculating the curve; in the above equations, these forces have as components the quantities $c^2\phi_i$.[21]

Now, all our results of §§ 2, 3, 5–8 were derived without the assumption that $\phi_i = \dfrac{\partial \phi}{\partial x_i}$ $(i = 1, 2, \ldots, n)$, so that all these results are valid if trajectories and trajectory surfaces are replaced by velocity curves and their corresponding surfaces.

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[21] Cf. the author's *Note on velocity systems in a curved space of N dimensions*, Bull. Am. Math. Soc., Vol. 27, 1920, pp. 71–77.

For a geometric characterization of velocity systems see the author's paper referred to in footnote 9.

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PIEZOELECTRIC CRYSTAL RESONATORS AND CRYSTAL OSCILLATORS APPLIED TO THE PRECISION CALIBRATION OF WAVEMETERS.

By George W. Pierce.

PIEZOELECTRIC CRYSTAL RESONATORS AND CRYSTAL OSCILLATORS APPLIED TO THE PRECISION CALIBRATION OF WAVEMETERS.

BY GEORGE W. PIERCE.

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I. INTRODUCTION.

Professor W. G. Cady's paper [1] on the use of the mechanical vibration frequencies of piezoelectric crystals as standards of frequency of electric oscillations came to my attention while I was engaged in attempting a precision calibration of the wavemetric standards of Harvard University.

Professor Cady's suggestions proved to be of great value in this work, and it is thought that the interest and utility of the piezoelectric devices justifies the publication of this report.

Piezoelectric Resonators and Oscillators.— Cady's paper is devoted chiefly to the *piezoelectric resonator*, which gives a *transient* response when an electric oscillating circuit is made to pass through a period synchronous with the mechanical period of a parallelopiped cut from a piezoelectric crystal. He gives, however, a description also of the *piezoelectric oscillator*, which gives *sustained* oscillations. In Cady's apparatus this oscillator consists of a parallelopiped of quartz crystal with two pairs of electrodes,— one to be connected into the first grid circuit and the other into the last plate circuit of an amplifier train with resistance coupling between stages. With this device he obtained *sustained oscillations* with a period fixed by the mechanical period of the quartz alone.

In my experiments I have been able by a change of connections to use a plate of piezoelectric quartz connected to a single vacuum tube (with or without inductance or capacity in the circuit) to constitute a *piezoelectric oscillator* (described in II) capable of giving sustained electric oscillation with a constant frequency determined by the frequency of the crystal alone. By means of this single fixed frequency and its harmonics used in coöperation with a variable electric oscillator and its harmonics I have found it possible to calibrate completely and by fine steps a series of wavemeters extending in range from 50 meters (6000 kilocycles per second) to 50000 meters (6 kilocycles per second), and to obtain finally the absolute period of the crystal oscillator itself in terms of a standard clock by the intermediation of a tuning fork.

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[1] W. G. Cady: The Piezoelectric Resonator, Proc. Inst. Radio Engineers, Vol. 10, p. 83, 1922.

Having obtained this calibration with an accuracy of about one-tenth of one per cent over the entire range, a number of crystal resonators (made according to Cady's description) have been standardized as to fundamental and partial frequencies.

II. Improved Piezoelectric Oscillator.

The piezoelectric oscillator here employed has the circuit shown in Figure 1. The plate circuit of a vacuum tube contains posts, shunted by the bypass condenser C, of about .001 microfarad, to which a telephone or the *input* of an audio frequency amplifier may be connected, in series with a B-Battery and a resistance load of 30,000 Ω. The grid is connected to the filament by a leak resistance of about one megohm; and the piezoelectric crystal provided with electrodes is connected between the plate and grid.

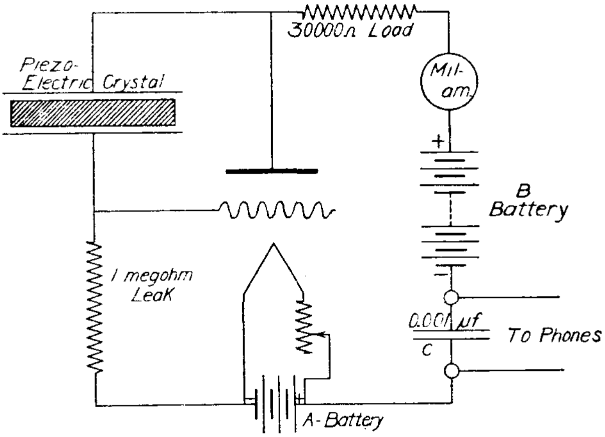

Figure 1. Diagram of Connections of Improved Piezoelectric Oscillator.

In practice the 30,000 Ω plate resistance, which will be called the "Load," may be replaced by an inductance of any value if large enough (say greater than 10 milhenries for a crystal of period 420,000 cycles per second). This inductance may be used to couple the piezoelectric oscillator to any other circuit. Beats with another circuit are, however, often loud and clear with no coupling other than static effects, so that the resistance " load," or even a very large iron-core inductance may be used at the position marked " load."

Mounting for Quartz Crystal of Piezoelectric Oscillator.— A plate of quartz of thickness 0.5 cm., width 4.5 cm. and length 6.2 cm. on top surface and 7.0 cm. on bottom surface, was mounted, as in Figure 2, loosely within a hard rubber frame which was a little thicker than the quartz. Sheets of brass to serve as electrodes were screwed to the two faces of the frame.

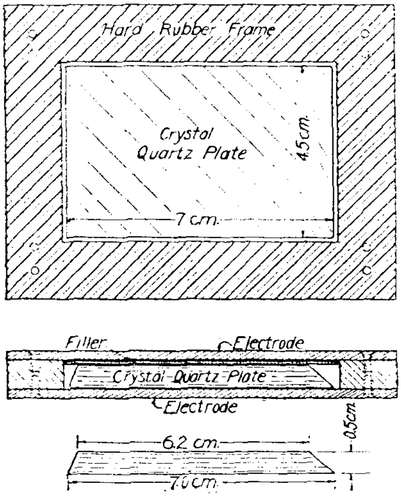

FIGURE 2. Mounting of Quartz Plate.

A thin sheet of spring brass (marked " filler ") was inserted between one of the electrodes and the quartz, in such a way that only a light pressure was exerted on the quartz. The particular piece of quartz shown in Figure 2 was supplied to me by Professor E. L. Chaffee from his optical collection, and was employed without modification. This accounts for its peculiarly beveled ends, which are for present purposes accidental.

The optic axis of the specimen was determined experimentally to be parallel to the 7-cm. edge. Other facts regarding the original orientation of the plate in the natural crystal have not been determined.

The specimen when tested as a piezoelectric *resonator* exhibits six or eight prominent resonant frequencies and many hundred resonant frequencies of weaker response. When connected between the plate and grid of the vacuum tube as in Fig. 1, so as to constitute an *oscillator*, the apparatus gives persistently sustained electric oscillations of frequency 419,640 cycles per second, as was found by methods described below. This frequency is probably the mechanical frequency of elastic vibration of compression in the direction of the shortest dimension of the plate, although this period is also conditioned somewhat by the length and breadth of the plate.

The persistent oscillatory current in the circuit is evidenced by a drop of the current in the plate milammeter when the crystal is connected, or by heterodyne effects when used in proximity to another oscillator.

Different tubes and plate voltages were employed. For wavemeter calibration a Radiotron UV 199 with a 44-volt plate battery was highly satisfactory. With a power-tube and a 500-volt plate battery several watts of oscillatory power could be obtained.

Constancy of Frequency.— The quartz oscillator was found not to change its frequency by as much as three one hundredths of one per cent for a change of temperature from 8°C to 24°C. Also a weight of 2 Kg. applied as a pressure to one of the electrodes did not change the frequency perceptibly. Variation of filament current, plate voltage, resistance, inductance, capacity, or any kind of electrical tuning of the piezoelectric oscillator had no effect as great as three one hundredths of one per cent on the frequency of the oscillation.

III. Precision Calibration of Wavemeter.

The general arrangement of apparatus for the calibration of a wavemeter in terms of the constant oscillation frequency of the piezoelectric oscillator is shown in Fig. 3. At WM is shown the wavemeter consisting of a General Radio Precision Condenser, Type 222, and having at L, interchangeable coils known in this report as coils A, B, C, D, E, F, G, and No. 5. At *O* in the figure is shown an *electric oscillator* having a "VT 1" vacuum tube, a 44-volt B-Battery, interchangeable three terminal coils at *M* used in a Hartley circuit, a gear-driven air condenser and vernier condenser for tuning, with the control handles (not shown) of these condensers projecting through the metal shield in the figure. The electric oscillator *O* had a milammeter (total scale 5 milamperes) in circuit with the grid. This milammeter was read by a large fixed lens, or reading glass, and was used to show reaction when the wavemeter *WM* was brought into resonance with the oscillator *O*. At *S* is shown the *piezoelectric oscillator* of Figure 1, used with a two-stage audio amplifier and loud speaker. The primary coil of the input transformer of the audio amplifier is connected to the terminals marked "To Phones" in Figure 1. This permits the use of a loud speaker (Magnavox or Western Electric) so that head receivers with their uncertain effects on frequency need not be worn by the observer. The coil *N* has an inductance of 20 or 125 millihenries (either could be used without any change of frequency of the piezoelectric oscillator *S*), and was connected as "Load" in Fig. 1, to replace the 30,000 ohms of resistance.

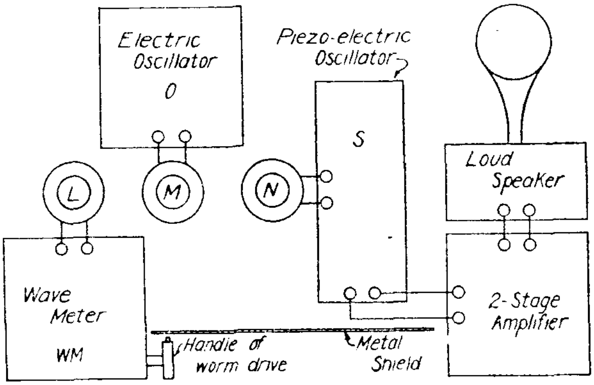

FIGURE 3. Arrangement of Apparatus for Calibration of Wavemeter.

The procedure of calibration was as follows: the electric oscillator *O* was adjusted in frequency by varying its condenser, beats were obtained between the frequency of *O*, or one of its harmonics, with the frequency of *S*, or one of its harmonics, and the adjustment was brought to the beat zero.

The wavemeter at *WM* having been stationed far enough away from *M* to give only a slight reaction was open circuited and left in position during the preceding adjustment of *O*. Its circuit was then closed and adjusted to resonance with *O*. The resonance was indicated by a small decrease of deflection of the grid milammeter of *O*, or by the absence of a beat note in the loud speaker, whereas for a slight departure from resonance the reaction of the wavemeter upon the coil *M* caused a note. After proper arrangement of the metal shield and control handles so as to keep the observer at a distance from the electric oscillator O, readings consistent to about one tenth of one per cent could be obtained.

Identification of Inter-harmonic Resonant Frequencies.—Beats occurred when any harmonic of the electric oscillator O (up to the 70th harmonic) was nearly of the same frequency as any harmonic of the piezoelectric oscillator S (up to the 29th harmonic), and a beat zero was obtained when the frequencies of these two harmonics (or fundamentals) were the same.

To identify these harmonics note was taken of the intensities of the beats, and such values as $n=1$, 2, $\frac{1}{2}$, $\frac{1}{3}$ etc., where $n=$ ratio of wave-length of O to wave length of S, were easily identified by their great intensity and their sequence.

The more complicated resonance relationships, such as $n = 8/29$, indicating that the 8th harmonic of O was resonant with the 29th harmonic of S, could be identified by the use of the interpolation formula

$$\lambda^2 = aC^\circ + b, \tag{1}$$

or

$$n^2 = \alpha C^\circ + \beta, \tag{2}$$

where

λ = wave length of oscillator O,
C° = divisions of wavemeter condenser,
n = ratio of wave length of oscillator O to wave length of oscillator S,
a, b, or α, β = constants determined by two simple values such as values of C° for $n = 1/6$ and $n = 1/3$.

This formula (1) or (2) holds only for the parts of the condenser scale where the capacity calibration against divisions is linear. Outside of this range values for different wavemeter coils were intercompared.

Further facts regarding the interpolation formula are treated later in this report.

Example: Calibration of Wavemeter with Coil B and C.—As an example of this operation, data for coil " B " and " C " are given in Tables I and II. The first column contains settings of the wavemeter condenser (in divisions) corresponding to various adjustments of the oscillator O which gave beat zero with the oscillator S. The second column contains the values of n finally arrived at. The determination of these values of n for C° greater than 3.5 was effected by using formulas of the type of (2) which, with constants determined from easily recognizable values of n, are

TABLE I.

Coil B.

C° Wavemeter Condenser Divisions	n	n^2 observed	n^2 calculated by (3b)	Percentage Departure
3.530	3/11	.07437	.07433	− .06
3.863	2/7	.08163	.08170	+ .12
4.135	8/27	.08779	.08790	+ .14
4.230	3/10	.09000	.09004	+ .03
4.440	4/13	.09467	.09480	+ .14
4.783	8/25	.1024	.1024	.00
5.173	1/3	.1111	.1112	+ .10
5.603	8/23	.1210	.1209	− .08
6.122	4/11	.1322	.1325	+ .20
6.685	8/21	.1451	.1451	.00
7.356	2/5	.1600	.1602	+ .10
7.948	5/12	.1736	.1735	− .06
8.117	8/19	.1773	.1773	.00
8.405	3/7	.1837	.1837	.00
9.023	4/9	.1975	.1976	+ .05
9.442	5/11	.2066	.2070	+ .20
10.090	8/17	.2215	.2216	+ .05
11.361	1/2	.2500	.2501	+ .04
12.886	8/15	.2844	.2843	− .03
13.450	6/11	.2975	.2970	− .16
13.936	5/9	.3086	.3079	− .23
14.769	4/7	.3265	.3266	+ .03
15.362	7/12	.3403	.3399	− .13
16.240	3/5	.3600	.3596	− .11
17.052	8/13	.3787	.3778	− .24
18.240	7/11	.4049	.4045	− .10
20.017	2/3	.4444	.4444	.00
22.052	7/10	.4900	.4900	.00
23.811	8/11	.5289	.5295	+ .12

TABLE II.

Coil C.

C^c Wavemeter Condenser Divisions	n	n^2 observed	n^2 calculated by (3c)	Percentage Departure
.638	4/13	.09467	These values of n were identified by intercomparison with Coil B.	
.979	8/25	.1024		
1.158	1/3	.1111		
1.335	8/23	.1210		
1.480	4/11	.1322		
1.651	8/21	.1451		
1.841	2/5	.1600		
1.945	7/17	.1695		
2.059	8/19	.1773		
2.304	4/9	.1975		
2.571	8/17	.2215		
2.896	1/2	.2500		
3.268	8/15	.2844		
3.754	4/7	.3265	.3263	.06
4.130	3/5	.3600	.3604	.1
4.322	8/13	.3787	.3778	.3
5.055	2/3	.4444	.4441	.07
5.800	5/7	.5120	.5115	.1
6.007	8/11	.5289	.5303	.25
6.368	3/4	.5625	.5625	.00
7.211	4/5	.6400	.6394	.1
8.872	8/9	.7901	.7898	.04
11.195	1	1.000	1.0001	.01
15.18	7/6	1.362	1.362	.00
16.050	6/5	1.440	1.440	.00
17.39	5/4	1.563	1.561	.13
19.78	4/3	1.778	1.778	.00
21.830	7/5	1.960	1.962	.1
22.675	10/7	2.041	2.040	.05

$$n^2 = .022444C^\circ - .0049, \qquad \text{and} \qquad (3b)$$
$$n^2 = .090551C^\circ - .0136, \qquad (3c)$$

for coils " B " and " C," respectively.

Values of n^2 calculated from these formulas are recorded in column four, and the simple common fractions with squares nearest to them are inserted in column two. It is seen that with the exception of the values at two or three points the two values of n^2 check within two tenths of one per cent. Since n^2 is proportional to λ^2, this identifies λ to 1/10 of one per cent in its relation to the fundamental wave length of the crystal oscillator.

In case of coil " *C* " values of n for which C° is less than 3.5 were identified comparison with results obtained by the use of the smaller wavemeter coil " B," with which they came within the range of condenser settings for which the linear formula (3b) held.

Similar observations were made with coils " A," " D," " E," " F," " G " of the wavemeter. Finally by the use of coil " No. 5 " ($\frac{1}{2}$ henry) the calibration was extended into the range of audible frequencies, so that the absolute value of the fundamental frequency of the piezoelectric oscillator could be obtained by comparison with a vacuum-tube driven tuning fork.

Concerning Complete Range.— If

λ_0 = electric wave length corresponding to the fundamental mechanical frequency of the piezoelectric oscillator *S*, and

λ = any wave length of the oscillator *O* which gives a beat zero, then

$\frac{\lambda}{\lambda_0} = n = \frac{p}{q}$, say, where p and q are integral numbers. This means that the pth harmonic of the electric oscillator *O* is of the same frequency as the qth harmonic of the piezoelectric oscillator *S*.

In Table III, some of the combinations of p and q that were actually employed are indicated by an " x "; for example, an " x " at the intersection of the row for $p = 9$ with the column for $q = 11$ indicates that a wave length $\lambda = \frac{9}{11}\lambda_0$ was a calibration point. It is seen that the table extends from $\lambda = \frac{\lambda_0}{13}$ to $\lambda = 70\,\lambda_0$ and this will be seen to be

TABLE III.

Showing What Wavelength Ratios Were Employed.

		q												
		1	2	3	4	5	6	7	8	9	10	11	12	13
p	1	x	x	x	x	x	x	x	x	x	x	x	x	x
	2	x	x	x	x	x	x	x	x	x	x	x	x	x
	3	x	x	x	x	x	x	x	x	x	x	x	x	x
	4	x	x	x	x	x	x	x	x	x	x		x	
	5	x	x	x	x	x		x		x	x	x	x	
	6	x	x	x	x	x	x		x	x	x	x	x	
	7	x	x	x	x	x	x	x			x	x	x	
	8	x	x		x	x	x		x		x	x	x	to $q-29$
	9	x	x	x	x	x	x			x			x	
	10	x	x	x	x	x	x	x	x		x		x	
	11	x	x	x		x		x				x		
	12	x	x	x	x		x		x	x	x		x	
	13	x	x				x		x					x
	14	x	x		x		x	x	x		x		x	
	15	x	x	x		x	x	x		x	x		x	
	16	x												
	17	x												
	18	x												
	19	x												
	.	x												
	70	x												

from 50 meters to 50430 meters as soon as λ_0 is obtained in the next section.

The range can very easily be extended particularly in the direction of the shorter waves, and this will be done as soon as the proper wavemeter coils are constructed.

The oscillators for this work have been made up in a permanent form so that these wavemeters may at any time be recalibrated by a series of observations requiring about one-half hour for each wavemeter coil and condenser.

Determination of Absolute Value of Frequency of Piezoelectric Oscillator.— A vacuum-tube drive tuning fork was investigated for me by Professor F. A. Saunders by the use of a phonodyke and found to have a frequency F = 996.3 cycles per second at 21°C. Professor E. L. Chaffee and I measured the same fork by driving an iron gear wheel containing 40 teeth near the poles of a telephone receiver and controlling the speed of the driving motor by a Leeds and Northrup tuning-fork control-apparatus, adjusted so that the frequency of current generated by the toothed wheel synchronized with the vacuum-tube driven fork. The motor speed was then chronographed 3000 seconds. By this we obtained $F = 995.9$.

The average of our value and Professor Saunders's value is

$$F = 996.1 \ (\pm .2) \text{ cycles per sec.}$$

The vacuum-tube driven fork here employed differs from that of Professor Eccles in that a microphone button was used to modulate the current to the grid circuit. The advantage of the microphone control is its simplicity of construction and the absence of the possibility of self-excitation by induction independent of the fork. This microphone button was of the inertia-back type and was mounted on the heavy block to which the base of the tuning fork was attached, so that changes of the distribution of mass in the microphone did not affect the period of the fork.

The essentials of the circuit employed with the vacuum-tube driven fork are shown in Figure 4. The apparatus was made by an easy modification of a General Radio 1000-cycle microphone-driven fork. The microphone button was taken from the prong of the fork where it occurs in the General Radio apparatus, and was mounted at its base, marked " Mic." in Figure 4. A 4-volt battery was connected in series with the microphone button and the low side of a modulation transformer. The high side of this transformer was connected to the grid and filament of the vacuum tube, of which the plate circuit contained the B-Battery and driving coil (wound to a high resistance). A condenser of about .2 microfarad was connected in shunt to the driving coil.

In the work of the absolute calibration it was found convenient to dispense with the tube whose elements are shown at *P*, *G*, and *F*,

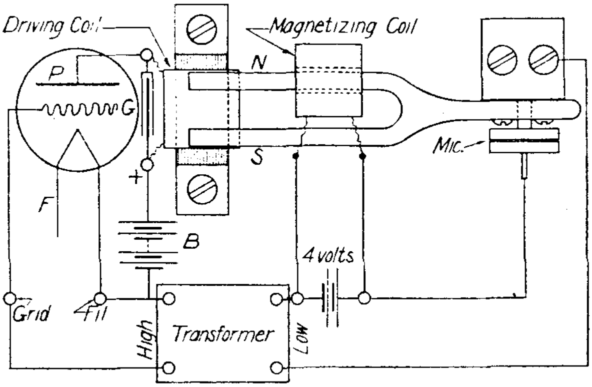
Driving Coil
Magnetizing Coil
N
P
G
S
Mic.
F
+
B
4 volts
Grid
Fil
High
Transformer
Low

FIGURE 4. Circuits of Vacuum-tube Driven Tuning Fork.

Figure 4, and drive the fork with the two-stage amplifier needed for other purposes with the apparatus, as is shown in Figure 5.

The high-side of the modulation transformer of the tuning fork (see Fig. 4) was connected in series with the plate circuit of the electric

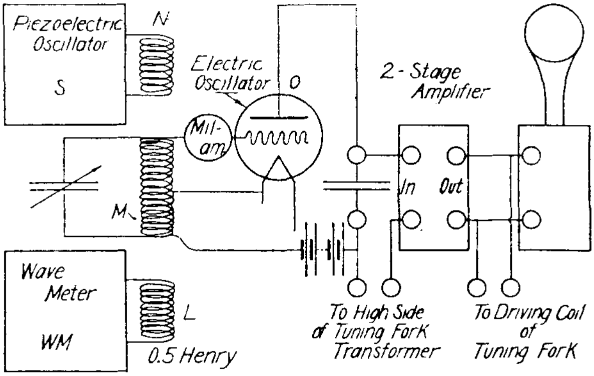
Piezoelectric Oscillator
S
N
Electric Oscillator
O
2-Stage Amplifier
Mil-am
In
Out
M
Wave Meter
WM
L
0.5 Henry
To High Side of Tuning Fork Transformer
To Driving Coil of Tuning Fork

FIGURE 5. Arrangement of Apparatus for Absolute Standardization of Piezoelectric Oscillator.

oscillator *O* and the input terminals (a transformer primary) of the amplifier (see Fig. 5). The terminals of the driving coil of the tuning fork (Fig. 4) were connected to the output terminals of the amplifier in parallel with the coil of the loud speaker. With this arrangement the amplifier amplified the output of oscillator *O* and also maintained the vibration of the tuning fork.

In Figure 5 approximate positions of the wavemeter *WM* and the piezoelectric oscillator *S*, relative to the electric oscillator *O* are shown. The coil *L* of the wavemeter in this case was $\frac{1}{2}$ henry; the condenser of the wavemeter was the General Radio Precision Air Condenser used in the work described above. The purpose of the oscillator *O* and of the wavemeter in Figure 5 is to permit interpolations between the various multiples of the frequency of the tuning fork and the various fractions of the frequency of the Piezoelectric Oscillator *S*. The procedure was as follows:

The approximate wavemeter setting for the 6th harmonic of the tuning fork (called 6F) was identified, and the oscillator *O* was adjusted so that its fundamental gave a beat zero with this 6th harmonic. The wavemeter *WM* was then resonated to the oscillator by giving its condenser the value 21.817 divisions. We then proceeded with other harmonics of the tuning fork obtaining Table IV.

Following the interpolation law of equation (1) we find that the data of Table IV satisfy with an accuracy of $\frac{1}{10}$ of one per cent the equation

TABLE IV.

Wavemeter (Coil No. 5) in Terms of Tuning Fork.

m F Frequency	C° Wavemeter Condenser Divisions	*m* observed	*m* calculated
6 F	21.817	6	6.000
7 F	15.529	7	7.003
8 F	11.477	8	8.001
9 F	8.693	9	8.999
10 F	6.715	10	9.990
11 F	5.232	11	10.99
12 F	4.104	12	12.00
13 F	3.216	13	13.01

$$\frac{1}{m^2} = .0011758\,C^\circ + .002126, \tag{4}$$

where C° = divisions of wavemeter condenser,
m = coef. of F in Column I (Table IV),
F = frequency of the tuning fork,

as may be seen by a comparison of the third and fourth columns of Table IV.

The next operation consisted in obtaining beat zeros between harmonics of the Electric Oscillator O and the fundamental of the Piezoelectric Oscillator S in the range of frequencies covered by Table IV. This gave the results recorded in Table V.

Here $f_0/70$ means that the 70th harmonic of the electric oscillator gave a beat zero with the fundamental frequency f_0 of the piezoelectric oscillator.

In this table the values of k in the first and third columns were identified by counting beat zeros all the way up from $k=1$. Their identity was established by two independent runs. The second column contains observed values of C° the resonant settings of the wavemeter condenser in divisions, when tuned to the electric oscillator O.

The observations satisfy the empirical interpolation equation

$$k^2 = 208.7\,C^\circ + 377.7, \tag{5}$$

as is seen by a comparison of the third and fourth columns. It is to be noted that an error of 0.2% in k^2 makes an error of only 0.1% in frequency.

If now we divide (5) by f_0^2 and divide (4) by F^2 we have

$$\frac{k^2}{f_0^2} = \frac{208.7C^\circ}{f_0^2} + \frac{377.7}{f_0^2}, \tag{6}$$

and

$$\frac{1}{m^2F^2} = \frac{.0011758C^\circ}{F^2} + \frac{.002126}{F_0^2}. \tag{7}$$

The left hand side of both of these equations is the square of the reciprocal of the frequency corresponding to a given value of C°; hence, equating coefficients, we have

$$\frac{208.7}{f_0^2} = \frac{.0011758}{F^2} \tag{8}$$

TABLE V.

Frequency of Wavemeter (Coil No. 5) in Terms of Fundamental Frequency of Piezoelectric Oscillator.

f_0/k Frequency	C° Wavemeter Condenser Divisions	k^2 obs	k^2 calc	f_0/k Frequency	C° Wavemeter Condenser Divisions	k^2 obs	k^2 calc
$f_0/70$	21.688	4900	4904	$f_0/50$	10.152	2500	2496
69	21.022	4761	4765	49	9.694	2401	2401
68	20.325	4624	4620	48	9.227	2304	2303
67	19.633	4489	4475	47	8.760	2209	2206
66	19.066	4356	4357	46	8.333	2116	2117
65	18.397	4225	4217	45	7.881	2025	2022
64	17.803?	4096	4093	44	7.462	1936	1935
63	17.191	3969	3965	43	7.047	1849	1848
62	16.608	3844	3844	42	6.598	1764	1755
61	16.013	3721	3720	41	6.257	1681	1684
60	15.420	3600	3596	40	5.849	1600	1598
59	14.852	3481	3477	39	5.501	1521	1526
58	14.298	3364	3362	38	5.113	1444	1445
57	13.68?	3249	3233	37	4.753	1369	1370
56	13.194	3136	3131	36	4.388	1296	1294
55	12.665	3025	3021	35	4.071	1225	1227
54	12.131	2916	2909	34	3.725	1156	1155
53	11.650	2809	2809	33	3.391	1089	1085
52	11.141	2704	2703	32	3.084	1024	1021
51	10.640	2601	2598	31	2.795	961	961

and

$$\frac{377.7}{f_0^2} = \frac{.002126}{F^2}. \tag{9}$$

Equation (8) gives $f_0 = 421.28\ F.$ (10)

Equation (9) gives $f_0 = 421.50\ F.$ (11)

Giving greater weight to (10) we may consider

$$\begin{aligned} f_0 &= 421.3\ F \\ &= 421.3 \times 996.1 \\ &= 419650 \text{ cycles per second;} \qquad (12) \end{aligned}$$

whence $$\lambda_0 = 714.9 \text{ meters.} \qquad (13)$$

In these equations f_0 is the frequency of mechanical vibration of the piezoelectric oscillator and λ_0 the corresponding wave length in free space of the electric wave of frequency f_0.

IV. THE LINEAR INTERPOLATION FORMULA.

Expression of Results of Calibration in Terms of Condenser Divisions.— Having now obtained the absolute value of the frequency f_0 of the piezoelectric oscillator and of the corresponding wavelength λ_0, and having found that an interpolation formula of the type of equation (2) expresses the results with an accuracy of one tenth of one per cent for values of condenser for which its capacity is linear in terms of division, C°, we may change equations such as (3b) and (3c) and (5) into equations for λ^2 by multiplying both sides of these equations by $\lambda^2_0 = (714.9 \text{ meters})^2 = (.7149 \text{ km.})^2$.

This gives within the limits prescribed, the following results for the several wavemeter coils (Table VI).

TABLE VI.

Equations Connecting Wavelength λ in Kilometers with Condenser Divisions C° Between C° = 3.50 and C° = 23.00.

Coil	Equation
A	$\lambda^2 = .0024778\ C^\circ - .0005724$
B	$\lambda^2 = .011471\ C^\circ - .002504$
C	$\lambda^2 = .046279\ C^\circ - .006951$
D	$\lambda^2 = .2181\ C^\circ - .030$
E	$\lambda^2 = 1.1084\ C^\circ - .024$
F	$\lambda^2 = 4.4597\ C^\circ + .2555$
G	$\lambda^2 = 9.5868\ C^\circ$
No. 5	$\lambda^2 = 106.66\ C^\circ + 193.35$

For values of C° outside of the specified range at the heading of Table VI, values were obtained also by intercomparison of different coils, and the whole result has been compiled in tables not here given.

Expression of Results of Calibration in Terms of Condenser Capacity.— Within the range of wavemeter condenser No. 14 between

3.50 and 23.00 divisions its capacity C in micromicrofarads in terms of divisions C° to better than .2% is given by the linear equation

$$C = 59.60C^\circ - 19.60, \tag{14}$$

so that the equations of Table VI, with C° in divisions changed to equivalent values in terms of C (in micromicrofarads) take the values of Table VII.

TABLE VII.

Equations Connecting Wavelength λ in Kilometers with C in Micromicrofarads Between C = 189 and C = 1455.

Coil	Equation
A	$\lambda^2 = .00004157\ C + .0002422$
B	$\lambda^2 = .0001925\ C + .001267$
C	$\lambda^2 = .0007765\ C + .00826$
D	$\lambda^2 = .003659\ C + .0417$
E	$\lambda^2 = .01860\ C + .3404$
F	$\lambda^2 = .07483\ C + 1.722$
G	$\lambda^2 = .16085\ C + 3.152$
No. 5	$\lambda^2 = 1.7897\ C + 228.6$

Coefficients of Table VII Examined in Their Relation to Self-Inductance and Natural Wavelength of the Coils.— If the equations of Table VII held for condenser capacity zero the final term of each equation should be λ^2_0 of the coil alone. Also, if the equations held for very large values of C, the coefficients of C should be $4\pi^2 \times 9 \times 10^{-2} L$, where L is the self-induction of each of the coils in henries. These relations give Table VIII.

TABLE VIII.

Regarding Coefficients.

Coil	L Henry Table VII	L measured at 1000 cycles/sec	λ^2_0 natural by Table VII	λ^2_0 natural observed	Tolerance in λ^2_0	Percentage Departure of L
A	.00001170	.0000137	.0002422			
B	.00005417	.000054	.001267			
C	.0002185	.000217	.00826			.70
D	.0010299	.001018	.0417	.0425	.0012	1.18
E	.0052341	.005174	.3404	.3798	.007	1.14
F	.021060	.02079	1.722	1.785	.03	1.28
G	.04527	.04462	3.152	3.338	.07	1.44
No. 5	.5037	.5000	228.6	233.17	1.20	.70

In the next to the last column the item marked "Tolerance in λ^2_0" is the amount by which λ^2_0 may be increased or diminished without affecting the agreement between observed and calculated values of λ within the calibrated range by more than 0.1%. From the fact that the observed and calculated values of λ^2_0 differ by more than the amount of this tolerance it is apparent that the linear law of Table VII does not extend to extremely small values of C.

Regarding L the measured values at 1000 cycles per second for coils "A" and "B" are not accurate beyond the two significant figures given. The percentage by which L calculated by Table VII exceeds the 1000-cycle of L is given in the column marked "Percentage Departure of L."

This Table VIII gives an idea of the error with coils of various sizes and shapes that would arise in the computation of λ from values of L and C measured at 1000 cycles per second and λ_0 measured by direct observations on the free wavelength of the coils.

All of the coils except No. 5 were banked windings designed to minimize distributed capacity. No. 5 was a coil of Maxwellian shape with square cross section closely wound with silk-covered wire.

In practice it is much easier to measure the frequencies and wavelength than the capacities and inductances employed.

Further Note on the Linear Relation of λ^2 to C.—Table IX was obtained with a special single layer coil wound on a glass tube, and with an enormous range of attached capacities. Observed values of λ are compared with values computed from the linear equation

$$\{\lambda_{km}\}^2 = .007349 C_{\mu\mu f} + .0577. \tag{15}$$

The table shows that with this particular coil this linear equation holds with an accuracy in λ of about 0.1% for values of C ranging from 17 micromicrofarads to 1404 micromicrofarads, and that the range may be extended to an upper value of C equal to 200000 micromicrofarads with an error in λ not greater than about $\frac{1}{2}$ of 1%. This may be largely the error of our measurements of C.

In fact the accurate measurement of C over a wide range presents a difficulty so great that I should recommend the use of the linear relation merely as an interpolation formula to be employed only over a range limited to about 10 to 1 in Capacity.

TABLE IX.

Wavelength for Various Capacities Attached to a Single Layer Coil No. G 3 (with L = .002048 henry).

	n	Resonant value of C in micromicro-farads	λ meters observed	λ meters calculated by (15)	Percentage Departure in λ
Mica Condenser		300400	46430	46990	1.20
		199600	38190	38300	.28
		100800	27030	27220	.70
		20000	12170	12130	.33
G. R. Precision Air Condenser No. 14	9/2	1404	3217	3221	− .12
	4	1105	2860	2860	.00
	7/2	845.6	2502	2504	− .04
	3	618.4	2145	2145	.00
	8/3	487.9	1906	1906	− .15
	5/2	427.3	1787	1788	− .06
	2	271.0	1430	1432	− .14
	7/4	205.4	1251	1252	− .08
	5/3	185.8	1192	1193	− .09
	3/2	149.0	1072.3	1073.6	− .12
	4/3	116.0	953.1	954.0	.10
	5/4	100.7	893.6	893.1	.05
Vernier Condenser	1	61.63	714.9	714.6	.04
	5/6	40.17	595.7	594.1	.26
	3/4	31.23	536.2	536.9	.06
	2/3	23.13	476.6	477.2	− .13
	3/5	17.22	428.9	429.2	− .07
		13.95	395.2	400.3	−1.2
Coil Alone		0.00	202.6	240.4	−20.0

V. Apparatus for Persistent Oscillations of Two Frequencies With a Single Vacuum Tube.

Incidentally it may be noted that the piezoelectric oscillator may be made to give persistent oscillations of two frequencies, with a single vacuum tube, by combining with the crystal a tuned electric circuit of a different frequency. Circuits for attaining this result are shown in Figure 6. This is similar to Figure 5, except that an inductance L_1 and a capacity C_1, in parallel are introduced in series with the piezoelectric crystal, between plate and grid. The inductance L_1 may be in inductive relation to the plate inductance L_2, although in practice I have found that the mutual inductance between L_1 and L_2 may be very small or zero. By adjusting L_1 or C_1 the electric system may be tuned over a wide range of frequencies, in which the system oscillates, while at the same time it is oscillating continuously with a second frequency; namely, the frequency of the piezoelectric crystal.

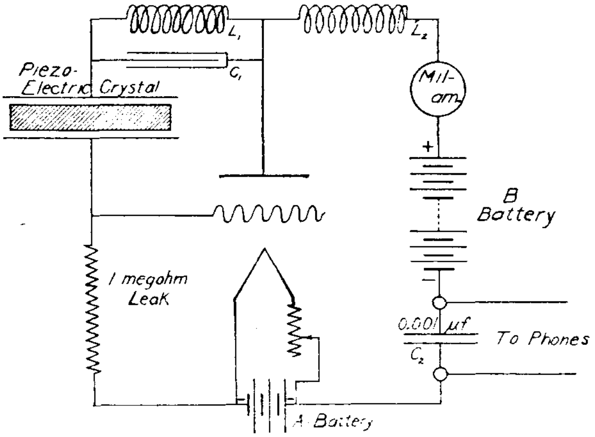

FIGURE 6. Circuits for Persistent Oscillations of Double Frequency.

Beats between those two frequencies, coexisting in the same circuits, occur whenever the fundamental or any harmonic of the electric frequency is made to be nearly equiperiodic with the fundamental or any harmonic of the mechanical period of the crystal; and a beat zero occurs when these frequencies are made exactly equiperiodic.

Thus a single circuit with a single tube may be made to replace the more elaborate combination of circuits shown in Figure 3, but in the practical calibration of a wavemeter I find Figure 3 somewhat more flexible.

Important applications of Figure 6 are, however, possible. For example, if one wishes to use the beat frequency as a source of audio frequency oscillations, one may cover a large range of audio frequencies with a very small and compact apparatus.

It seems likely that the tuned electrical circuit of Figure 6 may be replaced by one or more piezoelectric crystals, either in series or in parallel with the one shown, so that several frequencies may be produced piezoelectrically with a single vacuum tube. Such a device would meet with wide application.

Rochelle Salt Crystals of very small dimensions (2 x 4 x 10 millimeters) have been successfully employed in the piezoelectric oscillator of Figure 1, to replace the large quartz plate. We are now growing and studying these crystals with respect to their constancy and with respect to their use as sources of multiple frequency with a single vacuum tube.

VI. PIEZOELECTRIC RESONATORS.

We return now to the subject of piezoelectric resonators. As mentioned in the introduction, Professor Cady's paper deals chiefly with this subject. Following his construction the piezoelectric resonator consists of a parallelopiped of quartz one or two square millimeters in cross section and of length dependent on the frequency required.

As to orientation in the crystal, the long dimension [2] of the parallelopiped was, in the natural crystal, oriented perpendicular to two opposite of the six natural faces which hexagonally envelope the crystal. One of the shorter dimensions was in the direction of the optic axis of the crystal (which is perpendicular to the hexagonal base). In this direction the specimen is piezoelectrically *inactive*. The third dimension was perpendicular to the other two dimensions. These three dimensions will be called l, a, and b, respectively.

The parallelopiped is mounted as in Figure 7. Two metal electrodes E are pinned to a hard-rubber base so that they are separated by a space a little wider than the dimension b of the specimen. The crystal specimen is placed between these two electrodes and covered with a hard-rubber cover containing a cavity large enough to leave the crystal loose in the cavity. The inactive dimension *a* is in the direction perpendicular to the static field between the electrodes. A number (e.g. 13.1 in Fig. 7) stamped on each holder designates the approximate length of the several resonators here employed.

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[2] In disregard of strict usage I have here employed the word "dimension" to denote a line parallel to an edge of the parallelopiped.

As Professor Cady points out, the resonator electrodes may be brought into contact with various parts of an oscillating electric circuit and will give a response. Figure 8 shows a spring clip within which the electrodes of the resonator may be placed so that the crystal is parallel to the tuning condenser of a Hartley circuit.

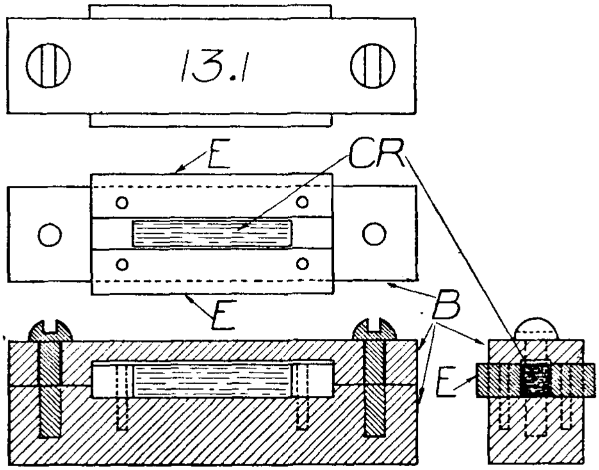

FIGURE 7. Piezoelectric Resonator.

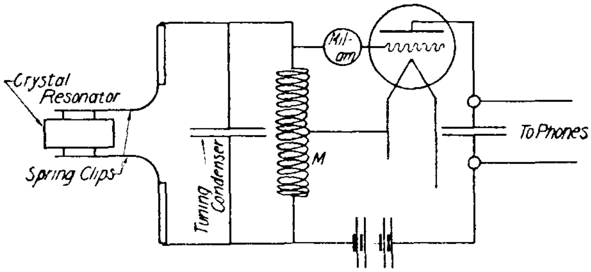

FIGURE 8. Resonator in Clip in Circuit.

With the crystal resonator in place a distinct click or " chirp " is heard on varying the tuning condenser whenever the electrical period of the electrical circuit passes through equiperiodicity with the mechanical period of the crystal resonator. This chirp is a transient phenomenon and by means of it the state of equiperiodicity may be determined with great accuracy.

Table X contains the wavelength λ of the electric wave in free space corresponding to the frequency of the electrical oscillator when it is resonant to the fundamental mode of vibration of several Crystal Resonators of designation given in column I and of actual length given in column II.

TABLE X.

Fundamentals of Crystal Resonators.

Designation No.	Length of Crystal in mm.	meters	meters per mm. length
13.1	13.08	1451	111
12.9	12.90	1429	111
10.82	10.82	1214	112
10.11	10.11	1122	111
8.76	8.67	973.0	112
H 11	8.48	894.9	105
7.62	7.62	856.1	112
7.04	7.04	788.9	112
6.02	6.02	684.3	113
3.1	3.10	332.4	107
		Average	110

In the last column is given the ratio of λ in meters to crystal length in millimeters. This quantity, since determined by the mechanical frequency of the crystal specimen, depends to a slight extent on the width and thickness of the specimen; but is of the order of 110. This gives the velocity of sound in quartz as approximately 5454 meters per second at the high frequencies employed, where no account is taken of end corrections and corrections for the thickness of the specimen.

In addition to the response corresponding to the fundamental mode of vibration of the crystal resonators, with a node in the middle, they also respond at frequencies corresponding to other modes of vibration having more than one node in the longer dimension, and also to modes of vibration corresponding to the propagation of sound in the direction of the static field as well as in the direction of the length. Table XI gives, for each resonator designated No. 13.1, etc., a column of the several wavemeter readings λ in meters of the electric oscillator when resonant to the crystal resonator. The letters S, W, M, indicate whether the response of the crystal is strong, weak or medium, respectively. The table is not complete, as the readings might have been extended much further in the direction of the shorter waves.

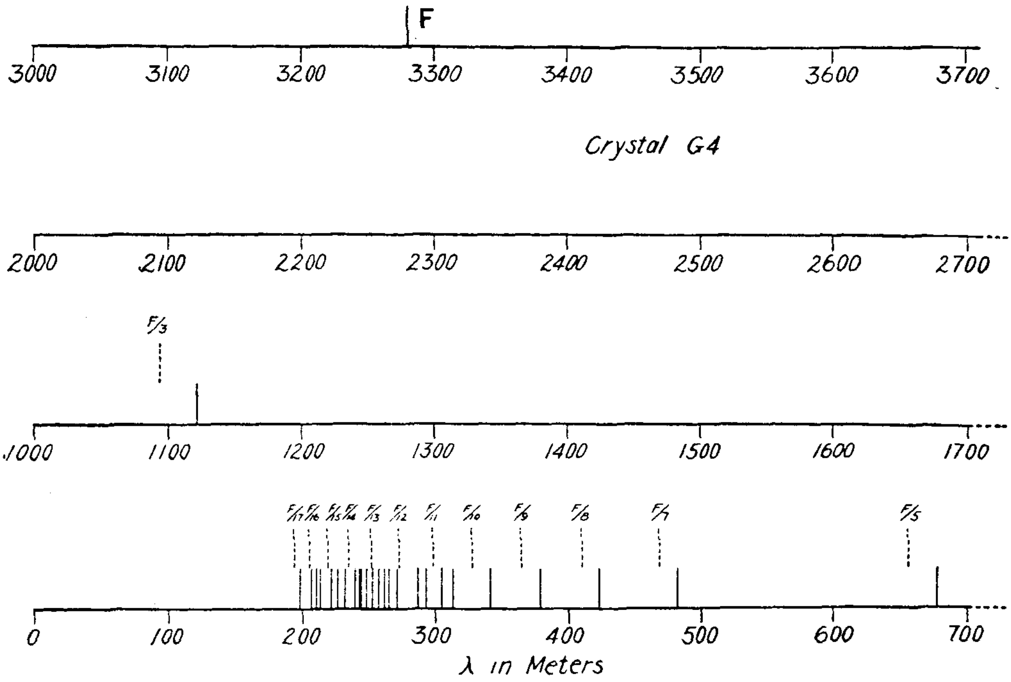

FIGURE 9. Electrical Spectrum of Crystal Resonator G4.

The results of a similar set of observations with a crystal receiver, lent me by Professor Cady, and designated "G4" are plotted as a spectrum in Figure 9.

TABLE XI.

Wavemeter Readings Resonant to Fundamental and Partial Vibrations of Various Crystal Resonators.

No. 13.1	No. 12.90	No. 10.11	H 11	No. 7.04
1450 S	1428.8 S	1121.9 S	893.8	788.9 S
491.2 S	719.8 W	395.8 S	303.6	709.8 W
351.0 S	485.7 S	362.5 W	270.6	697.4 W
346.2 S	358.7 W	348.0 S	219.8	394.7 W
342.9 W	355.9 S	335.8 W	199.3	355.3 W
338.1 W	350.2 W	322.2 W	180.5	350.8 S
334.1 M	283.1 W	255.3 W		347.6 W
305.5 W	270.1 W	222.8 W		333.1 S
284.9 S	258.6 W	212.3 W		327.9 W
281.4 W	256.0 S	203.2 W		311.9 W
270.9 W				276.9 W
269.0 M				262.7 W
265.8 S				260.0 W
263.4 W				258.2 W
				248.9 W
				236.3 S
				230.1 W
				228.3 W
				224.5 W
				219.5 W
				210.9 W
				209.5 W
				201.3
				198.0
				:
				:
				:
				154.8 S
				147.7

S = Strong
W = Weak
M = Medium

The continuous lines at $\lambda = 3280.2$, 1120.5, etc., indicate observed values. For comparison the dotted lines marked F/3, F/5, etc., are values obtained by dividing F = 3280.3 by 3, 5, etc., and show where the lines would fall if they were precise submultiples of a single wavelength F. It will be seen that the resonator frequencies are not proportional to the integral members, whereas, on the other hand, the piezoelectric oscillator, as shown in the results above, gives a sustained periodic vibration exactly analyzable into a fundamental frequency and integral multiples thereof.

When a resonator has its lines once tabulated as in Table XI, all the values given may be employed in the subsequent standardization of wavemeters or frequency meters.

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THE COMPRESSIBILITY AND PRESSURE COEFFICIENT OF RESISTANCE OF RHODIUM AND IRIDIUM.

By P. W. Bridgman.

THE COMPRESSIBILITY AND PRESSURE COEFFICIENT OF RESISTANCE OF RHODIUM AND IRIDIUM.

BY P. W. BRIDGMAN.

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INTRODUCTION.

THE data of this paper are supplementary to determinations of compressibility and pressure coefficient of resistance which I have previously given for a number of substances.[1] It is my intention to keep this work as up to date as possible by making new determinations either for new materials or else for materials of greater purity than have hitherto been available. I have not hitherto made any measurements on either rhodium or iridium, and I cannot find any records of others, so that these results are quite new.

The two metals were obtained from Johnson and Mathey, London. They were provided in the form of round wires approximately 10 cm. long and 0.08 cm. in diameter. They were shaped by swaging at a red heat, it not being possible to draw them, and therefore they were not perfectly uniform in diameter. The extreme variation from the average diameter of the rhodium was 6.5% and of the iridium 3.2%. Departures from geometrical perfection are entirely without effect on the measurements of pressure coefficient of resistance or of compressibility, but if the departures are too large, the accuracy of the specific resistance, which was determined incidentally, may be somewhat affected. For these two wires, however, the departure from uniformity was so small that no error in the specific resistance is to be feared.

I have no chemical analysis of the materials, but some idea of the probable purity may be obtained from the temperature coefficient of resistance at atmospheric pressure. That of the rhodium is high, and this material is probably as pure as any on which previous measurements (specific resistance and temperature coefficient of resistance) have been published. The temperature coefficient of the iridium, on the other hand, is low, and its purity is probably not as high as might possibly be obtained.

The methods of measuring both pressure coefficient of resistance and compressibility were the same in all respects as those used on other materials, and which have been previously described. The same apparatus was used, and the usual pressure range of 12000 kg/cm². Resistance was measured by the potentiometer method adapted to specimens of small resistance (the resistance of these specimens was of the order of 0.01 ohm). The current leads and the potential leads were small copper wires soft soldered to the specimen. For the compressibility measurements the specimens were mounted as tension specimens in the lever apparatus for long specimens. Before measurement, the wires were annealed to nearly a white heat in a bunsen flame.

The results follow.

RHODIUM.

Compressibility. The compressibility was measured at 30° and 75°. The points obtained were entirely regular, and all were used in the computation without discarding any. The results are expressed in the formulas:

$$\text{At } 30°, \frac{\Delta V}{V_0} = -10^{-7}[3.72 - 2.67 \times 10^{-5}p]\,p,$$

$$\text{At } 75°, \frac{\Delta V}{V_0} = -10^{-7}[3.81 - 2.67 \times 10^{-5}p]\,p.$$

At 30° the average deviation of the 14 observed points from a smooth curve was 0.28% of the maximum observed effect, and at 75° 0.19%. These deviations are on the measured difference of compressibility between rhodium and iron; the corresponding accuracy of the actual compressibility (assuming no error in the value used for iron) is about three times greater.

The compressibility is very nearly the same as that of molybdenum It is to be noticed that the departure from linearity, shown by the second degree term in the formula above, is greater than it is for iron. This is not usual, and is the first example I have found for a metal, it being the usual rule that the departure from linearity is greater for those metals with the greater compressibility.

Resistance. The electrical resistance was measured as a function of pressure at 30° and 65°. An attempt at 95° did not give good results. The dimensions of the specimen were not as advantageous to accurate measurement of the resistance as of the compressibility, and the results obtained were not as good. The results are expressed in the following formulas, which give the fractional change of resistance at atmospheric pressure and the temperature in question as a function of pressure:

$$\text{At } 30°, \frac{\Delta R}{R_{0,30°}} = -10^{-6}\,[1.738 - 9.7 \times 10^{-5}\,p]\,p$$

$$\text{At } 65°, \frac{\Delta R}{R_{0,65°}} = -10^{-6}\,[1.776 - 10.1 \times 10^{-5}\,p]\,p.$$

At 30° the average departure of the observed points from a smooth curve, (no discards) was 0.60%, and at 65°, 0.95% of the maximum effect.

At atmospheric pressure the specific resistance was calculated from the actual resistance and the dimensions and was found to be 4.90×10^{-6} at 0°. This is somewhat higher than 4.70×10^{-6}, the value found by Broniewski and Hackspill.[2]

Between 0° and 100° the resistance changes linearly with temperature at atmospheric pressure, and the mean coefficient over this range, in terms of the resistance at 0°, is 0.00399. From the values of Broniewski and Hackspill for the specific resistance I calculate for their temperature coefficient 0.00404. The agreement is within the limit of error indicated by their significant figures.

The pressure coefficient of resistance is of the order of 10% lower than for the other two metals of this class previously measured, platinum and palladium, and the temperature coefficient is slightly higher than for the best platinum, indicating high purity.

IRIDIUM.

Compressibility. The compressibility was measured at 30° and 75°. The results are reproduced in the formulas:

$$\text{At } 30°, \frac{\Delta V}{V_0} = -10^{-7}\,[2.68 - 1.3 \times 10^{-5}\,p]\,p,$$

$$\text{At } 75°, \frac{\Delta V}{V_0} = -10^{-7}\,[2.81 - 2.2 \times 10^{-5}\,p]\,p.$$

The average deviation of the observed points from a smooth line (no discards) was 0.094% of the maximum effect at 30°, and 0.16% at 75°. This deviation is on the difference of compressibility between iridium and iron; the error on the absolute compressibility is about one half of this.

The departure from linearity is here less than it is for iron, which is as to be expected. The variation with temperature of the second degree term is unusually high, as is also the temperature coefficient of the initial compressibility. It will be found on making the computation that the total change of volume between atmospheric pressure and 12000 kg. is only 1% greater at 75° than at 30°. Possibly the formula gives a temperature variation of both first and second degree terms larger than it should be, one compensating the other.

The absolute compressibility of iridium is smaller than any metal hitherto measured; the lowest value hitherto found was for swaged tungsten, whose absolute compressibility at 30° was 2.93×10^{-7}.

Resistance. The electrical resistance was measured as a function of pressure to 12000 kg/cm² at 30°, 65°, and 95°. The results are reproduced by the formulas:

$$\text{At } 30^\circ, \frac{\Delta R}{R_{0,30^\circ}} = -10^{-6}\,[1.353 - 4.0 \times 10^{-6}\,p]\,p,$$

$$\text{At } 65^\circ, \frac{\Delta R}{R_{0,65^\circ}} = -10^{-6}\,[1.280 - 3.9 \times 10^{-6}\,p]\,p,$$

$$\text{At } 95^\circ, \frac{\Delta R}{R_{0,95^\circ}} = -10^{-6}\,[1.340 - 3.9 \times 10^{-6}\,p]\,p.$$

At 30° the deviation from a smooth curve of the observed points (no discards) was 0.8% of the maximum effect, at 65° it was 1.1%, and at 95° 2.1%. The formulas show that the initial coefficient has a minimum value at 65°. This is unusual, but other examples have been found. The temperature variation seems to be outside the limits allowed by the probable error, and is probably real for this material. Whether it would be found for absolutely pure iridium is another question.

The pressure coefficient of resistance is materially lower than that for rhodium, as would be expected from its smaller compressibility and higher melting point, and is close to that previously found for tungsten.

The specific resistance at atmospheric pressure, found from the measured resistance and the dimensions, was 6.61×10^{-6} ohms per cm. cube, considerably higher than 6.10×10^{-6}, the value of Broniewski and Hackspill.[2]

The temperature coefficient of resistance at atmospheric pressure between 0° and 100° I found to be 0.00322. The relation between temperature and resistance is not quite linear, but the average coefficient between 0° and 50° is 0.8% less than between 0° and 100°. This is the normal direction for departure from linearity, and is the reverse of the direction shown by platinum. I calculate from the values of Broniewski and Hackspill for the specific resistance that the temperature coefficient of their iridium between 0° and 100° was 0.00361.

This is too much above my value to be accounted for by errors of measurements, and doubtless indicates a perceptible amount of impurity in my iridium. The effect of this impurity on the compressibility is probably very small, on the pressure coefficient of resistance somewhat larger, and on the temperature coefficient of resistance largest of all, bringing the value from 0.0032 to somewhere around 0.0040. It is to be anticipated, judging from experience with other metals, that the compressibility of pure iridium will be slightly less than the value given above, and the pressure coefficient of resistance somewhat greater numerically.

Discussion.

A comparative study of the properties of the three series of chemically related elements Fe, Co, Ni; Ru, Rh, Pd; and Os, Ir, Pt is of interest. In Table I are collected various data for these elements, namely atomic weight, melting point, atomic volume, compressibility at 30°, and pressure coefficient of electrical resistance at 30°. The two latter data are missing for Os and Ru. The table discloses irregularities in the progression of the properties; what the exact significance of these irregularities is I shall not attempt to discuss. In the first place the order of atomic weights in the first series is not the same as that of the atomic numbers, as is well known, there being an inversion between Co and Ni. This inversion is not found in the other two series, but it will be noticed that in the second series the difference of atomic weights between Ru and Rh is much less than between Rh and Pd, whereas in the third series the atomic weights are almost exactly evenly spaced. There is here evidently some sort of progressive change as the outer electron structure of the atom becomes more complicated. In each series the melting point progresses regularly, being least for the member with greatest atomic number. The atomic volumes, on the other hand, do not progress regularly, but there is a reversal in passing from the light to the heavy series; in the first series the lightest member has the greatest volume, and in the third the heaviest, the order being irregular in the second series. The compressibility also shows no regular order, but does follow the order of atomic volumes as far as the data are known, the element with the greatest atomic volume in any series also having the greatest compressibility, which is what would be expected. This would lead us to expect that the compressibility of Os would be even smaller than that of Ir, and the smallest known for a metal. It is even not unlikely that

TABLE I.

	Fe	Co	Ni
Atomic weight	55.84	58.97	58.68
Melting point	1530° C	1480	1452
Atomic volume	7.1	6.8	6.7
Compressibility at 30°	5.87×10^{-7}	5.39×10^{-7}	5.29×10^{-7}
Pressure coefficient of resistance at 30°	-2.42×10^{-6}	-9.3×10^{-7}	-1.90×10^{-6}
	Ru	**Rh**	**Pd**
Atomic weight	101.7	102.9	106.7
Melting point	2450?	1950	1550
Atomic volume	9.0	8.5	9.2
Compressibility at 30°		3.72×10^{-7}	$\left.\begin{matrix}5.19\\5.28\end{matrix}\right\}\times10^{-7}$
Pressure coefficient of resistance at 30°		-1.74×10^{-6}	-1.96×10^{-6}
	Os	**Ir**	**Pt**
Atomic weight	190.9	193.1	195.2
Melting point	2700	2350	1755
Atomic volume	8.5	8.6	9.2
Compressibility at 30°		2.68×10^{-7}	$\left.\begin{matrix}3.60\\3.05\end{matrix}\right\}\times10^{-7}$
Pressure coefficient of resistance at 30°		-1.35×10^{-6}	-1.95×10^{-6}

it may be less than for diamond. The pressure coefficient of resistance in each of the series decreases numerically from the third to the second member of the series. In the Fe, Co, Ni series the coefficient increases again on passing from Co to Fe; the data have not yet been obtained to show whether the behavior in the other two series will be the same.

SUMMARY.

Measurements by previous methods of compressibility and pressure coefficient of electrical resistance have been extended to rhodium and iridium. The compressibility of rhodium is about that of molybdenum, and that of iridium is somewhat less than that of tungsten, and is the lowest yet measured for a metal. The pressure coefficient of resistance of rhodium is of the order of that of platinum and palladium, and that of iridium is close to that of tungsten. Measurements are also given of the specific resistance at 0° and the mean temperature coefficient of resistance between 0° and 100° at atmospheric pressure. In the discussion attention is called to certain irregularities in the progression of properties in the three chemically related series Fe, Co, Ni; Ru, Rh, Pd; Os, Ir, Pt.

I am indebted to my assistant Mr. I. M. Kerney for making many of the readings.

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THE EFFECT OF TENSION ON THE THERMAL AND ELECTRICAL CONDUCTIVITY OF METALS.

By P. W. Bridgman.

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Introduction.

The principal object of this research was to determine the effect of tension on thermal conductivity, the flow of heat being in the direction of the tension. This has been previously measured by only two observers, A. Johnstone and N. F. Smith,[1] who both found that thermal conductivity is increased by tension. This result is not what would be expected, particularly in view of the fact that electrical conductivity is known to decrease under tension. On carefully examining the experiments of these two observers, it appeared that their results were subject to exceedingly large irregularities, there being discrepancies in the individual determinations of several fold and sometimes even of sign, and furthermore it seemed to me that there was a possible constant source of error in each piece of work which might give the wrong sign to the effect. In view of the theoretical interest of the matter, I have reëxamined the question by a different method, which I believe to be free from large error, and certainly from the possibility of a consistent error of sign. The result of my observations is that for a number of the more easily handled metals the effect of tension is to decrease the thermal conductivity by amounts small, but greater than can be accounted for by the change in dimensions under tension. The only exception is nickel, which shows also an abnormal increase of electrical conductivity under tension.

I have also measured the effect of tension on the electrical conductivity of the same samples of metal. This has been determined previously by only a few observers,[2] with not very concordant results, although there is no question as to the sign of the effect.

The question of chief theoretical interest here is whether the Wiedemann-Franz ratio of electrical to thermal conductivity will remain constant under tension; that is whether the change in electrical will be the same as the change in thermal conductivity. The constancy of this ratio for a number of metals would perhaps lead one to expect that the ratio would be unaffected by tension, although my measurements [3] on the effect of hydrostatic pressure, showing that electrical and thermal conductivities change under pressure by different amounts, would prepare one to expect different tension effects. This is the case; the two conductivities are not changed by the same amount, and the relative change of one or the other may be greater, depending on the metal.

Experimental Method.

Thermal Conductivity. The apparatus for measuring thermal conductivity is shown in Figure 1. The specimen is in the form of a drawn rod about 0.125 inches (0.317 cm.) in diameter and 6 inches (15.2 cm.) long. The two ends are threaded and held in conical grips of copper which automatically grip more tightly when the tension increases. These two grips of copper are seated in two massive cylindrical copper blocks, which slide closely in a brass tube 1 inch (2.54 cm.) in internal diameter. The brass tube is surrounded by a water bath at room temperature which was constantly stirred. The use of a temperature regulator did not prove necessary. Stuffing rings prevent the water from entering the brass tube or the space about the specimen, which contains only air at atmospheric pressure.

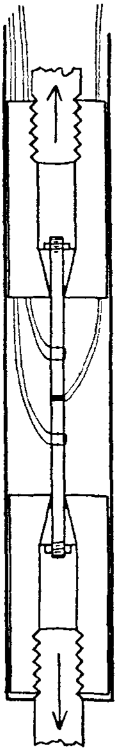

Figure 1. The apparatus for measuring thermal conductivity under tension.

A small heating coil is wound about the middle of the specimen. The heat input may be determined in terms of the current and the resistance of the heater. The heat flows along the axis of the specimen, into the copper blocks through the grips, and thence into the water bath through the brass tube. The dimensions are such that practically all the heat input is conducted into the bath through the metal parts, the loss laterally through the sides of the specimen being negligible.

The temperature gradient in the specimen accompanying the flow of heat is measured by two sets of thermo-couples about 3 cm. apart situated on both sides of the heating coil. The use of couples on both sides of the heater is one of the features of this new design. One is sure in this way of catching all the heat input on its way out, and errors which might arise from asymmetric change in the goodness of the thermal contact between specimen, grips, and bath, and a consequent asymmetric change in the heat flow, are thereby avoided. The thermo-elements were of copper-constantan. Each element consists of a piece of constantan 0.006 inch (0.0152 cm.) in diameter, 1.25 inches (3.17 cm.) long, silver soldered at either end to a length of copper wire of the same diameter. The joint is a butt joint and is perfectly smooth to the touch. The element is insulated with six to eight coats of insulating enamel baked on at 210° C, thereby increasing the diameter to 0.007 inch (0.178 mm.). The element is then threaded through two holes 0.008 inch (0.203 mm.) in diameter drilled transversely through the specimen and separated along the axis by about 6.5 mm., and is then pulled through by the right amount so that each junction between copper and constantan is situated on the axis of the specimen. The two copper ends of the couple are soldered to copper leads, connecting to a potentiometer with which the e.m.f. of the couple may be measured. It will be seen that what is determined in this way is the difference of temperature between two points on the axis 6.5 mm. apart. From this the temperature gradient may be calculated, using in the final calculations the mean obtained from the two couples on the two sides of the heater. The heat input is known, so that the data are at hand for computing the thermal conductivity. Repetition of the measurements under tension gives the tension change of conductivity.

The constantan wire between the junctions is five times as long as the distance between the two holes. The extra constantan wire was tightly wound about the specimen, as were also several turns of the copper wire of the element, and wrapped with fine silk thread. In this way error from heat leak into the junction along the wire is avoided, the wire of the elements near the junction being at the temperature of the specimen at the junction. Thermal contact between the element and the specimen inside the holes was ensured by filling the holes with oil before threading the element through them, so that there is a film of oil occupying the annular space 0.0005 inch wide between the wire of the element and the specimen.

The heater consists of a fine wire of high specific resistance, nichrome in some of the experiments and constantan in others, butt silver soldered between copper leads, exactly like the thermo-elements, insulated with enamel, and wound tightly about the center of the specimen, and further wrapped with fine silk thread to ensure good thermal contact. To ensure electrical insulation of all the parts the entire center portion of the specimen nearly up to the grips was covered with a couple of coats of enamel baked on. This was about 0.0001 inch thick.

The electrical measurements included measurements of the heating current and of the e.m.f. of the thermo-couple. The method has been described in detail elsewhere,[4] and employed the same potentiometer as that used in determining the effect of pressure on thermal conductivity of liquids and in other previous investigations. It is a null method. Balance is secured by varying a rheostat, and both current and thermal e.m.f. are obtained in terms of the resistance necessary to balance. The sensitiveness was such that both current and e.m.f. could be determined to a few hundredths of a per cent. The heating currents used were of the order of 0.3 amp. and the temperature difference of each couple of the order of 0.°7 C. The method was used to give comparative values only; the dimensions or the resistances were not determined accurately enough to give absolute conductivities.

The tension was applied to the specimen by a simple lever multiplying scheme, with a counterpoise arrangement for neutralizing the weight of the lever, which need not be described in detail. The load was a weight, which could be applied or removed through a pully so arranged as to avoid shock of sudden application. The specimen was hung from the ceiling by a rod which passed through the upper of the heavy copper blocks, and the lever acted on a rod attached to the lower copper block, and passed into the water bath at the bottom through a stuffing box so designed that there should be no perceptible friction.

At each load four readings were made for the current and each thermo-couple, twelve in all. These readings were made with both directions of flow of the heating current, and both directions of polarity of the balancing e.m.f. of the potentiometer, in this way eliminating parasitic e.m.f.'s or the effect of any failure of insulation between different parts of the apparatus. The essentially variable readings were those of thermal e.m.f. The heating current was maintained approximately constant with a storage battery and a ballast lamp, and was measured at every load merely by way of precaution; it did not show any variation with load, as of course it should not.

The effect of tension is very small, and to obtain good results it was necessary to make repetitions, taking the averages. The effect is so small that it was not possible in most cases to study satisfactorily the way in which the effect varies with the load, but I had to content myself with the effect at full load, as did also Johnstone and Smith. For iron, however, I was able to make readings at full and half load. Care was taken that the full load should be considerably below the elastic limit of the material. This condition was not realized for aluminum, which has a very low limit, and the figures in the following apply to aluminum which has been hardened by overstrain.

The procedure consisted in making a set of readings at zero load, applying the load and waiting for equilibrium, repeating the set of readings, removing the load, and again repeating after equilibrium etc. Because of the small dimensions of the apparatus and the good thermal contact between the parts, thermal equilibrium was reached in a very few minutes after changing the load, and the long and tedious waits of previous work, which are conducive to error because of accidental changes in the surrounding conditions, were avoided. Enough readings were taken to give for each metal between ten and twenty values for the change produced by load. The average of these changes was taken. The probable error of the average was computed by the usual least squares formula for the error of the mean, and is tabulated along with the other results. In taking these averages, no single readings were discarded, but all the readings used were obtained consecutively.

As compared with the two former methods, this method should offer the following advantages. Johnstone measured the flow along a rod, heat being put in by a heater wound on a micanite frame slipped over the rod. The temperature was measured by platinum resistance thermometers also wound on micanite frames and slipped over the rod. These frames were rigid, and did not follow the changes of dimensions of the rod when tension was applied. Under tension the rod contracts slightly, the thermal contact becomes poorer, with a resulting smaller flow of heat, and therefore a simulation of the effect of better thermal conductivity. This error is avoided in the present design by winding the heater directly onto the rod, and holding it tightly in contact with the rod by a wrapping of thread. The thermo-couples are also in better contact by my method of attachment, and are in no danger of being displaced by the load. Further, Johnstone measured the flow on only one side of the heater, instead of catching it all on both sides, as above. This I think to be a quite important feature, as sometimes the effects were quite unsymmetrical on the two sides, and in extreme cases one side might be of the different sign from the other.

The method of Smith was one in which one end of the rod was maintained at a constant temperature (heat input was not measured) and heat allowed to flow along the rod, partly flowing out at the distant end to other solid parts of the apparatus, and partly escaping laterally through the surface of the rod by conduction or convection through the surrounding air or by radiation. No account was taken of the possible change of lateral loss by tension, and most important of all, an improvement of the thermal contact at the distant end by tension, which almost certainly occurs, will account for the wrong sign of the effect.

An additional advantage over both of these methods I have already stated to be that my temperature differences and the whole scale of the apparatus were smaller so that equilibrium was reached much more quickly (minutes against hours) with less possibility of accidental variation of the external factors.

Electrical Conductivity. The same specimens were used, and tension was applied in the same apparatus as that used for thermal conductivity. The electrical measuring apparatus was the same as that which I have previously described in connection with determinations of the effect of pressure on the electrical resistance of specimens whose electrical resistance is small.[5] The general device is a potentiometer with a null method. Current was passed lengthwise through the specimen, and the potential difference determined between two needle points pressed into contact with the specimen by a simple spring arrangement. Electrical balance was obtained by changing the position of a slider on a slide wire. The sensitiveness was entirely ample, the change of resistance produced by the load causing displacements of the slider of the order of several cms., and each setting could be made to 0.1 mm. The sensitiveness was much greater than that of the thermal conductivity measurements, although the probable error as shown by the least squares formula was not always as much less as one might have expected; the errors were mostly due to changes in the external parts of the apparatus. The error was low enough to permit determinations at full and half load for four of the seven metals, instead of the one possible with thermal conductivity.

Readings were made as for thermal conductivity: first without load, then with load, and so on, until ten to twenty determinations were obtained of the change produced by load. The succession of readings was then repeated with the direction of current through the specimen reversed to eliminate parasitic effects. The series was then repeated with the second load. Equilibrium was attained about as fast as it was possible to apply or remove the load; the heating effects of the load were perfectly perceptible and fairly large, but disappeared very rapidly, thanks to the small dimensions of the apparatus.

Previous measurements on the effect of tension on resistance have probably been adequate as far as the electrical part of the measurement goes; the chief advantage in repeating them was to have measurements on both conductivities on the same specimens.

THE MATERIALS.

I used most of the cubic metals that can be obtained in a state of purity and homogeneity, and which are at the same time of sufficient mechanical strength to stand the handling and the tension; there are not as many of these metals as might be supposed. Non-cubic metals I did not think it worth while to trouble with, because I have recently been finding that there are very large differences in different directions. Such metals should be examined in the state of single crystal grains, and I hope that sometime I may be able to do it.

In order to give as much information as possible about the metals, I give in the following the values which I found for the specific resistances. These were computed from the resistance and the dimensions. The accuracy is not as high as might have been attained if this question was being investigated for its own sake, and there may be errors of the order of 1%. The temperatures were within one or two degrees of 20° C.

Copper was from the Bureau of Standards, a piece of the same sample as that whose pressure coefficient of thermal conductivity was measured,[6] and contains not more than 0.005% of impurity. It was annealed at a red heat after drawing. Due to an accident I have not a good value for the specific resistance of this rod of copper.

Iron was from the same bar obtained from the American Ingot Iron Co. from which I have cut specimens for a number of other measurements;[7] the impurity is not over 0.03%. It was annealed after machining to size. Its specific electrical resistance was 11.5×10^{-6} ohms per cm. cube.

Aluminum was of unusually high purity, total impurity 0.035%, and as was from the same original casting as the compressibility samples recently measured.[8] The rod was formed from the casting by hot extrusion with a reduction of diameter of about twofold. The extruded rod was quite crooked; it was straightened by rolling between two hot iron plates, filed as close to geometrical regularity as possible, and annealed for several hours at 300°. The same specimen was not used for thermal and electrical conductivity measurements because the elastic limit of the thermal conductivity piece was inadvertently exceeded on setting up for the electrical measurements, and it was badly stretched. Another sample was made from the same original stock and treated in the same way. Its specific electrical resistance was 2.47×10^{-6} ohms per cm. cube.

Silver was obtained from Baker and Co. and was stated by them to be of high purity, although I have no analysis. It was annealed to redness. Its specific electrical resistance was 1.61 × 10^{-6} ohms per cm. cube.

Platinum was also from Baker and Co., guaranteed to have less than 0.1% impurity and probably less than 0.01%. It was annealed to a bright red before measurements. Its specific electrical resistance was 10.1 × 10^{-6} ohms per cm. cube.

Palladium was also from Baker and Co., simply stated to be of high purity. It was annealed to redness before use. The purity of this is quite questionable to my mind, however, because I found a very high value for the specific resistance, 15.9 × 10^{-6} per cm. cube, against 10.7 × 10^{-6} by Jaeger and Diesselhorst.[9]

Nickel was commercial nickel obtained from the International Nickel Co., with slightly less than 1.0% of impurity. It was annealed like the other metals before using. Its specific resistance was 11.0 × 10^{-6} per cm. cube. It is unfortunate that I could not obtain for this purpose any of the exceedingly pure nickel for which I measured the compressibility and electrical resistance under pressure and tension,[10] but the size of the sample required for this work is somewhat greater than that yet possible in the very pure material. The measurements which I have already made of the effect of tension on the resistance of pure nickel included a careful study of the effect of loads over the entire range up to the elastic limit, and at several temperatures. It was not necessary to repeat this here. In order to afford a comparison with the thermal conductivity measurements, I merely measured the effect of tension on the resistance of the new sample at a single load, the same load as that of the thermal conductivity measurements. The effect of tension on thermal conductivity is so small that it was not possible to study the effect of varying loads, although it is to be anticipated that the effect will not be linear, as it was not for electrical conductivity.

EXPERIMENTAL RESULTS.

The experimental results are collected and summarized in Table I. In column 1 is the name of the metal, in column 2 the load in kg/cm², in column 3 the results for thermal conductivity, and in column 4 for electrical conductivity. The results given are the percentage changes of the actually measured conductivity under the loads listed, that is the thermal or electrical conductivity of a definite piece of metal with thermo-couples or potential terminals attached to fixed points, and with no correction for change of geometrical dimensions under ten-

TABLE I.

Collected Results.

Metal	Load kg/cm^2	Thermal Conductivity		Electrical Conductivity		Proportional Change of Conductivity per kg/cm^2		$\frac{1+2\sigma}{E}$	Proportional Change of Specific Conductivity per kg/cm^2	
		Percentage change	Probable percentage error of change	Percentage change	Probable percentage error of change	Thermal $\times 10^6$	Electrical $\times 10^6$	$\times 10^6$	Thermal $\times 10^6$	Electrical $\times 10^6$
Al	545	$-.34$	2.4			-6.25	-4.2	2.4	-3.8	-1.8
	393			$-.165$	5.0					
Cu	1080	$-.34$	7.8	$-.327$	2.4	-3.4	-3.03	1.33	-2.1	-1.7
	580			$-.154$	7.6		-2.66			-1.3
Fe	2050	$-.534$	3.8	$-.518$	2.2	-2.61	-2.53	.71	-1.90	-1.82
	1025	$-.32$	4.5	$-.257$	1.9	-3.08	-2.51		-2.37	-1.80
Pd	770	$-.133$	7.7	$-.235$	3.2	-1.73	-3.05	1.53	-0.20	-1.52
	457			$-.146$	1.3		-3.19			-1.66
Pt	790	$-.189$	4.5			-2.39		1.04	-1.35	
	770			$-.209$	2.9		-2.71			-1.67
	458			$-.119$	1.8		-2.60			-1.56
Ag	790	$\left.\begin{matrix}-.23\\-.33\end{matrix}\right\}$	10. 15.			$\left.\begin{matrix}-2.9\\-4.2\end{matrix}\right\}$		2.19	$\left.\begin{matrix}-0.7\\-2.0\end{matrix}\right\}$ 1.2	
	770			$-.43$	3.4		-5.6			-3.4
	460			$-.23$	4.0		-5.0			-2.8
Ni	1900	$+.076$	23.	$+.48$	.8	$+.40$	$+2.53$	.78	$+.48$	$+3.3$

sion. Two sets of figures are given in columns 3 and 4; the first set is the observed change of conductivity and the second is the most probable least squares error in this change. In columns 5 and 6 are given the changes in the two thermal conductivities for a load of 1 kg/cm^2, obtained from the figures in columns 3 and 4 by dividing the total change by the load in kg/cm^2. In the last three columns the reduction is made from change in observed conductivity to change in specific conductivity, taking account of the change of dimensions with tension. It may be readily seen that the change of longitudinal dimension (a stretch) and that of the lateral dimension (a contraction) both work in the same direction in decreasing the conductivity. The correction is proportional to the change of length and the change of area of cross section, and is given by $(1 + 2\sigma)/E$, where σ is Poisson's ratio, and E is Young's modulus, expressed for kg/cm^2 units. The values of $(1 + 2\sigma)/E$ are listed in column 7 of the table; special determinations of the constants σ and E were not made for this purpose, but the values were taken from Kaye and Laby's table of constants. In column 8 are the changes of thermal conductivity per kg/cm^2 corrected by this factor, and in column 9 the change of electrical conductivity similarly corrected. The probable percentage errors in the figures of columns 8 and 9 would be considerably higher than those in columns 3 and 4, because the former are differences.

The average probable error of the uncorrected tension coefficient of thermal conductivity is of the order of 5%. Apart, therefore, from the possibility of overlooked constant errors, there can be no question whatever as to the sign of the effect, which is a decrease of conductivity for all these seven metals except nickel.

The most important previous work on the effect of tension on electrical conductivity is by Tomlinson.[2] In Table II are compared his results and mine for the metals which we have measured in common. The differences are not important except for Al; here the difference may be well due to the much greater purity of my metal. It is to be remarked that although the cubic metals would be expected to show much less variation with direction than the non-cubic, still there is no reason to think that the results obtained with such a crystalline aggregate as a wire would not vary somewhat with the average orientation of the crystals. Perfectly definite and reproducible results are to be expected only with single crystals. A complete study would be very complicated even for so simple a stress as a pure tension; in general the direction of the current in a stretched cubic crystal need not be the direction of the applied e.m.f., although current and e.m.f. always coincide in an unstretched cubic crystal.

TABLE II.

COMPARISON OF PRESENT RESULTS WITH TOMLINSON'S FOR EFFECT OF TENSION ON ELECTRICAL RESISTANCE.

Metal	Change of Resistance per kg/cm² Tension Tomlinson	Bridgman
Al	1.89×10^{-6}	4.2×10^{-6}
Cu	2.21 2.40	2.9
Fe	2.10 2.20	2.53
Pt	2.28	2.65
Ag	4.27	5.3

With regard to the proportionality of the effects to the tension, the change of resistance appears to be proportional to the tension within the probable limits of error except possibly for copper and silver. As is well known, however, the "probable" error really means "probable" and must not be treated as though there were any absolute significance to it, so that in spite of the small variations found, I believe that these figures alone without further experiment would not justify any other expectation than that the effect is proportional to the load. The same is true for the variation of the tension coefficient of thermal conductivity of iron; under full and half load two figures were found which differ by somewhat more than the amount allowed by the probable error, but nevertheless I do not believe that we should expect anything else than proportionality.

Nickel is an exception to the rules found above both thermally and electrically, but more of an exception electrically, for the change of electrical conductivity with tension is about six times larger than the change of thermal conductivity.

The relations between the changes of thermal and electrical conductivity are summarized in Figure 2, where the circles show for each substance the change of thermal conductivity per kg/cm² (not corrected for change of dimensions), and the crosses the changes of electrical conductivity. The vertical lines drawn through the circles or crosses show the range within which the least squares errors indicate that the true value is most probably found. The figure brings out two things; in the first place the tension coefficients of the two conductivities are in general different by amounts much beyond the possibility of experimental error, and in the second place there is no universal rule as to which change is the greater numerically, but one may as well be greater as the other.

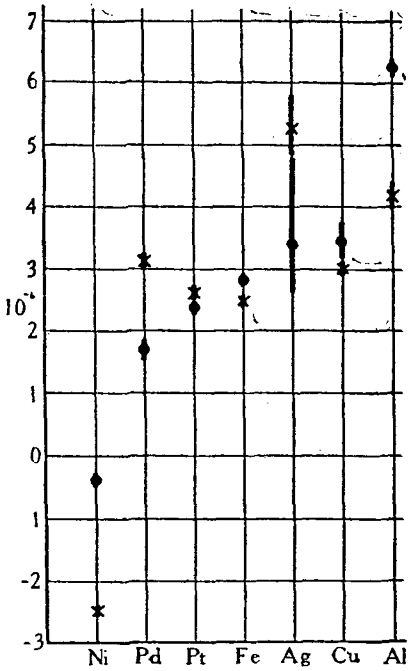

FIGURE 2. Summary of results. The circles show the fractional decrease of thermal conductivity for a tension of 1 kg/cm², and the crosses the corresponding decrease of electrical conductivity. The lines drawn through the circles and crosses show the probable spread of the results as calculated by least squares.

THEORETICAL DISCUSSION.

The classical theory of conduction by free electrons formulated by Drude, Lorentz and others, which accounts for the Wiedemann-Franz ratio in terms of a transfer of both electrical current and thermal energy by the same free electrons, would lead us to expect that thermal and electrical conductivity would change by the same amount under tension, for the reason that the ratio of one conductivity to the other at a fixed temperature is the same for all metals, and the same metal under two different tensions is merely a special case of two different metals. We have just seen experimentally that this expectation is not realized.

The same sort of considerations would apply to any sort of free path theory in which thermal and electrical conduction are performed by the electrons. The fact that the tension effect on the two conductivities is not the same is not to be taken as proof that a free path theory cannot be correct, but a resolution of the difficulty may be found in a consideration of the contribution to the total thermal conductivity made by the atoms, a part which is neglected in the classical theory as usually developed. I have already considered this question in connection with my measurements of the effect of hydrostatic pressure on the thermal conductivity of metals.[3] It appeared in that discussion that there is theoretical reason for believing that perhaps as much as one third of the entire thermal conductivity of a metal may be due to the atoms. Furthermore, it appeared that there is no reason for expecting the effect of pressure on the atomic part of thermal conductivity to be the same as on the electronic part, but that the atomic part may be either greater or less than the electronic part according to the details of the structure of the metal. In the same way we may expect the tension effect on the atomic part to be either greater or less than on the electronic part, and so the net effect of tension on thermal conductivity to be either greater or less than on the electrical.

Considering now only electrical conductivity, the change under tension is a subject on which there seems to be no published speculation, at least for a normal metal, but is one on which we can obtain at least a little more light. Let us compare the effects of tension and hydrostatic pressure. The important experimental fact seems to be that the change of resistance under pressure is a much larger numerical multiple of the change of linear dimensions than is the change with tension.

We can see roughly why this should be so. The following discussion will be concerned only with the geometrical factors; there are in addition specific effects of tension or pressure, the most important of which is probably the change of amplitude of atomic vibration. Under tension, in particular, the amplitudes of vibration in different directions will presumably be differently affected. At present we apparently do not possess the means for computing this non-isotropic effect. These geometrical considerations can therefore tell only part of the story.

The view of conduction which I have developed in several papers[11] regards the electrons as travelling in free paths long compared with the distance between atomic centers. The path is most simply thought of as through the substance of the atoms; for the present purposes we may very roughly regard the electrons as travelling in "grooves" fixed relatively to the atoms. The length of the path is determined by the ease of the electron's jumping from atom to atom, and involves the distance between atomic centers and the amplitude of atomic vibration. Now for changes at constant temperature we may suppose as a first approximation that the proportional increase in the length of free path in a definite direction is proportional to the fractional decrease in distance between atomic centers, or the linear strain, in that direction. This presupposes that the change of amplitude of atomic vibration in any direction depends only on the change in atomic separation in that direction, irrespective of whether that change is brought about by a tension or a pressure, and thus neglects any specific tension effects, as explained above. This is very much the same thing as supposing the material has no rigidity, but only volume compressibility.

In terms of this simplified picture, we can see why the ratio of pressure effect to change of dimensions is larger than the ratio of the tension effect. When a metal is subjected to hydrostatic pressure, all lengths are equally shortened, all free paths are equally lengthened, and all electrons make an equal proportional contribution to the increased current carrying capacity. But under tension, the length is increased along the line of tension but decreased at right angles. The paths in the direction of the tension are shortened, but those making an angle with the direction of the tension sufficiently close to a right angle are lengthened. Hence although the electrons moving in the direction of the tension contribute to a decrease of current carrying capacity, their effect is diminished by some of the opposite sign, so that the outstanding net effect is diminished.

This point of view may be formulated mathematically as follows. We need in the first place the expression given by simple elastic theory for the elongation, ϵ, of a linear element making an angle θ with the direction of tension. This is:

$$\epsilon = \frac{1}{E}\{1 - \sin\theta\,(1 + \sigma)\},$$

where E is Young's modulus and σ Poisson's ratio. Consider the case only where the applied e.m.f. is along the tension, that is, in the direction $\theta = 0$. Our hypothesis now is that the mean free path, and hence the contribution to the total current, of an electron travelling in the direction θ is diminished by an amount proportional to ϵ, or to $1 - \sin\theta\,(1 + \sigma)$. The total change in conductivity is to be found by summing for all directions. The absolute contribution to the conductivity by an electron moving along θ is proportional to $\cos^2\theta$. The reason for this is that the component of e.m.f. along the path is proportional to $\cos\theta$; this component produces along the path a velocity of drift proportional to $\cos\theta$, and the component of this drift along the direction of the total current or the e.m.f. is again diminished by the factor $\cos\theta$. Hence the contribution to the diminished conductivity made by the electrons along θ is proportional to

$$\cos^2\theta\,\{1 - \sin\theta\,(1 + \sigma)\}.$$

Summing this for all directions, we have

$$\int_0^{\frac{\pi}{2}} \cos^2\theta\,\{(1 - \sin\theta\,(1 + \sigma)\}\,2\pi\sin\theta\,d\theta,$$

which may be evaluated to

$$2\,\pi\left\{\frac{1}{3} - \frac{\pi}{16}(1 + \sigma)\right\}. \tag{A}$$

The same analysis applied to the case of hydrostatic pressure, or a uniform change of dimensions in every direction, would give

$$2\,\pi\left\{\frac{1}{3}\right\}, \tag{B}$$

the second term in expression A, which arises from the change of strain with direction in the case of tension, being absent.

Now if we take a mean value for σ, namely 1/3, we shall find that the ratio of B to A is 4.7. That is, considering the geometrical factor only, and also considering only that part of the change in specific conductivity produced by a change in the free path, a change of linear dimensions in the direction of the current when produced by hydrostatic pressure should be 4.7 times as effective in changing resistance as the same change of linear dimensions produced by tension.

In Table III are given the figures for comparing with experiment. In the first part of the table are the data for hydrostatic pressure,

TABLE III.

Comparison of Effects of Hydrostatic Pressure and Tension on Electrical Resistance.

Metal	Data for Pressure						Data for Tension				
	I $\frac{1}{w}\frac{\partial w}{\partial p}$	II $\frac{1}{v}\frac{\partial v}{\partial p}$	III $\frac{1}{l}\frac{\partial l}{\partial p}$	IV $\frac{1}{\rho}\frac{\partial \rho}{\partial p}$	V $\frac{1}{\rho'}\frac{\partial \rho'}{\partial p}$	VI Ratio of V to III	I′ $\frac{1}{\rho}\frac{\partial \rho}{\partial T}$	II′ $\frac{1}{l}\frac{\partial l}{\partial T}$	III′ $\frac{1}{\rho'}\frac{\partial \rho'}{\partial T}$	IV′ Ratio of III′ to II′	V′ Ratio of VI to IV′
Al	-4.42×10^{-6}	-1.34×10^{-6}	$-.45\times10^{-6}$	-4.87×10^{-6}	-3.53×10^{-6}	7.85	$+1.8\times10^{-6}$	1.39×10^{-6}	1.35×10^{-6}	.97	8.1
Cu	-1.96	$-\ .72$	$-.24$	-2.20	-1.48	6.17	1.6	.80	1.3	1.59	3.9
Ag	-3.63	$-\ .99$	$-.33$	-3.96	-2.97	8.99	3.1	1.24	2.8	2.26	4.0
Fe	-2.42	$-\ .59$	$-.20$	-2.62	-2.03	10.1	1.8	.46	1.6	3.5	2.9
Pd	-1.94	$-\ .53$	$-.18$	-2.12	-1.59	8.83	1.6	.87	1.4	1.61	5.5
Pt	-1.95	$-\ .36$	$-.12$	-2.07	-1.71	14.2	1.6	.60	1.5	2.50	5.7

and in the second for tension. Column 1 gives the change per kg/cm^2 of the observed resistance w (terminals fixed to the wire, with no correction for change of dimensions), column 2 the volume compressibility, column 3 the linear compressibility, column 4 the change per kg/cm^2 of the specific resistance ρ (column 1 corrected by the linear compressibility), column 5 the change per kg/cm^2 pressure of that part of the specific resistance to be attributed to the change in the free path (column 4 corrected by the volume compressibility)* and column 6 is the ratio of 5 to 3, or the factor by which the change of linear dimensions is to be multiplied to give the "free path" change of conductivity.

In the second part of Table III, column 1′ gives the tension coefficient of specific resistance (listed in Table I), column 2′ the change of length with tension, column 3′ the change with tension of that part of the specific resistance due to change of free path (column 1′ corrected by column 3, since the change of volume under 1 kg/cm^2 tension is 1/3 of the change under 1 kg/cm^2 pressure), column 4′ is the ratio of 3′ to 2′, or the factor by which the change of linear dimension when produced by tension is to be multiplied to give the "free path" change of conductivity, and column 5′ is the ratio of 6 to 4′.

The geometrical analysis above has suggested that the figures in column 5′ should be 4.7. The experimental figures vary on both sides of it. With the exception of Al, the variation is within a range of 30 or 40%, and I believe is not more than the rough nature of the considerations would lead us to expect.

The result of a similar analysis from the classical electron theory point of view is of interest. In this theory the free path of the electron is entirely free, there is nothing analogous to a "groove" in which it moves, and an applied e.m.f. will produce the full drift in its own direction irrespective of the relative direction of the path. The above analysis is to be modified by omitting the factor $\cos^2\theta$. It will be found, contrary to what one might think at first, that the theory does give the correct sign to the effect. The reason is that the electrons inclined to the tension at an angle approaching a right angle are so much more numerous than the others that their contribution to a diminished conductivity more than wipes out the contribution of the others to an increased conductivity. The ratio which we found above to be 4.7 now comes out numerically to be 21. However, the fact that the classical theory gives the correct sign to the tension effect is to be considered as a curiosity only, for it gives the wrong sign to the hydrostatic pressure effect, unless hypotheses concerning the change in the number of free electrons under pressure are made *ad hoc*.

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* The total conductivity is proportional to the number of electrons per cm^3 × free path × velocity of electrons. The velocity is constant at constant temperature. Our hypothesis has been that the ratio of the number of electrons to the number of atoms is constant. Hence the number of electrons per cm^3 changes as the volume of the metal, accounting for the compressibility as a correction term.

SUMMARY.

An improved method has been developed and the thermal conductivity under tension of seven metals measured. The change of electrical conductivity under tension of the same metals has also been measured. With the exception of nickel, which is abnormal with respect to both thermal and electrical conductivity, the effect of tension is to diminish thermal conductivity, contrary to the results of the only two previous observers, who found an increase of thermal conductivity with tension. The decrease of thermal conductivity may be greater or less than the decrease of electrical conductivity, that is, the Wiedemann-Franz ratio may either increase or diminish with tension.

In the theoretical discussion it is suggested that the contribution of the atoms to thermal conductivity may be responsible for the varying signs of the tension coefficient of the Wiedemann-Franz ratio, in the same way that I have previously suggested that it may be accountable for the varying sign of the pressure coefficient. The theory of electrical conduction which I have been developing suggests that the fractional change of electrical conductivity produced by hydrostatic pressure should be a considerably higher multiple of the change of linear dimensions than the change produced by tension, and analysis gives a numerical value for the ratio of these two effects which agrees as closely with experiment as the rough nature of the considerations would suggest. The classical theory, on the other hand, would give the wrong sign for this effect.

I am indebted to my assistant Mr. H. B. Curry for making many of the readings of this paper.

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THE THERMAL CONDUCTIVITY OF LIQUIDS UNDER PRESSURE.

By P. W. Bridgman.

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TABLE OF CONTENTS.

Introduction 141
Apparatus and Method 142
Corrections 148
Procedure 152
Detailed Data 155
  Choice, Source, and Purity of Materials 156
  Discussion of Results 158
    Temperature Coefficient of Conductivity 160
    Pressure Effects 160
Theoretical Considerations 162

Introduction.

In this paper I give the values of the thermal conductivity of fifteen liquids as a function of pressure between atmospheric and 12000 kg/cm^2 at two temperatures, 30° and 75°. The liquids chosen for investigation are for the most part those for which I have previously measured the compressibility and thermal expansion over the same range of pressure.[1] As far as I know, there have been no previous determinations of the effect of pressure on the thermal conductivity of liquids, so that the results of this paper are essentially new.

The whole field of the thermal conductivity of liquids is still largely unworked. With regard to experimental technique, there is the fundamental question of separating the part of the heat carried by radiation through the liquid from the part carried by conduction proper. This is discussed in the following. It will be seen that error from this effect may have considerably affected many previous determinations; it may well be intimately connected with the failure of many previous experimenters to agree as to the sign of the temperature coefficient of thermal conductivity. Theoretically, the thermal conductivity of liquids has been even more neglected. I have not been able to find a single attempt to deduce theoretically an expression for the thermal conductivity of a liquid, and there seems to be only one empirical expression, that of Weber.[2] In the following I give an expression for thermal conductivity which rests on a theoretical picture of extreme simplicity, and which, considering its simplicity, is in surprisingly good agreement with the facts.

APPARATUS AND METHOD.

The method to be employed is restricted by the requirement that the apparatus be of small enough dimensions to be contained in the pressure cylinder, and also by the requirement that the heat leak through the pressure transmitting liquid be small. This last requirement rules out a number of methods applicable at atmospheric pressure in which the heat leak from one part of the apparatus to the other is through the air and is for this reason small; such a method adapted to pressure would give a much larger heat leak because the air must be replaced by the pressure transmitting liquid.

The method adopted was one of radial flow between concentric cylinders, and is somewhat like that which I formerly used for the more poorly conducting metals.[3] As far as I can find, this method has not been previously applied to measuring the thermal conductivity of liquids; it seems to offer considerable advantages over nearly all other methods, and may well be used to give more accurate results than have hitherto been attained at atmospheric pressure, as it is not at all restricted to use under pressure. The method is very rapid as well as accurate, and is particularly adapted to giving relative conductivities referred to some standard substance. Its chief disadvantage is that considerable mechanical skill is necessary in the construction of the apparatus.

Two years ago I made a preliminary attack on this problem with an apparatus somewhat like the final one, but in which the heating element and thermo-couples are both directly immersed in the liquid. This led to short circuiting effects, and furthermore, there was no adequate method of preventing the pump liquid from mixing with the liquid under measurement. With this apparatus I obtained rough values for the pressure effect on kerosene, petroleum ether, methyl alcohol, and ethyl alcohol. These results I described at the spring meeting of the National Academy of Sciences in Washington 1921, but I have published only the values of the relative conductivity of petroleum ether,[4] which were needed as a correction term in my experiments on the thermal conductivity of metals under pressure.

The final apparatus is shown in Figure 1. The liquid to be measured, shown shaded, is contained in the annular space between two copper cylinders A and B. The outer copper cylinder B is placed inside the pressure cylinder, in which it is a fairly good fit. The pressure cylinder is filled with a liquid transmitting pressure equally to all parts of the apparatus. Thermal contact between B and the pressure cylinder is further secured by an arrangement of flat strips of spring copper (not shown in the diagram) like those which have been described in a previous paper.[5] The source of heat is at the axis, H, of the inner cylinder, where there is an insulated wire carrying a current of electricity. Heat flows radially from this wire through the cylinder A, the layer of liquid, the cylinder B to the pressure cylinder, and out through its steel walls to the temperature bath. The bath is rapidly stirred and maintained at a constant temperature by a regulator.

The thermal conductivity of the liquid may be calculated when the dimensions of the apparatus are known, the rate of heat input, and the difference of temperature between the outer and inner surfaces of the liquid. The formula for this is

$$k = \frac{Q}{2\pi(\theta_1-\theta_2)} \log\frac{r_2}{r_1},$$

where k is the thermal conductivity of the liquid, Q the heat input per unit of time per unit of length along the axis, $\theta_1-\theta_2$ the difference of temperature between the outer and inner surfaces of liquid, and r_2 and r_1 are the outer and inner radii of the layer of liquid.

Because the thermal conductivity of the copper is so much greater than that of the liquids (2500 times greater for the ordinary liquid) the temperature difference $\theta_1-\theta_2$ may be obtained with sufficient accuracy by measuring the temperature of the metal of the cylinders A and B at points near the liquid. Practically the entire drop of temperature which drives the heat flow takes place across the layer of liquid.

The heat input Q is measured in terms of the resistance of the wire per unit length and the current. The difference of temperature is determined from the e.m.f. of

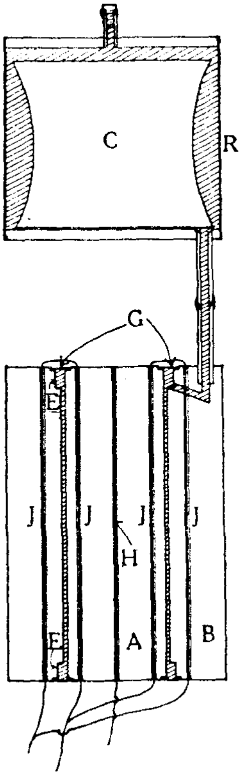

FIGURE 1. Section of apparatus for measuring thermal conductivity of liquids. The liquid is shown shaded. The upper part is a reservoir to supply the volume lost by compression. The conductivity is measured in the lower part. Heat flows radially from the heater H across the copper cylinder A across the concentric layer of liquid (shaded), across the outer copper cylinder B, and out through the surrounding pressure cylinder (not shown).

thermo-couples of copper-constantan, one of whose junctions, J, is in A and the other in B. Three of these couples were used, situated at angular intervals of 120° and connected in parallel. For convenience in drawing, two couples are shown in Figure 1. In this way the mean of three temperature differences was obtained, thus correcting for any slight geometrical irregularities.

The apparatus for measuring the heating current and the e.m.f. of the couples was the same potentiometer arrangement as that used in previous work.[6] Its mode of construction as well as the various corrections necessary in its use have been fully described previously and need not be entered into here. All measurements could be made to considerably better than 0.1%.

The heating current was of the order of 0.3 amp.; this was maintained at the requisite constancy by an independent storage battery in series with a General Electric Co. iron-in-hydrogen ballast lamp. The resistance of the heater was such that the heat input per unit length was of the order of 0.06 watts. For the ordinary liquids, such as the alcohols, this produced between A and B a temperature difference of the order of 0.6°, giving a thermal e.m.f. of 25 micro volts.

The thermo-couples were of copper-constantan 0.006 inches (0.015 cm.) in diameter. They were composed of three pieces of wire, a central length of constantan flanked by two lengths of copper. Connections from copper to constantan were made by butt silver soldering, the entire couple after soldering being perfectly uniform in size, so that it was not possible to detect any irregularity by running the wire through the fingers. The construction of such couples has been previously described.[7] The length of the constantan was so chosen that the two junctions copper-constantan were situated at the center of the length of the cylinder, as shown in the drawing. Situated here, any correction for end effects is a minimum. The couples were insulated with seven or eight coats of insulating enamel baked on at a temperature of about 210°. A simple calculation will show that because of the small diameter of the wire any heat leak along the constantan from the junction in A to that in B is entirely negligible.

For the heater, the high resistance alloy known as "Chromel" was used. In the earlier work a wire 0.0035 inches (0.0102 cm.) in diameter was used. This wire was originally bare, and it was insulated with enamel baked directly onto it. The mechanical strength of the insulation did not prove sufficient, however, and there were occasional short circuits. The original heater was therefore replaced in most of the work by a heating unit of 0.005 inch (0.0127 cm.) Chromel wire, double covered with silk insulation, which was strengthened by insulating enamel baked onto it for a number of hours at a temperature low enough not to char the silk. Connection was made to the heating element at the lower end of the cylinder by a thin strip of copper, prevented from grounding to the cylinder by a thin washer of mica, and at the upper end it was grounded to the cylinder by a thin strip of German silver to which it was soldered. In this way the heater could be made accurately of the same length as the cylinder. Connections were made to the measuring apparatus through the same three terminal plug as was used in the previous work with the metals. Two of the three insulated terminals were for the thermo-couple connections, and the third was for that end of the heater which was not grounded.

Various details of the mechanical construction require comment as the successful performance depends essentially on them. In the first place, and most important, in order to secure freedom from convection currents in the liquid, particularly as the cylinders are used in a vertical position, it is necessary that the annular space between the cylinders be small. This distance was only 1/64 inch (0.040 cm.), the outer diameter of the inner cylinder being 3/8 inch (0.95 cm.), and the inner diameter of the outer cylinder 13/32 inch (1.03 cm.). In the early days of thermal conductivity measurements a number of radial flow methods between concentric cylinders were used [8] (without however using an axial source of heat as above) which have been later more or less discredited because of error due to convection. The layer of liquid used in this present apparatus is many times thinner than the thinnest used previously. Convection is further reduced by the unusually small temperature difference between the inner and the outer cylinders. The heat conveyed convectively varies, other things being equal, as the square of the temperature difference, so that it is important to keep this difference down. The thinness of the layer of liquid and the smallness of the temperature difference should together make any convection errors entirely negligible. That the performance of the apparatus in this regard was satisfactory may be seen by comparing the values of the absolute conductivity found with it with the best of previous measurements.

A further advantage of the thin layer is that the proportional part of the total heat transfer due to direct radiation through the liquid from the wall of A to that of B is thereby made a minimum. This will be discussed later.

The length of the cylinder should be as great as possible subject to the restrictions of construction and its use in the pressure cylinder.

The length actually used was 1.25 inches (3.18 cm.), 80 times as long as the thickness of the layer of liquid.

The inner cylinder was supported concentrically with the outer by rings of German silver, G, at top and bottom, 0.002 inch (0.005 cm.) thick. It is here that the difficulty of mechanical construction enters. These rings were first spun into shallow grooves turned on the two cylinders, and were then made leak tight by a very small amount of soft solder. I am indebted to the skill of Mr. David Mann for the construction of these cylinders. There is of course some heat leak from inner to outer cylinder at the two ends across the German silver rings. This is small because of the thinness of the German silver and its poor thermal conductivity. The leak was further reduced by increasing the radial separation of the two cylinders at the two ends, so that the conduction from one to the other through the liquid was decreased. By choosing the dimensions of the enlargement, E, correctly it is possible to make the decreased conductivity through the liquid exactly compensate the increased conductivity through the German silver. The dimensions were so chosen that this compensation should be exact for the average of the liquids used.

The heating unit and the thermo-couples were threaded through the cylinders in copper tubes 0.016 inch (0.041 cm.) inside diameter and 0.040 inch (0.102 cm.) outside diameter. These copper tubes were sweated into holes 0.040 inch (0.102 cm.) in diameter which were drilled lengthwise of the cylinders at the proper places, shown sufficiently to scale in the diagram. Because of the relatively low thermal conductivity of the liquids, no trouble was ever experienced arising from poor thermal contact in the sweated joints such as I had found previously with metals.

Because of the volume compression of the liquid as pressure is raised, it is necessary to provide a reservoir of the liquid outside of the space between the concentric cylinders. This was accomplished as follows. The reservoir, R, was made of a thin collapsible tube of tin 0.75 inch (1.78 cm.) in diameter, soldered at top and bottom with an easily fusible solder to end discs of brass. It was filled with liquid and connected with the annular space between the cylinders through a German silver tube. In order to cut down the volume compression as much as possible, the interior of the collapsible tube was filled with a metal core, C, of the shape shown, of such dimensions that when the tube was entirely collapsed against the core there should be a loss of volume of perhaps twice the volume compression of the liquid between the cylinders under the maximum pressure. This scheme was entirely successful in separating the liquid to be measured from the surrounding liquid, petroleum ether or kerosene, by which pressure was transmitted from one part of the apparatus to another, and in transmitting the pressure freely to the liquid between the cylinders. The thin rings of German silver separating the inner from the outer cylinder never showed the slightest sign of deformation after a run, which, in view of their extreme thinness, is good evidence that the pressure in the liquid between the cylinders must have been very nearly the same as that in the liquid outside.

Usually it was possible to complete a set of readings on one substance, which consisted of runs at two temperatures, without rupture of the tin container, and therefore without mixing of the two liquids. At first I was afraid that there might be slight mixing, which might introduce error, and a special procedure was adopted to eliminate it. This consisted in making nearly all the readings with increasing pressure, at intervals of one thousand kilograms, instead of the usual procedure of reading at the even thousands going up and the odd thousands coming down. The purpose of the usual procedure is to eliminate error from hysteresis. By changing the procedure, any error due to a leak of the outside liquid into the reservoir and a mixing of the two liquids on releasing pressure seemed more likely to be avoided. Furthermore, because these measurements are on liquids, instead of on solids as usual, no hysteresis is to be expected, and therefore there is no reason why the procedure should not be changed. Further, check readings were always taken at 6000 and 0 after the succession of readings with increasing pressure; these check readings were found to agree with the others. The revised procedure therefore turned out after all not to be necessary. However, as the measurements had been begun in this fashion, they were continued so to the end. Even in those few cases in which the tin tube was found ruptured after a run with some escape of the inner liquid, the mixing must have been confined to the reservoir, and could not have reached to the region between the cylinders, as was proved by the consistency of the check readings.

A new tin reservoir was made for each new liquid, and connected to the copper cylinder by a soldered joint in the German silver connecting tube. The apparatus was filled in vacuum. A glass reservoir was connected to a tube at the upper end of the tin reservoir, and the whole apparatus was placed under the receiver of an air pump, and exhausted to the boiling pressure of the liquid in question. Air was then admitted, driving the liquid into the reservoir and between the copper cylinders, and then the filling was completed by several successive boilings in vacuum, followed by admission of air. The upper end of the tube leading from the tin reservoir was then made tight by a screw soldered into place with a touch of easily fusible solder, just enough of the liquid in the upper end of the tube having been removed to make this possible.

The copper cylinders were cleaned after a run with one liquid ready for the next by several washings with ether and dryings in vacuum at room temperature or temperatures up to 130°, depending on the liquid used. There was some chemical action between the liquids and the copper, because the surface of the copper was found blackened on one occasion when an accident made necessary reassembling the cylinders, but such action must have been very slight as shown by the consistency of the check readings and the complete clearness of the liquids removed from the apparatus after a run.

CORRECTIONS.

There were the usual corrections to be made to the pressure and other electrical measurements; these need not be described in detail again here. In addition there were several peculiar to this work.

The only correction arising from the change of dimensions of the apparatus is that due to the change of length. Since the geometrical figure remains the same, the ratio of the inner and outer diameters of the layer of liquid is not altered by pressure. The change of length enters in determining the resistance of the heating element per unit length. It was assumed that the heating element retains the length of the cylinder, an assumption justified by the observation that the wire is not stretched perfectly tight, but has enough curvature to rest in spots against the sides of the containing tube. The correction for the change of length is only 0.3% at the maximum pressure of 12000 kg/cm^2. The resistance of the heating element changes under 12000 kg. by about 0.15%; its change with temperature between the extremes of these experiments, 30° to 75°, is about 0.7%. These corrections had been determined by previous measurements.[9] The thermo-electric force of the couple of constantan-copper decreases under 12000 kg. by 0.77% at 30° and 0.73% at 75°. This again was determined in former work.[10]

The change of the thermo-electric power of the couple with temperature is a much more important factor than any of the above, and must be accurately determined if the temperature coefficient of thermal conductivity at atmospheric pressure is to be obtainable from these measurements. In order to get this correction I made special measurements on a couple constructed of wire cut from contiguous lengths of the same spools as those from which the couples for use under pressure were made. The thermal e.m.f. of this couple was measured as a function of temperature between 20° and 95°, one of the junctions being maintained continually in ice. The measurements were checked within the limits of error by readings with both increasing and decreasing temperature. With an error at no point greater than 0.1% of any single reading, dE/dt over this range is a linear function of the temperature, and is given by the formula

$\frac{dE}{dt} = (38.26 + .0862\,t) \times 10^{-6}$ volts, where t is the temperature in degrees Centigrade.

This may be compared with results for other samples of constantan and copper. The figure in which we are most interested is the ratio of the thermo-electric powers at 30° and 75°. For the present material this is 1.0949. For another couple of different size wire I have previously found 1.0974,[9] and Adams for another sample gives figures from which I calculate the ratio 1.0902.[11] The differences are small, but it is nevertheless evident that there are slight real differences between couples of different construction (the difference is quite as likely to be in the copper as in the constantan), and that for accurate determinations of temperature variations the particular material must be independently calibrated.

In addition to these corrections, there is a correction for heat leak from the inner to the outer cylinder due to the short circuiting action of the pressure transmitting medium in which the whole apparatus is bathed. The magnitude of this correction was determined by measuring the apparent thermal conductivity of the most poorly conducting of the liquids used in this work, namely ethyl iodide, for which the effect is greatest, first when the apparatus was mounted in the pressure cylinder without any transmitting liquid, and then when the pressure transmitting liquid was admitted. The heat leak in the first case is through the air only and may be neglected. In this first case good thermal contact between thermo-couples and heating element and the copper cylinders was ensured by a film of oil in the tubes carrying them. The correction so found was 3.5%. It is proportionally less for the better conducting liquids. This heat leak correction changes with pressure, because the effect of pressure on the thermal conducting liquid is in general different from that on the liquid under investigation.

At 30°, where it was necessary to use petroleum ether as the transmitting liquid in order to avoid mechanical deformation from increased viscosity under pressure, this correction in all except two cases increased with rising pressure. This is because the increase of thermal conductivity of petroleum ether is greater than that of the liquid being measured. It was assumed that the change in the correction was proportional to the relative change in conductivities. Thus if under 12000 kg. the increase of thermal conductivity of petroleum ether is 50% greater than that of the liquid being measured, then the heat leak at 12000 was 50% greater than at atmospheric pressure. At 75°, however, where the transmitting liquid was kerosene, the heat leak usually decreased with rising pressure. Special measurements were made on petroleum ether and kerosene to determine their conductivity under pressure.

It will be noticed that all the corrections of this method are unusually small, whether we are discussing the corrections to be applied under pressure in determining relative changes of conductivity, or the corrections to be applied at atmospheric pressure in calculating the absolute conductivities. In fact, in no case did the corrections amount to more than 4% of the effect measured. This is much smaller than for most methods at atmospheric pressure; it is gratifying that it was possible to keep the pressure corrections down to such a low value. The relations with these liquids are very much more favorable than they were for the metals previously measured, when the corrections might become as large as the effect sought.

There is another entirely different sort of correction which may well play an important part in thermal conductivity in liquids, although it has received exceedingly meager discussion; this is the effect of heat transfer by heat radiation through the liquid. There are two kinds of radiation effect to be considered. The first is the transfer of heat by radiation through the liquid from one of the walls of the containing vessel to the other, in the case above from the outer wall of the inner cylinder to the inner wall of the outer cylinder. This effect is important only in layers of liquid so thin that a large fraction of the radiation emitted from the walls is transmitted through the liquid without absorption. The thicknesses ordinarily used in thermal conductivity measurements of liquids seem to come within this range. The data do not seem to exist by which the correction can be calculated exactly. What is needed are the radiation emissivity constants of the metal of which the walls are composed at temperatures in the neighborhood of room temperatures and at those wave lengths which correspond to the maximum emission at such temperatures. An upper limit to the correction can be obtained, however, by assuming that the walls radiate like a black body. If the two walls of the container are fairly close together, we may treat the radiation from each wall like that from an infinite plane. The net energy radiated per unit time from unit area of that wall whose temperature is $\Delta\theta$ higher than the other is given by the formula

$$u = 4K\,\theta^3\,\Delta\theta,$$

where K is the radiation constant, and is 5.72×10^{-5} in Abs. C. G. S. units.

For 30° C. and 75° C. the formula gives the numerical values: 63.6×10^2 and 96.7×10^2 ergs, respectively, per unit temperature difference. This radiated heat is independent of the distance of separation of the walls. If now the distance between the walls is l, the amount of heat conducted from one wall to the other through the liquid in unit time is $k\,\frac{\Delta\theta}{l}$. The ratio of this to the heat transferred by radiation is

$$f(\theta)\,\frac{k}{l}.$$

With the dimensions of the apparatus used above this gives for water a ratio of 240 at 30° and 157 at 75°, or corrections for radiation of 0.42% and 0.64%. For methyl alcohol the corrections are proportionally larger, being 1.22% and 1.85% respectively.

What data there are on the emissivity of metals at low temperatures[12] would indicate that this correction, calculated as an upper limit, may be many fold too large. In consideration, therefore, of the uncertainty, I have not applied any correction at all in the following for this effect. It is to be emphasized, however, that the correction is proportional to the thickness of the layer of liquid, and that therefore the measurements here described are more advantageously situated in this regard than any previous ones. It seems to me very probable that this correction may well have a quite perceptible effect on some of the previous work; the correction is in such a direction as to make the previously calculated values larger than they should be. Since the correction increases with rising temperature, this means that too rapid an increase of conductivity with temperature may have been found previously.

The second way in which heat may be transferred by radiation is by internal radiation, from one part of the liquid to another. Every volume element of the liquid radiates in virtue of its temperature, the radiation being greater the higher the temperature, and is a function of the liquid. Radiational equilibrium is maintained by the absorption in every volume element of a portion of the radiation from other parts of the liquid which is passing through it. If there is a temperature gradient in the liquid, there will be a net transfer of heat across any surface by radiation from the side which is hotter to the side which is cooler. Because the radiation at ordinary temperatures is not large, and because most liquids are fairly transparent, it is probable that this effect is small compared with conduction proper, and that it reaches its maximum amount only in masses of fluid so large that all the energy radiated from an element is reabsorbed before reaching the boundaries. Theoretically it does not seem certain whether there is any difference in principle between such a transfer of heat by internal radiation and the ordinary transfer by conduction. It may be that the conduction transfer is the same in kind, except that the transfer is by elastic waves from atom to atom, and that the absorption of these elastic waves is very much greater than that of the ether radiational waves, so that the effect reaches a limiting value for thicknesses of the film of liquid which are below our powers of observation. In any event, it seems that there are questions here which have not hitherto been considered with sufficient care. Experimentally it would be interesting to determine whether a liquid may not have two apparent thermal conductivities, one obtained in the limit for thin layers, and the other as the limit for thick layers. Even the possibility must be recognized that there may be no exact limit for thin layers, but that the elastic waves which convey the ordinary conduction heat are so long that the effect of their existence may be perceptible in measurable dimensions.

PROCEDURE.

The conductivity apparatus was filled with the liquid to be measured in the manner already described. The apparatus was then attached to the three terminal plug, and connections made between its terminals and the thermal junctions and heating element by soldering. The plug with the conductivity apparatus was then screwed as a single self contained piece into the pressure cylinder. Connections were made between the external leads and the leads to the measuring apparatus by soldering. The external leads were protected by a length of flexible rubber tubing pulled over them and tied to the steel cone which formed the lower part of the three terminal plug. The temperature bath of water was then placed around the pressure cylinder, the regulator adjusted, and a sufficient time allowed to elapse for the apparatus to come to thermal equilibrium. This was shown by the vanishing of the parasitic e.m.f.'s in the thermo-couple circuit. The heating current was then switched on, and again sufficient time allowed to reach a steady state. This was now shown by the constancy of the thermo-couple readings. Regular readings were now made. These consisted of two readings of the resistance of the manganin pressure gauge on the Carey Foster bridge, and four readings each of the heating current and the thermo-couples. These four readings were made for both directions of the heating current, and for both polarities of the balancing e.m.f. In this way the effect of parasitic e.m.f.'s in the parts of the circuit external to the temperature bath were eliminated, and also the effect of any defective insulation, should there be such, between the heating circuit and the thermo-couples. All the runs retained in the final calculations were ones in which the insulation was perfect as far as could be determined. The insulation of the plug was further tested independently after every run, with the connections unsoldered, and several times the insulation of the plug was tested independently under pressure, with no leads connected. Once in a great while there was failure of the insulation, which was then repaired by re-enamelling the thermo-couples, or else installing new ones. The insulation of the heating element did not give trouble. The four readings obtained for heating current and thermal e.m.f. do not give evidence as to the freedom of the circuits inside the pressure apparatus from parasitic e.m.f.'s. This check could not be conveniently made during the course of the measurements, as has already been explained in connection with the measurements on solids. The independent test before applying the heating current should give sufficient security on this point.

After the readings of pressure, heating current, and thermal e.m.f. were completed, pressure was increased by 1000 kg., and after conditions had again become steady, the same succession of readings was repeated at the higher pressure. The time required to reach equilibrium was about ten minutes. This is very much less than in the majority of thermal conductivity experiments; it is due to the thinness of the layer of liquid, the smallness of the apparatus, and the fact that much metal was used in the construction.

The succession of readings described above was repeated at increasing intervals of 1000 kg. to a maximum pressure of 12000 kg. Check readings were then made at 6000 and zero again. This was done for two temperatures, 30° and 75°, so that we have results for the temperature coefficient of the pressure coefficient, and also for the temperature coefficient of conductivity at atmospheric pressure, for which there are only a few previous data.

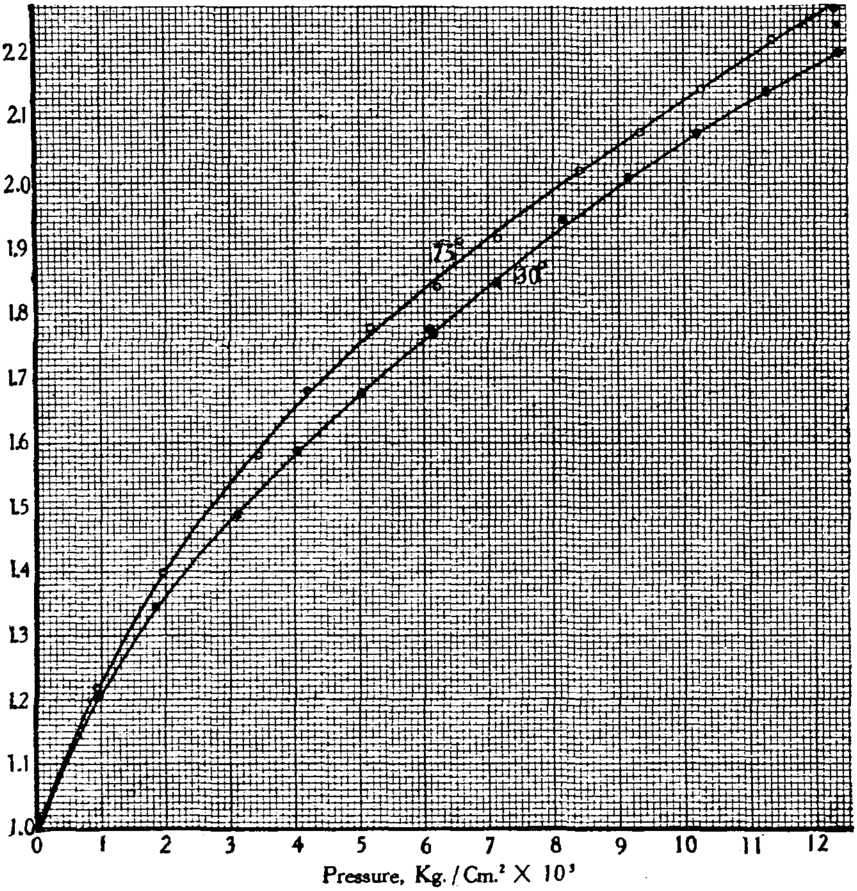

FIGURE 2. Shows the relative changes at 30° and 75° of the thermal conductivity of iso-propyl alcohol as a function of pressure. The points shown are observed points, with small corrections still to be applied.

The run at 30° was made with petroleum ether as the pressure transmitting medium. At 75° this was replaced by kerosene, because the low boiling point of the petroleum ether, 40°, would have given inconvenience. The kerosene was introduced into the apparatus in place of the petroleum ether merely by raising the temperature at atmospheric pressure, allowing the ether to boil out, and then forcing kerosene into its place with a very slight pressure. For those liquids which boil at atmospheric pressure below 75°, the lowest pressure reading at 75° was made under a pressure of 100 kg. or so, and the results which are tabulated in the following were obtained by extrapolation to zero, the amount of the extrapolation being usually only two or three per cent.

Nearly all the points obtained lay on smooth curves within a small limit of error. In fact, the regularity of the results was very gratifying. Occasionally, however, a point was obtained which lay very widely off the curve. This was almost always due to failure of the temperature regulator of the bath to function properly. If the temperature of the bath got away from its steady value by as much as one tenth of a degree, as it did sometimes from accidental causes, a sufficient temperature gradient was set up in the leads of the three-terminal plug, which consisted of piano wire and copper, to introduce comparatively large parasitic e.m.f.'s, and so give bad points. These points were of course discarded in taking the final results, and the fact that they were found at all has nothing at all to do with the validity of the rest of the measurements.

In view of the regularity of the results it does not seem necessary to reproduce in detail the data obtained for each substance. An idea of the consistency of the results may be obtained from Figure 2, which shows the *uncorrected* relative results obtained for iso-propyl alcohol, a substance for which the results were of average regularity.

DETAILED DATA.

Most of the results for the various substances can be collected into a systematic table, in which all the substances are treated alike. In this table will be given the absolute conductivities at atmospheric pressure at 30° and 75° (extrapolated values at 75° marked with an asterisk), the relative conductivity at intervals of 1000 kg. at each of these temperatures in terms of the conductivity at atmospheric pressure as unity (the changes are smoothed so as to give regular first differences), and finally at the end of each row of pressure data the average deviation from a smooth curve of the observed points (not including in the average the points discarded as already explained because of failure of the temperature regulator). The average deviation is given as a per centage of the maximum effect; it was obtained by adding graphically (with the dividers) all the deviations, and dividing this sum by the total number of summands and the maximum ordinate, a number usually in the neighborhood of 2.25.

In addition to the data which are of the same character for all substances and can be systematically treated in a table, there are various individual differences, which will be specifically mentioned in the following.

Choice, Source, and Purity of the Materials. In the first place, I endeavored to measure as many of those liquids as possible for which I had already determined the compressibility and other thermodynamic properties under pressure.[1] There were twelve of these liquids. Of these ten are studied here. Ethyl chloride was omitted because of the difficulty of manipulating it arising from its volatility, and phosphorus trichloride was omitted because of the difficulty of getting it pure and because of its corrosive action on metal. It was not feasible to use the same sorts of propyl and butyl alcohols in the two series of measurements. The thermodynamic data under pressure were determined for normal propyl and iso-butyl alcohols, whereas the thermal conductivity measurements of this paper were made on isopropyl and normal butyl. The two sets of measurements are not, therefore, exactly comparable. In addition to these ten liquids, petroleum ether and kerosene were measured, particularly because their properties enter the corrections, and water, toluol, and normal pentane. The list of easily obtainable liquids which may be studied over a wide pressure range is not large, because so many freeze under pressure. Thus at 30° the usual pressure range of 12000 kg. was curtailed by freezing in the case of water, acetone, and toluol. For toluol I was able to obtain a value for the thermal conductivity of the solid (there seems to be only one other determination of the thermal conductivity of an organic liquid in the liquid and solid states [13]). It would not have been possible to undertake a systematic study of this subject with the present apparatus because of danger of mechanical rupture when pressure is applied to the solid.

The following liquids I owe to the kindness of Professor Frederick Keyes of the Mass. Institute of Technology: methyl and ethyl alcohol, sulfuric ether, carbon bisulfide, toluol, and normal pentane. These were of extreme purity, and were purified by the method described by him.[14] The purity is so great that it is possible to use the freezing points as fixed points in low temperature thermometry. The liquids were sealed into glass containers immediately after purification, and the containers were not opened until immediately before the filling of the pressure apparatus. Iso-propyl and normal butyl alcohol I owe to the kindness of Professor Brinell of Bryn Mawr, by whom they had been especially prepared and purified. These were not sealed into glass, but after preparation were left standing in the dark in glass stoppered bottles for a year or so. The acetone (from bisulfite) was obtained from the Eastman Kodak Co., as were also the iso-amyl alcohol, and the ethyl bromide and iodide. These were shipped in glass stoppered bottles, sealed with celluloid, and were perfectly colorless, except the ethyl iodide, which showed a slightly brownish tinge. I redistilled this immediately before use in a Hempel column, completely removing the color. In spite of the color, the distillation took place completely at a temperature constant within 0.1°. The water was ordinary laboratory distilled water, distilled in a tin still. Kerosene and petroleum ether were commercial articles. These are probably not entirely reproducible, but pains was taken during the experiment to use only material from the same original lots, so that there can be no error in the corrections due to chance variations of the composition of either of these liquids.

The precautions described above to ensure pure materials are probably quite superfluous for this kind of work, because the thermal conductivity of all known organic liquids (excepting water) varies over a range of only two fold, and it has been proved by Lees [15] that the conductivity of a mixture may be calculated by the law of mixtures from the conductivities of the constituents.

For most of the substances only one set of runs, with a single filling of the apparatus, was made. The following substances are exceptions. A number of preliminary runs were made with methyl alcohol, with a poorly constructed preliminary apparatus, giving results which could not be retained. With the final apparatus, runs were made with two separate fillings, there having been difficulties due to leak in the pressure cylinder, and also leak from the tin reservoir. The less satisfactory of these two fillings gave results for the conductivity at atmospheric pressure differing from the more satisfactory one by 1% at 30° and 2% at 75°. Two fillings with ethyl iodide were necessary because of escape of the liquid from the tin reservoir. Check readings of the absolute conductivity at atmospheric pressure at 30° differed by 0.5%. Two complete sets of runs were made with water, not because of any difficulty, but because of the importance of the substance and in order to check the readings. The second run was made with a modified arrangement of the electrical measuring apparatus which did not allow the absolute values of the conductivity to be obtained with accuracy, but which did give accurately the relative values of the conductivity. The maximum difference between the smoothed results of these two sets of runs at any point was 0.8%, and the average arithmetical difference was 0.29%. Because of the high value of its absolute conductivity, the check afforded by the repetition is more severe in the case of water than for any of the other liquids.

It may be thought that the consistency of the results would justify one more significant figure than is given for the absolute conductivity. However this is probably not true, the absolute conductivities being limited by the accuracy of the measured dimensions of the apparatus.

Discussion of Results. The results are now given in Table I.

Before discussing the pressure effects, it will pay to consider the absolute values of the conductivities at atmospheric pressure, as this is a subject with regard to which previous measurements are considerably at variance, and the results obtained with this new apparatus should be of value from this point of view, as well as with regard to the pressure effects.

In Table II are collected for comparison my values of the absolute conductivity at atmospheric pressure at 30° and the best of previous determinations. Few of these previous determinations have been made at 30°; in the table I have indicated the temperature at which they were made. In most cases the temperature coefficient of conductivity is so small that the correction for temperature difference would be beyond the accuracy of previous work; in those cases where a temperature correction seems called for, the necessary data may be obtained from my values at 30° and 75°, assuming a linear relation between temperature and conductivity. (In the case of the data of Goldschmidt, I have placed in parentheses besides Goldschmidt's values for 0° the values which would be computed for 30°, using my value of the temperature coefficient.) The values listed in the Table are only the more recent ones, and those which apparently represent the most careful work. The discrepancies are very large, much larger than they should be for measurements on well defined substances. On the whole my agreement with Goldschmidt, the most recent as well as the most extensive of the previous work, is the best. The agreement with Lees is also good.

TABLE II.

Comparison of Thermal Conductivity at Atmospheric Pressure by Different Observers.

Liquid	Goldschmidt[16] 0°	Lees[15] 25°	Weber[2] 1880 0°	Weber[2] 1885 9°–15°	Graetz[17]	DeHeen[18]	Chree[19] 19°	Bridgman 30°
Methyl Alcohol	.000524 (522)	48		495		393	58	505
Ethyl Alcohol	.000446 (436)	43	49	423	55	347	59	430
Isopropyl Alcohol	.000368 (366)			373(?)				367
Isoamyl Alcohol				33		265		354
Ether	.000338 (333)		41	303	38			329
Acetone	.000423 (407)							429
Carbon Bisulfide	.000388 (375)		42		27		54	382
Ethyl Bromide				247				286
Ethyl Iodide				222				265
Water	.00150 (157)	136	124	136	157		136	144
Tolnol	.000349 (333)			307				364

Probably the most careful work on thermal conductivity of liquids has been by Milner and Chattock [20] and by Jakob [21]; both are, however, confined to the single liquid water. In the paper of Jakob will be found a careful critical survey of previous work, in which the various sources of error are pointed out. These possible errors seem so large that none of the work before Jakob can be relied on to give even the sign of the temperature coefficient of conductivity. (Milner and Chattock did not attempt to get the coefficient of water with any accuracy, but from the results of readings differing in mean temperature by only one degree they estimated that the coefficient would be found to be positive.) It is gratifying to note that these best previous values for water agree closely with that found above. I find at 30° 0.00144, Jakob gives 0.001444, and the value 0.001433 given by Milner and Chattock for 20° corrects to 0.001455 at 30°, using my value of the temperature coefficient. We may probably take at 30° as the most probably correct value to three significant figures 0.00144.

Temperature Coefficient of Conductivity. I have already stated that the discussion of Jakob makes probable that none of the work before his can be relied on to give the sign of the temperature coefficient. The previous experiments which make the most serious pretensions in this regard are of Lees [15] and of Goldschmidt.[16] Lees finds the temperature coefficient of four liquids, water, glycerine, methyl and ethyl alcohol, to be negative, and fairly large, varying from 0.3 to 0.6% per degree. Goldschmidt finds the temperature coefficient of those few liquids which he measured (he did not measure water) to be always negative, and something of the order of 0.2% per degree.

I find the temperature coefficient of all the liquids measured here with the exception of water to be negative, but considerably smaller than either Lees or Goldschmidt. It varies from −0.02% per degree for iso-propyl alcohol to −0.15% for toluol and kerosene, these coefficients being in terms of the conductivity at 30°. For water I find 0.22% per degree in terms of the conductivity at 30°. The only previous value seriously to be compared with this is 0.28% at 30° of Jakob. The agreement is much closer than that of any previous work, and perhaps one should be satisfied, but nevertheless it would seem that the agreement should be closer when the self consistency of either piece of work by itself is considered, or the estimates of either of us as to the probable error of our results. The difference is in the direction of the radiation effect, discussed above.

Pressure Effects. In general characteristics the effect of pressure is the same on all the liquids tried. The thermal conductivity increases under pressure by an amount for 12000 kg/cm^2 which varies from 1.5 fold for water to 2.7 fold for normal pentane. In general the effect is greater for those substances with the lower boiling or freezing points (these are also the more compressible substances). The effect is not at all proportional to pressure, but at high pressures a given increment of pressure produces a much smaller effect, both absolutely and relatively, than at lower pressures. It may be seen from the results in Table I that the second 6000 kg. produces an effect which is in general about half that of the first 6000 kg.

The figures given in the table for kerosene smooth out an effect whose existence seems pretty certain, but which could not be established quantitatively with accuracy. Between 6000 and 12000 kg. there seems to be a slight reversal in the direction of curvature of the curve giving conductivity as a function of pressure; this effect is roughly large enough to depress the ratio at 9000 kg. to 0.020 less than the value given. Similar effects have been found previously for the volume of kerosene as a function of pressure and temperature. The explanation is doubtless to be found in the gradual freezing out over a range of pressure of the various pure components of which kerosene is composed.

The relative effect of pressure is greater at 75° than at 30° by something of the order of 5 or 10%. This change with temperature is so large that at the upper end of the pressure range the temperature coefficient of conductivity is reversed, and all the liquids are more conducting at the higher temperature at high pressures. The pressure at which this reversal takes place varies somewhat with the liquid, but as a rough average it is about 3000 kg/cm^2. This reversal by pressure of a temperature relation is not the first instance, but I have previously found [1] that at high pressures the thermal expansion of all these liquids is greater at low temperatures rather than at high temperatures, which is the normal behavior at atmospheric pressure. It may be significant that the pressure of reversal of this effect was also roughly 3000 kg/cm^2.

The conductivity of solid toluol at 11000 kg. is about 28% greater than that for the liquid extrapolated to the same pressure. We would in general expect the solid to have the greater conductivity because its specific volume is less than that of the liquid and the velocity of elastic waves in it is greater, and this is also what has been found by Barus [13] for the only other organic substance which I know of which has been measured in the solid and the liquid. Barus found at atmospheric pressure that the conductivity of solid thymol was greater than that of the liquid. The physical constants of neither toluol or thymol in the solid state are known, so that we cannot go farther and compare the difference between the solid and the liquid with the difference to be expected in terms of the theoretical picture to be given in the next section.

THEORETICAL CONSIDERATIONS.

The thermal conductivity of liquids is a subject which has received very little theoretical attention. It has been recognized that the mechanism of conduction in a liquid must be different from that in a gas. In the gas there is an intimate connection between thermal conduction and viscosity; each is connected with the length of free molecular flight, conductivity being concerned with the transfer of energy and viscosity with the transfer of momentum. The relation between these is exhibited by the elementary formula

$$\kappa = \eta C,$$

η being viscosity and C specific heat. That there is no such relation for liquids may be shown in the first place by substituting numerical values into this formula; the thermal conductivity will be found to be of the order of ten fold too small. The failure of a relation between conductivity and viscosity has also been shown by experiments[22] in which the viscosity of a solution of gelatine in water has been varied by a factor of many fold, with only slight changes in the thermal conductivity. The same is suggested by the pressure experiments above; there is very good evidence that the change of viscosity of at least some of the liquids measured above is many fold greater than the change found in the thermal conductivity.

So far as I know, there is only one empirical relation connecting the thermal conductivity of liquids with their other properties. This is the formula of Weber,[2] first given as $\kappa/\rho c =$ Const, and later modified to

$$\frac{\kappa}{\rho c}\left(\frac{m}{\rho}\right)^{\frac{1}{3}} = \text{Const.}$$

Here ρ is the density of the liquid, c its specific heat, and m its molecular weight. The factor $\left(\frac{m}{\rho}\right)^{\frac{1}{3}}$, by which the modified formula differs from the original one, is seen to be proportional to the mean distance of separation between the centers of the molecules. Weber's own data indicated a surprising constancy of the modified expression, the average for most liquids being in the neighborhood of 0.210, with extreme variations from 0.271 for sulfuric acid to 0.186 for iso-butyl iodide. Later discussion, however, has disclosed considerably greater variations than supposed by Weber. Van Aubel [23] has shown that the constant varies with temperature considerably for a single substance, and that recent determinations of thermal conductivity by Lees,[15] which presumably are more accurate, do not fit as well. It seems probable that the relation discovered by Weber has no particular absolute significance. He gave no way of calculating the constant, which turns out to have the dimensions of $M^{-\frac{1}{3}} L^3 T \theta^2$. No simple relation between a constant of these dimensions and any of the fundamental constants of nature has suggested itself to me. It is to be expected *a priori* that it would not be difficult to discover relations like those of Weber, because the properties of ordinary organic liquids vary through only a comparatively small range.

In the course of speculations on the significance of the data above, I have come across an expression for thermal conductivity which I have not found mentioned before, and which is different from that of Weber in that it enables us to calculate completely the thermal conductivity, leaving no empirical constant unaccounted for. The relation is

$$\kappa = 2\alpha v/\delta^2,$$

where α is the gas constant, v the velocity of sound in the liquid, and δ the mean distance of separation of centers of molecules, assuming an arrangement cubical on the average, and calculating δ by the formula $\delta = \left(\frac{m}{\rho}\right)^{\frac{1}{3}}$, where m is the absolute weight in grams of one molecule of the liquid.

In Table III are shown the computed and observed values of thermal conductivity for those substances for which the necessary data have been procured. The velocity of sound has been directly determined for only a few liquids. In order to treat all liquids consistently, I give in the table the velocity calculated from my values for the compressibility. The compressibility that enters the formula for velocity is the adiabatic compressibility. This is given in terms of the isothermal compressibility, the thermal expansion, the specific heat, and the absolute temperature by the formula

$$\left(\frac{\partial v}{\partial p}\right)_s = \left(\frac{\partial v}{\partial p}\right)_\tau + \frac{\tau}{c_p}\left(\frac{\partial v}{\partial \tau}\right)_p^2.$$

Here the second term is a small correction term, and the values of thermal expansion and specific heat used in the computation of it were

TABLE III.

COMPARISON OF COMPUTED AND OBSERVED THERMAL CONDUCTIVITIES AT 30°.

Liquid	Velocity of Sound cm /sec.	δ^{-2} cm^{-2}	Thermal Conductivity Abs. C. G. S. Units Computed	Observed
Methyl Alcohol	1.13×10^5	6.00×10^{14}	27.4×10^3	21.1×10^3
Ethyl Alcohol	1.14	4.74	21.8	18.0
Propyl * Alcohol	1.24	3.94	19.7	15.4
Butyl * Alcohol	1.05	3.49	14.9	16.7
Iso-amyl Alcohol	1.24	3.13	15.7	14.8
Ether	.92	3.19	11.9	13.7
Acetone	1.14	4.00	18.5	17.9
Carbon Bisulfide	1.18	4.61	21.9	15.9
Ethyl Bromide	.90	3.97	14.5	12.0
Ethyl Iodide	.78	3.81	12.1	11.1
Water	1.50	10.4	63.0	60.1

* Computed for normal propyl and iso-butyl alcohols, observed for iso-propyl and normal butyl.

taken from tables of constants. The calculated velocity agrees with the directly determined velocity within a couple of per cent where the comparison is possible.

In view of the simplicity of the relation, the agreement between experiment and calculation shown in Table III is surprising. It is particularly surprising that the high conductivity of water is reproduced so closely. The high conductivity of water therefore does not appear to be due in any special way to the molecular peculiarities of water (two or more molecular species) but is directly referable to its low compressibility and the fact that the centers of the molecules are closer together than in the average liquid.

The physical picture in terms of which this expression for thermal conductivity may be obtained is as follows. Imagine the molecules in simple cubical array, the distance between molecular centers being δ. Let there be in the liquid a temperature gradient $d\theta/dx$. The energy of a molecule is $2\alpha\theta$ (half potential and half kinetic), where α is the gas constant, 2.02×10^{-16}, and θ is the absolute temperature. The difference of energy between neighboring molecules in the direction of the temperature gradient is $2\alpha\delta\, d\theta/dx$. This energy difference is to be conceived as handed down a row of molecules with the velocity of sound. The total energy transferred across a fixed point of any row of molecules per unit time is the product of the energy difference and the number of such energy steps contained in a row v cms. long, or $2\alpha\delta\,(d\theta/dx)\,v/\delta$. The total transfer across unit cross section is the product of the transfer across a single row and the number of rows, or $2\alpha v\,(d\theta/dx)\delta^{-2}$. But by the definition of thermal conductivity this transfer is also $\kappa\, d\theta/dx$. Whence, identifying coefficients, we have for the thermal conductivity

$$\kappa = 2\alpha v\delta^{-2}.$$

I gave this formula at the spring meeting of the American Physical Society in Washington 1923, but did not publish it as part of my abstract [24] because I discovered it too late.

Not only does this very simple expression give approximately the absolute magnitude of the thermal conductivity, but it also accounts for the sign, although not the numerical magnitude, of the temperature coefficient of conductivity. Thus it will be found that for the ordinary organic liquid both v and δ^{-2} decrease with rising temperature, giving a conductivity decreasing with rising temperature, as found experimentally. For water, on the other hand, v increases with rising temperature (both the isothermal compressibility and the thermal expansion of water changing abnormally with temperature) at a rate more than sufficient to compensate for the decrease of δ^{-2}, so that the net effect is an increase of conductivity, again agreeing with experiment.

In view of the success of this simple expression at atmospheric pressure, it was a surprise to me to find that it does not account at all well for the phenomena under pressure. It does, of course, give the correct sign for the pressure effect, for both v and δ^{-2} increase with pressure, so that an increase in conductivity is to be looked for. But the change given by the formula is greater than that found to be actually the case. The velocity of sound is the factor which changes most with pressure. I have already found that the ratio of the compressibility at 12000 kg. to that at atmospheric pressure for the normal organic liquid is of the order of 1 to 15. Combined with the change of density this results in a change of velocity of sound by a factor usually between 3 and 4. Combined with the factor δ^{-2} this demands a change in thermal conductivity at 12000 kg. of between three and four fold, whereas we have seen the experimental value is at most 2.7, and for the majority of liquids is in the neighborhood of 2. The computed change of conductivity with pressure for water agrees no better with experiment than it does for other liquids.

The conclusion is, therefore, that pressure produces some change in the structure of the liquid not taken account of in the simple picture above. It is not difficult to imagine what the nature of this effect may be. We have supposed the molecules to stand in more or less coherent columns or rows, so that the energy of thermal vibration may be handed on from one molecule to the next without hindrance or delay. The formation of these coherent rows we may suppose to take place naturally under the free play of molecular forces when there is no external compulsion. One is reminded of the way in which the molecules in the surface film of a liquid are known to allign themselves into rows. But when external compulsion is applied, as by the application of external pressure, the simple columnar arrangement is violated, the transfer of energy from molecule to molecule down a coherent row is interfered with, and the thermal conductivity decreases.

We may with some plausibility find support for this point of view in the reversal of the temperature coefficient of conductivity at high pressures. With increasing violence of temperature agitation the molecules are brought into closer contact during part of their vibrations, and so part of that ease of transmission of thermal energy is restored which was destroyed by the unnatural configuration forced by pressure.

A corollary of this is the suggestion that an increase of external pressure is not like in all respects the so-called internal pressure due to molecular attractions.

This simple picture of the mechanism of thermal conductivity is apparently applicable only to electrically non-conducting liquids. The thermal conductivity of rock salt calculated in this way is only one quarter of the actual conductivity, and for iron it is only 1/22 of that part of the total conductivity which may be ascribed to the action of the atoms (one third of the total conductivity). For the case of liquids, our picture is essentially one in which the free path for thermal transfer is the distances between molecular centers. The fact that the thermal conductivity of solids is greater than given by this expression means that for solids the free path for thermal transfer (or the extinction distance of an elastic wave) must be much greater than the distance between centers. Of course this has been known to be the case on other grounds. In fact, the formula given above may be shown to be the same, except for the numerical coefficient, as a formula given by Debye [25] for the conductivity of a solid, provided we assume that Debye's quantities take the proper values in a liquid. Debye's expression for the conductivity of a solid is $\kappa = ql\rho c/4$ where q is the velocity of a distortional wave, l is the mean free path of this wave defined as the distance within which the energy of the wave drops to $1/e$ of its initial value, ρ is the density of the solid, and c its specific heat. Now let us, in accordance with the suggestion above, put l equal to δ, the distance between molecular centers. For the thermal energy of unit volume we have $\rho c = 2\alpha N = 2\alpha\delta^{-3}$. Also put $q = v$. Substituting these values in Debye's expression gives $\kappa = \alpha v\delta^{-2}/2$, agreeing with my expression except for the coefficient 1/2 instead of 2. In Debye's application of the expression to solids, the calculation of l, the mean free path of the elastic wave, is a complicated matter, and finally an expression is obtained involving the compressibility, natural frequency of vibration of the molecules, the gas constant, and the absolute temperature.

A somewhat closer connection with experiment than that given by the formula above is obtained if we set the variation of thermal conductivity with pressure merely proportional to the velocity of sound, instead of to the product of the velocity and δ^{-2}. In a paper read at the Spring meeting of the National Academy of Sciences in Washington 1921, but never published, even in abstract, I called attention to the parallelism between the change in thermal conductivity with pressure and the velocity of sound, and expressed the opinion that the most important single factor determining the thermal conductivity of a liquid was the velocity of sound.

SUMMARY.

An apparatus is described for measuring the thermal conductivity of liquids both at atmospheric pressure and high pressures; the method uses the radial flow of heat across a thin layer of liquid contained between two concentric metal cylinders. The importance of eliminating the effect of radiation through the liquid from one wall to the other in all methods of measuring conductivity of liquids is emphasized; the effect may be minimized by making the layer thin. The method is applied to the measurement of fifteen liquids over a pressure range of 12000 kg/cm^2 at 30° and 75°. At atmospheric pressure the thermal conductivity of all the liquids except water decreases with rising temperature. Under pressure the conductivity increases by an amount varying from 1.5 to 2.7 fold for the entire pressure range. The effect is greatest for the most compressible liquids, and is least for water. The effect is far from linear with pressure, but the initial rate of increase of conductivity with pressure is several fold greater than the rate at 12000 kg. At pressures above roughly 3000 kg. the temperature coefficient reverses sign, so that all of these liquids are more conducting at high pressure at the higher temperatures. A simple theoretical formula for thermal conductivity of liquids has been obtained, in which the difference of temperature energy in a liquid in which there is a temperature gradient is represented as travelling through the liquid with the velocity of sound.

I am much indebted to my assistant, Mr. H. B. Curry, for assistance in making many of the readings.

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THE COMPRESSIBILITY OF FIVE GASES TO HIGH PRESSURES.

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TABLE OF CONTENTS.

Introduction 173
The Method 174
Corrections and Computations 182
Detailed Data 185
    Hydrogen 185
    Helium 188
    Ammonia 190
    Nitrogen 191
    Argon 194
General Survey of Results 195
Equations of State for High Pressures 202
Summary 210

Introduction.

The following represents the results of a first attack on the problem of determining the compressibility of gases to pressures of the same general magnitude as I have reached in my other high pressure work; in this case the pressures reached 15000 kg/cm^2 or more. It would probably be possible to improve on the results given here; my reason for giving results not the best that can be possibly obtained is that this matter of measuring the compressibility of gases to high pressures turned out to be of unexpected difficulty, and my various attempts to find a suitable method have already occupied three years. The error in the results to be given here I believe to be not more than a few per cent, and the results have seemed to me to be of sufficient interest to justify their publication, particularly as it may be some time before I am able to make a more successful attack on the problem.

The gases which I have measured are: hydrogen, helium, nitrogen, ammonia, and argon. The results for the monatomic gases helium and argon should be of particular significance in connection with recent speculations on the constitution of the atom.

The measurements given in the following are of the change of volume in cc. per gm. of gas as a function of pressure reckoned from 3000 kg/cm^2 as the initial point. For values of the absolute volume per gm.

the work of other observers at low pressures, up to 3000 kg/cm^2, must be relied on. Direct measurements seem to have been made only for hydrogen and nitrogen (by Amagat [1]); for the other gases more or less probable values of the absolute volume at 3000 kg. must be obtained from equations of state which fit the data as well as possible over the low pressure range previously measured.

THE METHOD.

It will perhaps be of some interest to outline briefly the methods which were tried at first without success.

The apparatus used in all the attempts at measuring the compressibility of gases differs from that used in measuring the compressibility of liquids or solids [2] in that it was considerably smaller. This was primarily for reasons of safety, so that it would not be necessary to handle large volumes of gas, with the danger from explosions to be expected from the ruptures that almost always take place in this high pressure work. There was further an incidental advantage in using small apparatus in that the cylinders could be heat treated to higher elastic properties, so that higher pressures could be reached. 15000 and sometimes 16000 kg/cm^2 were reached in this work, against the more usual 12000 of my previous work.

The first attempt employed the simplest possible modification of the method which I have already used for the compressibility of liquids.[2] The gas was enclosed in a steel cylinder provided with a pressure gauge, and the pressure was produced by pushing a leak-proof piston into the cylinder. The compressibility is given by the change of volume, which is given by the motion of the piston. In order to reach pressures at all high, it is of course necessary to start with the gas at a rather high initial pressure. The necessity of providing this initial pressure in the gas distinguishes in one respect the technique of gas compressibility measurements from that of liquids. In the first form of apparatus the gas at high initial pressure was admitted through a bye-pass situated at the upper end of the cylinder, and closed off by the advance of the piston. The original source of the gas was a steel bottle in which the pressure was about 150 kg/cm^2. This gas was admitted to an intensifier of the same design as that already used for liquids; the intensifier allowed an initial pressure of 2000 kg/cm^2.

Attempts were made to measure in this way the compressibility of hydrogen and air, since these could be easily obtained compressed into steel bottles. Almost immediately a fundamental limitation of the method was found for hydrogen, namely the steel cylinder will not withstand a pressure of more than about 9000 kg/cm^2 of hydrogen. Above this pressure the hydrogen escapes through the steel walls of the cylinder with explosive violence. The cylinders used had an external diameter of 2 inches and an inside diameter of $\frac{1}{4}$ inch. The first escape of the gas is through a fissure so fine that it is impossible to find it even with moderate magnifying power, and the correct explanation was not at once obtained. However, after recharging the cylinder a number of times, and repeating the effect every time, the fissure finally grew to a size large enough to see with the naked eye. The effect was repeated on other cylinders. There is no possibility of the effect being due to a flaw in the steel, because the cylinders had been previously tested with a liquid to a pressure of 25000 kg/cm^2. The action of the hydrogen is very similar to that which I have previously found with mercury.[3] The maximum pressure of mercury that steel will support is about 6000 kg/cm^2, lower than for hydrogen. The action in the case of mercury was due to a combination of an opening of the pores of the steel by the pressure and the same chemical affinity between mercury and steel which manifests itself in amalgamation, if the steel surface is perfectly clean. There is no action between mercury and steel at high pressures unless the pores of the steel are distended by the pressure so that the mercury can enter. Steel vessels containing mercury can be used with perfect freedom under high pressure provided that the vessel is subjected to a hydrostatic pressure all over, thus producing a uniform volume compression, with closing instead of opening of the pores. The same sort of an effect I anticipated for hydrogen, and it will be seen in the following that some success was obtained by enclosing the hydrogen in a vessel subjected to hydrostatic pressure all over. The fact that the pressure of failure with mercury is less than that with hydrogen is doubtless partly explained by the fact that the size of the hydrogen molecule is greater than that of the mercury atom (the molecular volume of hydrogen even in the condition of greatest density now known is greater than that of mercury at atmospheric pressure), but this is not the whole story, for the molecular volume of helium is less than either hydrogen or mercury, and there is no similar penetrating effect. The element of chemical affinity is probably also necessary.

After the failure of the simple method with hydrogen, I spent much time trying for measurements with air. There was no such rupture effect as found with hydrogen, but the results were always inconsistent, and frequently could be apparently explained only by a slight leak, although the entire apparatus was placed under water, and no leak could be ever detected externally. Finally, after being used a large number of times, the cylinder ruptured at about 13000 kg/cm^2. There seems to be an effect with air like that with mercury and hydrogen, but very much slower. It is probable that the irregular results which simulated leak were due to an actual absorption of the air by the steel walls of the cylinder. In addition to this effect, there is some chemical action, as was found in an attempt to measure the compressibility of air with the final form of apparatus. The high pressure air oxidizes the manganin of the pressure measuring coil, permanently changing its resistance, and also slightly oxidizes the silk insulation with which the wire is wound, giving a slightly charred appearance, and damaging the insulating properties.

The method having failed with hydrogen and air because the gas actually enters the pores of the metal, it appeared to me probable that the same effect would be found as a source of error with all gases, although very probably to a much less degree with some, and I discarded the method with regret, for its simplicity and directness.

The next method adopted was patterned in some ways after methods which have at times been used for the compressibility of both liquids and gases. Carnazzi,[4] for example, has employed a method for measuring the compressibility of liquids in which the liquid is compressed along a uniform tube by a rising column of mercury, and the height of the mercury is determined in terms of the resistance of a fine wire stretched along the axis of the tube and short-circuited by the advancing mercury and so reduced in resistance. It did not appear feasible to use a wire stretched along the axis of the tube in this case because of the necessarily small dimensions, and I modified the scheme by using a metal instead of a glass tube, and measured the total resistance of the tube between the two ends. The walls of the tube were thin, so that the resistance of unit length of mercury was much less than the resistance of unit length of the tube itself, thus allowing the position of the mercury to be obtained accurately from the measured resistance. It was necessary to fit the lower end of the tube with a reservoir into which a supply of gas at an initial pressure of several hundred kg/cm^2 was forced, the reservoir being of such dimensions that the gas should be all compressed into the capillary by 2000 kg. This reservoir had to be fitted with a valve, and a method of filling devised. The tube was a steel capillary of about 0.100 inch (0.254 cm.) inside diameter, and walls 0.003 inch (0.0076 cm.) thick, especially drawn for this purpose.

Preliminary tests of the apparatus at low pressures of a few atmospheres gave satisfactory results, but I was never able to get any consistent results at high pressures, although a number of modifications were made, and much time was spent at it. The trouble is that the contact between the mercury and the steel wall of the capillary is capricious. Amalgamating the inside of the tube did no good. Apparently the trouble is that at high pressures any oxygen or water vapor present as a slight impurity in the gas, and of course all the more if air or oxygen is originally used, attacks the metal wall of the capillary, or perhaps the advancing mercury column, so that a film of oxide is deposited between the walls and the mercury, preventing contact. The walls of the tube were almost always found dirtied after exposure to pressure, although there was no trace of such action near atmospheric pressure. The method was finally abandoned.

I finally returned to the method used for liquids, but modified as follows. The gas is placed in a separate container, initially under a pressure of 2000 kg/cm^2, and the container is surrounded by the kerosene with which pressure is transmitted. In this way direct contact of the gas with the walls of the cylinder and the consequent rupturing effect is avoided. The method is very much like that which I have previously used in determining the freezing curve of carbon dioxide.[5]

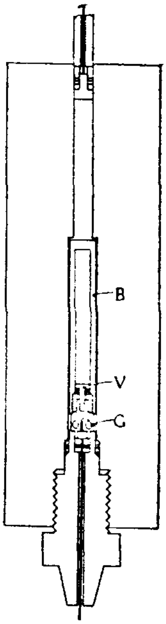

FIGURE 1. Cross section of apparatus for measuring compressibility of gases. The gas is contained in the bomb B. which is closed initially by the valve V. Pressure is measured with the manganin gauge G.

The final apparatus is shown in Figure 1. The gas container is the steel bomb B, closed at the lower end with an inward opening valve V. The bomb is charged to an initial pressure of about 2000 kg/cm^2 and the valve V closed, after which it maintains itself tight by the internal gas pressure. The quantity of gas is determined by weighing the bomb before and after filling. In order to obtain sufficient capacity, it is necessary that the walls of the bomb be as thin as consistent with safety. It can be understood that the explosion of the bomb, during the handling incident to filling and weighing, with an internal gas pressure of 2000 kg., might be a serious matter. The design must be such as to give as large a margin of safety as possible. The bomb was made of the same chrome vanadium steel as the outer cylinder, heat treated, and tested with fluid pressure to 4000 kg. I have never had trouble with the rupture of the bomb except when it had become weakened by contact with hydrogen, as will be explained later.

The bomb is filled with an auxiliary arrangement of design sufficiently obvious, so that it need not be described in full detail. The source of gas is a compressed steel bottle, with an initial pressure of between 100 and 150 kg/cm^2. The bottle connects with the rest of the apparatus through a valve block, by which the bottle may be disconnected. The valve block connects with an intensifier of the same design as that used in my regular high pressure manipulations, in which the multiplication ratio is four to one, driven by a hand pump capable of giving 1000 kg/cm^2. The pistons of the intensifier were packed with rubber, and were run dry, to avoid contaminating the gas. This packing has to be renewed very frequently, the wear on it being severe because of the lack of lubrication. The valve block also contains a manganin pressure gauge, connected with a Carey Foster bridge, with which the pressure in the gas produced by the intensifier may be measured. The valve block finally connects with another block in which is mounted the bomb, and which is provided with a stuffing box through which the valve at the lower end of the bomb is actuated by means of an extension stem screwing onto the thread at the lower end of the stem of the valve, shown in Figure 1, and projecting through the stuffing box. The valve has to be kept open with a screw during filling, as otherwise the internal pressure of the gas would close it. The procedure in filling is to open the connection to the gas bottle, with the piston of the intensifier in its extreme position, flush out the apparatus by admitting gas at the pressure of the bottle and blowing it out a couple of times at atmospheric pressure (or in the case of the rare gases argon and helium the interior of the apparatus is initially exhausted to a low vacuum through an auxiliary valve connection), the bottle is then shut off by the valve, leaving throughout the entire interior a pressure of 100 to 150 kg/cm^2, and then the intensifier is actuated with the hand pump until the desired pressure of 2000 kg. is reached, as indicated by the manganin gauge. If the gas is very compressible, or if the initial pressure in the bottle is not high enough, it is necessary to use two intensifiers in parallel to get the requisite 2000 kg. This proved necessary for hydrogen and helium. It is necessary to guard carefully against leak during this filling; it is not possible with a stuffing box and a rod projecting through it to use the automatically tightening packing that I use in most high pressure work,[6] and leak will occur unless care is taken with the packings. To guard against leak small air tight chambers are built around the outside of the strategic places, connecting with fine glass tubes which open under water, so that any leak is at once evident by bubbling through the water.

After filling to the required pressure the lower valve of the bomb is shut through the stuffing box, the bomb is removed from its receptacle, and the amount of gas determined by weighing. The weighing can be easily made to 0.0001 gm., the total weight being of the order of 30 gm., which means 0.1% on the weight of the lighest gas (weighing very nearly 0.1 gm.), and of course a greater proportional accuracy on the amount of the other gases. To avoid error in the weighing, the outside of the bomb is polished, and of course carefully cleaned before each weighing.

The bomb filled with gas is then placed in position in the pressure cylinder, which is now closed at the lower end by the plug with the manganin gauge, G, attached, as shown in Figure 1. The entire cylinder is now filled from the open upper end with the required amount of kerosene from a pipette provided with a stop cock. The filling takes place in three or four steps. A small amount of kerosene is first run in, and the cylinder exhausted through the upper end to a vacuum of about 0.5 cm. of mercury. When air is readmitted the kerosene is forced down into the crevices around the gauge and the bomb. This process is repeated, exhausting after every small addition of kerosene, until the required level is reached. The entire operation takes only a few minutes. The amount of kerosene is determined by weighing the pipette before and after filling. The kerosene used in all these measurements was taken from the same original lot, so that there is no error because of change in the quality of the kerosene.

The moving plug is now placed in the upper end of the cylinder. The packing of this plug is of the same design as used in my previous high pressure work, and is without leak. The only novel feature about the packing plug is that it is driven by a hollow piston. Through this piston projects a fine steel stem, soldered to the shank of the moving plug. This stem projects above the upper end of the hardened piston, and may be viewed with a reading telescope attached to a micrometer screw. A second reading telescope is set on a point fixed to the upper surface of the cylinder. The difference of reading of the two telescopes gives the amount of motion of the piston into the cylinder, and so the change of volume. Previously the motion of a point attached to the hardened piston was read.[2] The disadvantage of the former method was that any wear of the packing is included in the apparent motion of the piston, whereas this is obviously avoided with the present modification.

The cylinder, filled as just described, was next mounted in the hydraulic press with which pressure was produced by pushing the piston into the cylinder. This press was a small one especially made for this work. The diameter of its piston is 1.625 inches (4.13 cm.) and it is driven by a hand pump of 1000 kg. capacity. Since the diameter of the small piston of the high pressure cylinder is 0.25 inch (.635 cm.), the multiplying power is 42 fold, thus permitting a maximum pressure of 42000 kg/cm^2, without allowance for friction. The ram of the press is provided with the usual screw stop, so that the piston may be held in any desired position should the pressure of the hand pump be accidentally lost.

The lower part of the press with the cylinder in place dips into the temperature bath maintained at constant temperature by the conventional stirrer and regulator. The entire high pressure part of the apparatus is thus at the same temperature, and error is avoided due to volume changes when liquid passes from a part of the apparatus at one temperature to a part at another temperature.

Because of the small size of the apparatus, it was not possible to use the regular manganin pressure gauge which I have used in all the rest of my high pressure work.[7] To replace this gauge I procured from Baker and Co. some special American manganin wire 0.0035 inch (0.0087 cm.) in diameter and double silk covered, instead of the German manganin 0.0055 inch (0.0140 cm.) in diameter which I have previously used. Using the smaller size wire, it was possible to fit into the space at my disposal a coil of the usual resistance, 120 ohms. This American manganin was carefully seasoned before using by a long exposure to 140° and several applications of 16000 kg/cm^2, but in spite of every care in preparing the coils, it was always much less satisfactory in its behavior than the regular German manganin. This may possibly have been due to the fact that the German manganin was 25 or more years old, but I believe was more probably due to an intrinsic difference in the material.

Several coils were made of Baker's manganin, and they were calibrated against the standard at each of three temperatures, 30°, 65°, and 95°, at which it was intended to make the measurements. The inferiority of the new material showed itself in a shift of the zero after an application of high pressure. This shift persisted throughout all the work, and showed no tendency to become less as the coil became better seasoned. It was spasmodic and without predictable element. In extreme cases the zero shift might amount to the equivalent of 300 kg/cm² on the pressure, or 2% of the total pressure range. This amounts to only about 0.05% on the total resistance, but since this pressure gauge depends on small differences of resistance, it constitutes a serious objection to this material for pressure gauges. I attribute by far the greater part of the error in my final results to trouble with the pressure gauge. The remedy is to return to the German manganin gauge, using a shorter length of wire, and to improve the sensitiveness of the electrical measurements to compensate for the decrease of resistance of the coil. As a matter of fact, I have already done this, and have measured in this same apparatus the compressibility of some of the more compressible solids with much better results.

Runs were made at three different temperatures of the bath, as it was part of the program to obtain the thermal expansion of the gas as a function of pressure as well as the compressibility. At any fixed temperature the procedure consisted of first pushing the piston in by small amounts, starting with zero pressure, and plotting the piston position as a function of pressure. When the pressure in the kerosene rises as high as the pressure of the gas within the bomb, the valve at the lower end of the bomb opens inward, admitting the kerosene to compress the gas, which from here on takes part in the compression. The opening of the valve is shown by a discontinuity in the slope of the curve of volume against pressure, the gas of course being more compressible than the exterior of the steel bomb. After the valve opens into the bomb, pressure is pushed in a few large steps to the maximum to which it is intended to make measurements, in order to season the cylinder and take the give out of the various packing appliances. After the first maximum, pressure is then lowered in small steps to the pressure of discontinuity, or as near it as the friction of the moving plug will permit, and is then raised again in small steps to the maximum, determining at each of these steps the position of the piston as a function of pressure. By making readings with both directions of motion of the piston, error from hysteresis, which might be considerable, is avoided. The mean of readings with increasing and decreasing pressure is taken as giving the correct relation between pressure and volume. The same process is now repeated at another temperature, except of course the preliminary cautious manipulation in opening the bomb is no longer necessary. The pressure must not be allowed to fall so low after the valve has been once opened that the gas can expand into the part of the apparatus beyond the bomb.

CORRECTIONS AND COMPUTATIONS.

The change of volume given by the motion of the piston as a function of pressure consists of two parts, one part due to the gas, and a second part including the change of volume of the kerosene, of the steel of the bomb, and the distortion of the cylinder. This second part may constitute one half of the total effect, and must be eliminated. The same method was adopted for doing this that was previously used for liquids. Blank runs were made with the gas replaced by a dummy steel core. If the amount of kerosene is the same in the two runs, the difference of the piston displacement of the two runs gives at once the difference of compressibility between the gas and steel, except for the small correction for the elastic change of bore of the cylinder. The compressibility of the steel is known, so we have at once the compressibility of the gas. But it was not possible to use exactly the same amount of kerosene in the two runs, so that a correction was necessary for the difference. This was determined by making two sets of blank runs, with different sizes of steel cores, and different fillings of kerosene, giving by difference the correction for kerosene. This correction was in all cases small, amounting to not more than one tenth of the major correction. The blank runs were of course made at each of the three temperatures, and the corrections were determined as a function of temperature.

The cross section of the cylinder increases at the higher pressures. This correction I did not attempt to determine directly, but calculated it from the theory of elasticity; I have fully discussed it in a previous paper.[8] I used the same formulas as previously, and the same numerical values of the elastic constants, as these constants are known to change very little with the grade of steel. The correction at the maximum amounts to about 1.0%.

The manganin gauge was calibrated at the three temperatures; at 95° the proportional change of resistance for a fixed pressure was 1.3% greater than at 30°. The linear relation between pressure and change of resistance has been established by direct measurement only to 13000 kg.[7] Within this range the linearity holds to better than 0.1%. It was assumed that the relation would remain linear through the additional range of this work, to 15000 or 16000 kg. The extension is so relatively slight as to occasion no uneasiness on this score.

When I began the measurements by this method I anticipated a serious source of error in the solution of the gas into the kerosene. This did not turn out to be at all important, however, as was shown in two ways. In the first place, the position of the piston at the maximum pressure on the second application was almost exactly the same as on the first. The first maximum was reached rapidly, so that there was little time for the gas to diffuse through the liquid, whereas the second application was after an hour or more, when there had been more chance for diffusion. If the solution of the gas into the kerosene takes place with change of total volume, then the piston at the second maximum should not be in the position of the first, but would presumably indicate a smaller volume. In the second place, the amount of gas in solution would be expected to be a function of pressure, so that after every change of pressure there should be a gradual change in the amount of gas in solution, either coming out of solution or going in, according as the pressure change is a decrease or an increase. This change in the amount of gas in solution should be shown by a pressure creep. Such a pressure creep was never found, but equilibrium after a change in the position of the piston was always reached in a very short time, usually less than three minutes. The rapid attaining of temperature equilibrium after a change of pressure is due to the small size of the apparatus, and the fact that a large part of the interior of the cylinder is filled with metal; it constitutes an additional very considerable advantage of the small scale high pressure apparatus.

Because of the necessity of starting with an initial pressure of 150 kg., to carefully purify the gases would have required an especially elaborate arrangement, which was beyond my resources. I therefore used the best gases commercially obtainable, and corrected my results by means of the analysis of the gases, assuming in making the correction that the volume of the impurity was additive. This assumption should not be seriously wrong for those gases which have little tendency to chemical action on each other, and since the gases were quite pure to begin with, the error introduced in this way should in any event not be important. The details will be found under the separate descriptions.

The method of computation was as follows. The true motion of the piston was first found from the difference of reading of the upper end of the piston and the fixed point on the cylinder. The results at each temperature were then plotted on a large scale, and a single smooth curve drawn between the ascending and descending branches. The difference between the two branches, due to hysteresis in the metal parts, was of the order of 1.5% at the middle of the range, and of course vanishes at the two extremes. From the smooth curve the position of the piston was read at even thousand kilograms. The points obtained in this way from the curve were now smoothed so as to give smooth first differences corresponding to the number of significant figures retained (four). These positions were now subtracted from that at 3000 kg., which thus became the fiducial pressure from which the changes of volume were calculated. In most cases it was not possible to reach much below 3000, and in any case, because of the steepness of the curve, the results below 3000 were not as accurate as above. The piston displacements were now corrected with the results of the two sets of blank runs, as already described, so that the displacement of the piston due to the gas alone was obtained at even thousands, with 3000 as the fiducial pressure. This displacement is that due to the weighed amount of gas. This was converted into the amount per gm. by multiplying by the proper factor, and this again into cc. per gm. by the cross section of the cylinder corrected for the distortion under pressure. The cross section of the cylinder was obtained by weighing the mercury which filled it to various depths, a special arrangement having been made for filling different lengths of the bore full without error from capillary effects at the end surfaces. The bore proved uniform over the entire length to the order of accuracy.

This calculation of the change of volume in cc. per gm. from 3000 kg. as the fiducial point was repeated for each of the runs at different temperatures and for each of the fillings of the apparatus, when more than one filling was made, as was usually the case. I had hoped to get from the measurements both the compressibility and the thermal expansion, but unfortunately in many cases the agreement between the different runs was not sufficient to give acceptable values of thermal expansion. What was usually done was to take the mean of all the sets of values for different temperatures and different fillings to give the best relation between pressure and change of volume at the mean temperature, which is approximately 65°. This strictly assumes that the relation between temperature and volume at constant pressure is linear. The temperature range is so small and the pressures are so high that no appreciable error can be introduced in this way.

In this way a relation between volume and pressure was obtained representing the mean of six or more runs. The details of the data, and the probable error of the mean will be found in the discussion for the different gases. In some cases it was possible to obtain values for the thermal expansion to which some degree of credence may be attached.

DETAILED DATA.

Hydrogen. The material was obtained from the Boston Electrolytic Oxygen Co. The purity was high, it being stated that there was less than 0.1% of impurity. I made no analysis myself, but the general validity of this figure has been checked by analyses made in the Cryogenic Laboratory at Harvard University on hydrogen from the same source. The results given in the following are without correction for the effect of any impurity.

I have already given a brief account of the measurements on hydrogen in the Schreinemakers Festschrift.[9] The measurements gave much more trouble than with any other gas, the reason being the action between hydrogen and the steel cylinder. I have already described how it was impossible to use the hydrogen directly in the cylinder because of penetration of the steel by the hydrogen and rupture of the cylinder. The same effect was found to a much more limited extent in the final arrangement. Although the hydrogen is prevented by the kerosene from coming directly into contact with the steel, the hydrogen goes into solution in the kerosene and attacks the steel from solution. The action is much slower than when the hydrogen comes into direct contact, and it was possible to obtain a few measurements to high pressures, but not as many as with the other gases. The action between hydrogen and steel is not like in all respects that between mercury and steel. A piece of steel surrounded on all sides by mercury and exposed to hydrostatic pressure exerted by the mercury absorbs no mercury into its pores, which are closed by the pressure, and after removal of the pressure the steel returns to its original state in every respect. But if a piece of steel is surrounded by hydrogen, or by kerosene in which hydrogen is dissolved, and subjected to hydrostatic pressure, the hydrogen dissolves in the steel in spite of the closing of its pores, and very seriously weakens the steel. The action was shown by the weakening of the bombs in which the hydrogen was initially compressed to 2000 kg., and of other steel parts. After an excursion to high pressures the bomb was often weakened or even ruptured, so that it would not support a second filling. One of these bombs exploded during manipulation previous to weighing after the second filling, but luckily with no serious consequences, the fracture taking the form of an open seam down the side with no flying fragments. Once or twice the bottom was separated from the rest of the bomb. Four or five bombs were used altogether. Other parts of the apparatus were also weakened: the valve stem ruptured several times, the stem of the moving plug pinched off several times after a run with hydrogen, and finally the large outer cylinder itself ruptured after a number of runs to high pressures. At this point the experiments with hydrogen were discontinued, since I already had a few measurements to high pressures, and the uncertainty and danger in the experiment did not seem to justify further experiment. Steel which has been acted on by hydrogen is quite altered in appearance, the fracture being coarse and crystalline instead of fine and silky as is characteristic of high grade alloy steel.

Runs to high pressures were obtained with two fillings of the apparatus. The first filling gave complete runs between 2000 and 13000 kg/cm^2 at 30° and 65°, and also a fragment of a run at 95°, which was terminated by fracture of the stem of the moving plug. The second filling gave an incomplete run at 30° between 2000 and 14000 kg. This run was terminated by the rupture of the large cylinder; for this reason the high pressure results of this run are not to be accepted, as it is not unlikely that there was some unusually large distortion of the cylinder before rupture, but the measurements at the low pressure end of this run agreed exactly within the sensitiveness of the measurements with the other run. In fact, the agreement between the two runs is much better than for the average of the other gases, which gave no trouble with rupture.

The results are collected in Table I. The data obtained directly from experiment are the changes of volume from 3000 kg. as the fiducial pressure, which are shown in Columns 2 and 3. In order to obtain the actual values of volume I have assumed Amagat's volumes [1] at 3000 at 30° and 65°. The actual volumes calculated from Amagat's fiducial volumes are given in columns 4 and 5. In column 6 are given the values at 30° of pv, in order to facilitate comparison with the method of presentation of Amagat. The pressure is here expressed in kg/cm^2 instead of in atmospheres, as does Amagat, and the volume is the volume of that quantity of gas which at 0° C. and 1 kg. pressure occupies 1 cc.

There is some question as to whether the fiducial volumes taken from Amagat are correct. This is suggested by the fact that in the range between 2000 and 3000, in which our measurements overlap, our results do not agree. His change of volume between 2000 and 3000 is 2.50 against 2.25 of mine. Furthermore, if pv is plotted as a function of pressure it will be found that Amagat's value for 2000 lies off the smooth curve, although his 3000 point lies on it, whereas all of my points lie on the same curve. My value for the change of volume

TABLE I.

Results for Hydrogen.

Pressure kg/cm²	Change of Volume cc. per gm. from 3000 kg.		Volume cc. per gm.		pv at 65°	Volume cc. per mol	
	30°	65°	30°	65°		at 30°	at 65°
2000	−2.25		13.89		2.54	27.99	
3000	0.00	0.00	11.64	12.17	3.18	23.47	24.53
4000	1.12	1.14	10.52	11.03	3.83	21.21	22.24
5000	1.84	1.88	9.80	10.29	4.50	19.76	20.74
6000	2.35	2.44	9.29	9.73	5.08	18.73	19.62
7000	2.77	2.88	8.87	9.29	5.65	17.88	18.73
8000	3.09	3.21	8.55	8.96	6.23	17.24	18.06
9000	3.38	3.46	8.26	8.71	6.17	16.65	17.56
10000	3.63	3.68	8.01	8.49	7.29	16.15	17.12
11000	3.86	3.88	7.78	8.29	7.80	15.68	16.71
12000	4.09	4.04	7.55	8.12	8.25	15.22	16.37
13000	4.32	4.21	7.32	7.96	8.66	14.76	16.05

between 2000 and 3000 agrees exactly with that which would be calculated by an empirical equation of Keyes,[10] which fits the other low pressure observations within the limits of error. In addition, the equation of Keyes fits my observations very well to higher pressures, as I will describe more in detail later. Of course if it should be established certainly at any future time that Amagat's results at 3000 are in error, the values in Table I can be simply corrected by an additive constant.

The values of the thermal expansion, to be obtained from the Table by taking the differences of volume between 30° and 65° are to be used with a certain amount of caution. I have already stated that the thermal expansions in general do not have the accuracy which I had hoped. It is nevertheless probable that the values for hydrogen are more accurate than the average, indicated partly by the smoothness of the data, and partly by the fact that my change of volume between 30° and 65° at 3000 kg. agrees exactly with that of Amagat. The values of the Table show a remarkable feature in that the expansion at 13000 kg. is greater than at 2000. Such behavior is not without precedent, however, as I have already found it for a number of liquids.

I believe, however, that in theoretical speculations it would be safest to ignore this apparent abnormality of the thermal expansion at high pressures. It will be found on a careful analysis of the data that the anomaly is entirely due to the shape of the isothermal at 30°. If the compressibility is plotted as a function of pressure, it will be found that beyond 8000 at 30° the compressibility remains nearly constant independent of pressure, whereas at 65° it continues to fall over the entire pressure range as is normal for other gases. (See page 199 of the following.) For this reason I have ued in the following the isothermal at 65° as more probably accurate than that at 30°. Of course the possibility must be recognized that the abnormal isotherm at 30° may be connected with a real freezing of the hydrogen at high pressure, the freezing being very much rounded because of solution of the hydrogen in kerosene, but this does not appear to me to be a very probable explanation.

Further discussion of the results for hydrogen is reserved for the general discussion of all the data.

Helium. This was obtained from one of the cylinders prepared during the war for the American Navy. I have to thank Professor H. N. Davis for obtaining me this particular cylinder. It is known that the impurity in this helium is practically all nitrogen. Professor F. G. Keyes of the Institute of Technology was so kind as to analyze this gas for helium content, and found 95.8% by volume pure helium. I have assumed in applying corrections to the data that the remaining 4.2% was entirely nitrogen, and the corrections applied were obtained from the high pressure measurements on nitrogen. The correction amounts to about 20% of the observed piston displacements, since my results are given in terms of a weight instead of a volume analysis.

Three sets of runs in all were made with helium. The first did not give results of much value because the quantity of helium that could be got into the bomb was so small that it was not possible to reach more than 5 or 6000 kg. The reason for this was that the initial pressure in the steel bottle was much less than had been supposed, there having been obviously some leak after storing. This was remedied for the second and the third runs by using two intensifiers in parallel instead of one. The second filling gave enough gas to reach 12000 kg., but only the run at 30° gave reliable results. There was a leak at the end of the stroke at 30° due to my endeavor to reach too high pressures; in spite of the leak I continued the run at 65° and 95° hoping that the leak had been entirely kerosene, but the character of the results when finally computed showed that this could not have been the case, and only the 30° results of this filling were retained. Helium is quite different from the other gases measured in its continued compressibility at high pressures, so that my preliminary estimates of the dimensions required turned out to be incorrect. The device of partly filling the interior of the bomb with a plug of steel and compensating for the decreased volume by a higher initial pressure made it possible to reach a pressure higher than on the second run. The third filling gave results without accident to 15000 kg/cm^2 at 30°, 65°, and 95°. The two runs at 30° of the second and third fillings did not give consistent results, but the changes of volume of the second filling, calculated from 3000 as the fiducial point, were consistently almost exactly 10% less than those of the third over the entire pressure range. There seems to be no good reason why the second run, before the leak developed, should not be as good as the third, and I have therefore averaged the results in the following way. The average of the three runs at 30°, 65° and 95° of the third filling was taken as the best value at 65° as determined by this filling, and this was then corrected by subtracting 5% in order to allow for the results with the second filling.

The values obtained for the thermal expansion of helium bear internal evidence of being fairly reliable; this internal evidence is the comparability of the thermal expansion between 30° and 65° with that between 65° and 95°. These two expansions are roughly the same over the entire pressure range; at the lower end of the range the expansion over the lower temperature range is somewhat larger than over the upper range, while the behavior is reversed at the highest pressures. Probably the variations are beyond the limits of error, and I have not tried to retain them in the final results.

In Table II are given in column 2 the volume decrements at 65° in cc. per gm. with 3000 kg/cm^2 as the fiducial pressure (this is the average calculated as explained), in column 3 the total thermal expansion between 30° and 95° in cc. per gm., in column 4 the volume at 65° in cc. per gm. and in column 5 the volume in cc. per gm. molecule, using as the fiducial volume at 3000 kg. the value given by Keyes' equation of state [11] (there are no experimental values to this pressure). The value given by the equation, which amounts to a rather wide extrapolation, is in some doubt, and is, I believe, too high, so that it is probable that helium is even more compressed than appears in the table. In the last column of the table is given the product pv, pressure in kg/cm^2, and v the volume in cc. of that amount of helium which under 1 kg. pressure and at 0° C. occupies 1 cc. (1/7210 gm.) Further comment on the values in the table is reserved for the general discussion.

TABLE II.

RESULTS FOR HELIUM.

Pressure kg /cm²	Change of Volume cc. per gm. at 65°	Total Change of Volume 30° to 95° cc. per gm.	Volume at 65° cc. per gm.	Volume at 65° cc. per mol.	pv at 65°
3000	0.00	.613	5.54	22.16	2.31
4000	0.77	.598	4.77	19.08	2.64
5000	1.23	.589	4.31	17.24	2.99
6000	1.54	.584	4.00	16.00	3.33
7000	1.77	.581	3.77	15.08	3.66
8000	1.96	.579	3.59	14.36	3.98
9000	2.10	.578	3.44	13.76	4.29
10000	2.22	.576	3.32	13.27	4.60
11000	2.33	.575	3.21	12.84	4.89
12000	2.41	.574	3.13	12.52	5.21
13000	2.48	.572	3.06	12.24	5.52
14000	2.55	.571	2.99	11.96	5.82
15000	2.60	.570	2.94	11.76	6.11

Ammonia. I am indebted for the material to the kind offices of Professor A. B. Lamb, who made the suggestion, and Dr. A. T. Larsen of the Fixed Nitrogen Research Laboratory, who furnished the material. It was provided in a small steel bottle, and was of exceptional purity, the only impurity being not more than 0.001% of water vapor. Because ammonia is liquid at low pressures, it was not necessary to use the intensifier arrangement for filling, but the ammonia was distilled across from the bottle, which was maintained at 95°, into the bomb, which was maintained at 0°. The connecting pipes were of course exhausted before filling, and were also flushed out with a preliminary filling of ammonia.

Owing to unexpected chemical difficulties, only a few readings were obtained with this substance. At 30° with the first filling a run was made to 12000 kg. which appeared satisfactory in every respect. On trying for the run at 65° there were minor irregularities in the pressure readings, which became very much accentuated at 95°, the behavior being exactly like that when the coil has been accidentally short circuited with water, the resistance decreasing and there being polarization effects. A second filling was now made, and I tried the runs in inverse order, thinking that perhaps the polarization effect was something that gradually developed after the ammonia had stood in contact with the kerosene at high pressures. At 95°, however, there was this time at the very beginning, as soon as the apparatus was brought to temperature, and while the pressure was not more than a few hundred kilograms, a creep of the zero amounting to a pressure change of a couple of thousand kilograms. Equilibrium was reached after several hours, and a run was made, with no such trouble from polarization as had been found before. Then the temperature was lowered to 65°, when polarization appeared to such an extent that readings could not be made. The polarization persisted on lowering the temperature to 30°. The apparatus was taken apart, the ammonia of course evaporating into the atmosphere, and the polarization completely disappeared. It is evident that no water had accidentally got into the coil, but that the effect is in some way connected with the ammonia. The chemists with whom I have talked seem to think that no direct chemical action is to be expected, but the physical evidence seems strong that either the ammonia dissociated enough to conduct electrolytically, or else that some volatile compound was formed that is an electrolytic conductor.

The uncertainty of the pressure measurements made it possible to retain only the readings at 30° of the first run. The results of this run are given in Table III. At 30° the change of volume reckoned from 3000 kg. as the fiducial pressure is given in cc. per gm. and also in cc. per gm. molecule. It is to be noticed that it was possible with ammonia to make readings to considerably lower pressures than with the "permanent" gases, the volume readings reaching down to 1000 kg. The reason for this was that since ammonia is a liquid under slight pressure, the initial pressure in the bomb was low, and only a small pressure was required to open the valve. There seem to be no adequate low pressure measurements from which the fiducial volume at 3000 (or even at 1000) kg. may be extrapolated, so that I have not attempted to give in the table the absolute volumes.

Nitrogen. This was obtained from the Air Reduction Co., through the kindness of Professor H. N. Davis. It was of an especially pure grade, O_2 not more than 0.2% and argon probably about 0.16%. No corrections were applied for impurity.

Runs were made with four fillings of the apparatus. The first filling gave runs at 30°, 65°, and 95°, apparently successful in every way, and the best runs of the series. With the second filling I tried for the runs in inverse order, in order to eliminate as far as possible effects due to irregularities in the zero shift of the manganin pressure gauge. A successful run at 95° was obtained, but on passing to 65°, there was leak due to rupture of the stem of the moving plug, which had been rotted by previous exposure to hydrogen. The third filling gave another successful run at 95°, but again on trying for 65° there was leak, this time because the piston was pushed in too far in the attempt to get too high pressures. The fourth filling gave successful runs at 65° and 30°. These runs were not as consistent as would have been desirable. The greatest inconsistency was at 95°, where the extreme variation amounted to 11%, at 65° the extreme variation was 7%, and at 30°, 4%. The average of the seven successful runs was taken to give the best value of the compressibility at the mean temperature, 68°.

TABLE III.

Results for Ammonia.

Pressure kg/cm²	Change of Volume at 30°	
	cc. per gm.	cc. per mol.
1000	−.827	−14.1
1500	−.400	− 6.81
2000	−.217	− 3.70
2500	−.088	− 1.50
3000	.000	0.00
4000	.120	2.04
5000	.200	3.41
6000	.261	4.44
7000	.310	5.28
8000	.348	5.93
9000	.380	6.48
10000	.409	6.97
11000	.436	7.42
12000	.461	7.85

The results with nitrogen were not regular enough to allow satisfactory values of the thermal expansion. The first run was the only one that gives even moderately regular values, and here the difference between the mean expansion per degree between 30° and 65° compared with that between 65° and 95° is so large as to be improbable.

In Table IV are collected the final results. In this are given the mean change of volume in cc. per gm. and in cc. per gm. molecule at 68°, reckoned from 3000 kg. as the fiducial pressure. These changes of volume are converted into the actual volume by using Amagat's value for the volume of 1 gm. of nitrogen at 68° and 3000 kg., namely 1.290. In the last column of the Table is given the values of pv, the pressure being expressed in kg/cm², and v being the volume of that amount of nitrogen (0.001211 gm.) which at 1 kg. pressure and at 0° C. occupies 1 cc.

TABLE IV.

Results for Nitrogen.

Pressure kg/cm²	Change of Volume at 68°		Volume at 68°		pv at 68°
	cc. per gm.	cc. per mol.	cc. per gm.	cc. per mol.	
2500	−.066	−1.85	1.356	37.98	4.11
3000	.000	0.00	1.290	36.13	4.68
4000	.089	2.49	1.201	33.64	5.82
5000	.152	4.25	1.138	31.88	6.89
6000	.197	5.52	1.093	30.61	7.95
7000	.234	6.56	1.056	29.57	8.95
8000	.264	7.40	1.026	28.73	9.94
9000	.287	8.04	1.003	28.09	10.94
10000	.308	8.61	.982	27.52	11.91
11000	.326	9.14	.964	26.99	12.84
12000	.342	9.60	.948	26.53	13.78
13000	.357	10.00	.933	26.13	14.70
14000	.370	10.36	.920	25.77	15.60
15000	.382	10.70	.908	25.43	16.50

It appears that just as in the case of hydrogen, Amagat's volume at 3000 kg. may be in doubt. For the difference of volume between 2500 and 3000 kg. I find experimentally 0.066 against Amagat's 0.070. The difference is in the same direction as for hydrogen, but less in amount. That there is a discrepancy between Amagat's data at low and high pressures has been shown in a thesis by L. B. Smith, entitled "The Equation of State and Some Related Quantities for Pure Nitrogen, Gas Phase," written at the Mass. Institute of Technology under the direction of Professor F. G. Keyes. I am indebted to Professor Keyes for an inspection of the manuscript. It appears that an equation of state fitting the low pressure observations with high accuracy gives somewhat smaller volumes at the high pressures than given experimentally by Amagat.

Argon. This again I owe to the kindness of Professor H. N. Davis, through whom it was secured from the Air Reduction Co., Special analysis gave 0.1% O_2 and 1.0% N_2, unusually pure for a commercial product compressed into a steel bottle. In making the corrections, I have proceeded as if the entire impurity was 1.1% of N_2. Some such approximation is necessary in view of the fact that the compressibility of O_2 to high pressures is not known. The approximation must fall well within the limit of error of the measurements, considering the small amount of impurity and the similarity of the molecular weights of O_2 and N_2. The total amount of the correction was about 0.5%.

It was necessary to use a different high pressure cylinder for argon than for the other gases, the cylinder which had been used for the others having been ruptured by the last run with hydrogen. The new cylinder was of the same construction and dimensions as the first, and was seasoned and calibrated for bore and distortion by blank runs with fillings of steel and kerosene exactly as was the first cylinder.

Two fillings of the apparatus were made, and with each filling complete sets of runs at 30°, 65°, and 95° were obtained. The readings appeared perfectly smooth and in every way successful, but on working up the results numerically, considerable irregularities were evident. One of the runs at 95° lay so far away from the other, and what would be judged from the runs at the other temperatures to be the probably correct behavior, that it was discarded in taking the final average. There were left five runs, of which the average was taken to give the compressibility at the average temperature, 55°. The two runs at 30° differed by 7% from each other, and the two at 65° by 3%. The runs at the different temperatures are not consistent enough to justify the attempt to force from them numerical values for the thermal expansion. The following general statement about the behavior of the thermal expansion does seem to be justified, however, that the change of volume in cc. per gm. for a fixed temperature rise decreases very slightly at the high pressures, so that the relative thermal expansion increases at high pressures nearly inversely as the volume. This is the general type of behavior that has been found for hydrogen and helium.

The results for the change of volume in cc. per gm. and also in cc. per gm. molecule at the mean temperature 55°, reckoned from 3000 kg. as the fiducial pressure, are shown in Table V. In general, the volume falls off less rapidly at the high pressures than it does for hydrogen and helium, which is what would be expected from the larger atom of argon.

TABLE V.

Results for Argon.

Pressure kg/cm²	Change of Volume at 65° cc. per gm.	cc. per gm. atom
2000	−.083	−3.31
2500	−.034	−1.36
3000	.000	0.00
4000	.049	1.96
5000	.085	3.39
6000	.112	4.47
7000	.134	5.34
8000	.152	6.06
9000	.167	6.67
10000	.180	7.18
11000	.190	7.58
12000	.201	8.00
13000	.209	8.34
14000	.217	8.66
15000	.224	8.94

General Survey of Results.

In comparing the results for the different gases it is convenient to refer the measured changes of volume to the gram molecule (or gram atom) as the unit, since in the state of a perfect gas the volumes of all gases are equal per gram molecule.

In Figure 2 are shown the volumes per mol. of hydrogen, helium, and nitrogen as a function of pressure, these three being the only ones of the five gases measured for which we have a fairly reliable value of the volume at the fiducial pressure, 3000 kg. The absolute volumes of nitrogen and hydrogen are fairly certain, since these depend on the directly measured values of Amagat,[1] but the volume of helium at 3000 is much more in doubt, being taken from an equation of Keyes [11] reaching to only 66 atmospheres. This will be discussed in more detail later; it is sufficient to state here that the fiducial value at 3000 assumed for helium is in all probability too high, so that the actual curve for helium should run lower than shown in Figure 2.

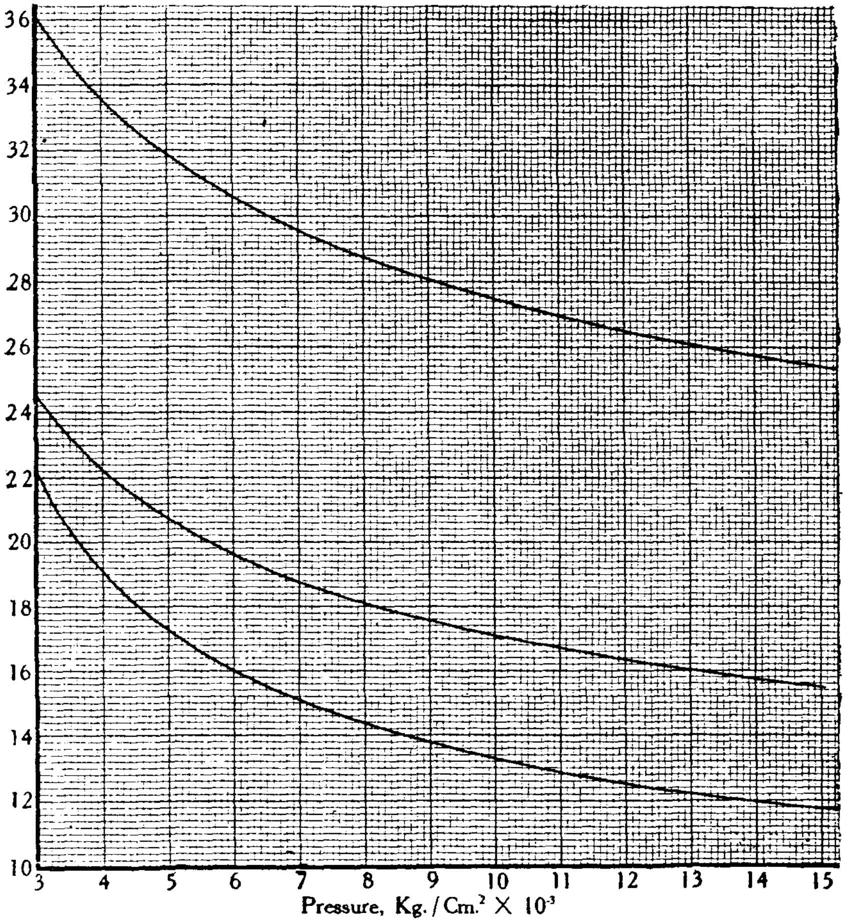

FIGURE 2. The volume in cc. per mol. as a function of pressure at 65° of nitrogen, hydrogen, and helium, reading from the top down.

Figure 2 brings out strikingly the great difference in the volumes per mol. of the different gases at high pressures, the variation from nitrogen to helium being by a factor of more than 2. The relative volumes are in the sequence which we would expect from the absolute dimensions of the molecules as given by the usual considerations of kinetic theory, as for instance, from the rates of diffusion. The diameter of the molecule of nitrogen thus found is the largest, and that of the atom of helium the least, and furthermore the difference of diameter between nitrogen and hydrogen is twice as great as the difference between hydrogen and helium, agreeing with the relative spacing of the curves shown in Figure 2.

It is interesting to compare the distances between the centers of the molecules (or atoms) at the highest pressure reached above, 15000 kg., with the molecular distances calculated from the ordinary kinetic theory. In Table VI are given the densities of the three gases at

TABLE VI.

MOLECULAR DISTANCES AT HIGH PRESSURES COMPARED WITH MOLECULAR DIAMETERS.

Substance	Density at 15000 at 65°	Distance between Molecular Centers		Molecular Diameter from Kinetic Theory	
		Spheres Close Packed	Spheres Normal Cubic	Boyle's Law	Free Path
Hydrogen	.1301	2.97×10^{-8}	2.65×10^{-8}	2.54×10^{-8}	2.72×10^{-8}
Helium	.340	2.87	2.56	1.98	2.20
Nitrogen	1.102	3.89	3.47	3.56	3.80

15000 (extrapolated from 13000 for hydrogen), the distance between molecular centers, calculated on the assumption that the molecules are perfect spheres and (1) packed as closely as possible or (2) packed in simple cubical array, and finally the molecular diameter from kinetic theory calculated (1) from the deviation from Boyle's law and (2) from free path data. It will be seen that the molecules are compressed very nearly into what would be actual contact if they behaved like rigid spheres. The approach to the dimensions of contact is most close for nitrogen and least close for helium (the distance for helium would be diminished by using a more probable value for the fiducial volume). At distances approaching so closely to contact it seems inevitable that the molecule ceases to behave like a rigid sphere, but must undergo considerable distortion which must be taken account of in framing a kinetic theory valid at such small volumes. It is also evident, of course, that the ordinary considerations of the kinetic theory valid at low pressures must be replaced by others of quite a different character. Thus the idea of a "free" path and simple molecular collisions is probably of little value when the molecules are so tightly packed.

By comparing the volumes of the gases under 15000 kg. with the volumes of the condensed phases at the triple points, we can set a lower limit to the compressibility of the solid or the liquid phase of these gases, a quantity which has never been measured experimentally, and for which we have no means, as far as I know, of guessing even the order of magnitude. The density of liquid hydrogen is given in tables as 0.070 at −252°, and that of the solid as 0.076 at −259.9°, both at approximately atmospheric pressure. Imagine that we compress solid hydrogen at a temperature of −259.9° to a pressure of 15000 kg., and then raise the temperature at 15000 kg. to 65° C. We arrive at a substance of density 0.1301. During the last increase of temperature we have increased the volume for two reasons, first the ordinary temperature expansion, and second an increase of volume when the substance passes from the solid to the gaseous phase. It follows that the density of the solid at 15000 kg. and −259.9° is certainly greater than 0.1301. Hence the compressibility of solid hydrogen is so great as to allow under 15000 kg. a decrease of volume to less (and probably much less) than 0.58 of its original value. This is certainly the highest compressibility that has ever been found in a solid; the greatest volume change in a solid that has hitherto been measured to my knowledge is that of potassium, which under 15000 would decrease to about 0.70 of its initial volume. In the same way by comparing the volume of the liquid at atmospheric pressure (density 0.070) with that of the gas under high pressure, we find that liquid hydrogen is so compressible as to allow under 15000 kg. a decrease of volume to less than 0.54 of its initial value. The most compressible liquid yet measured is ether;[12] this gives at 15000 (extrapolated from 12000) a decrease of volume to 0.66 of the initial value.

The compressibilities are even more extreme in the case of helium. The density of liquid helium at −271.6° is 0.1456, that of gaseous helium at 15000 kg. at 65° is 0.340 (or greater). Hence liquid helium at −271.6° is compressed by 15000 kg. to less than 0.43 of its initial volume, a truly astounding compressibility.

The figures for nitrogen are more conservative. The density of liquid nitrogen at −195° is 0.810 and at −205° 0.854. The density of the gas at 65° at 15000 kg. is 1.102. The compressibility of the liquid, making no allowance for temperature expansion, is thus very close to that of the more compressible liquids hitherto measured. Allowing for the temperature expansion, the compressibility is probably greater than that of the most compressible liquid yet measured, but not much greater.

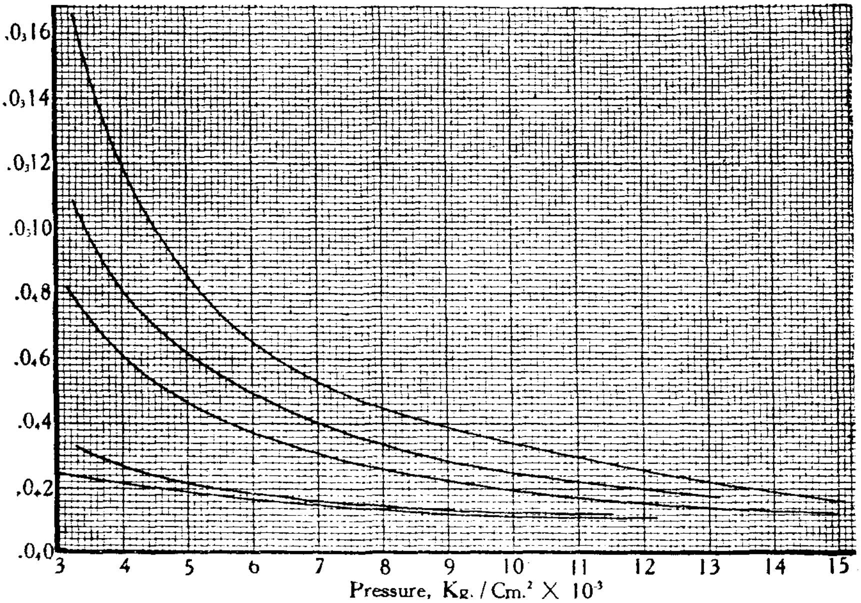

FIGURE 3. The compressibility, $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$, as a function of pressure, of helium at 55°, hydrogen at 65°, nitrogen at 68°, CS_2 at 65°, and water at 65°, reading from top down.

A further idea of the behavior of these high pressure gases as compared with the ordinary liquid is given by a study of the compressibility. This is shown as a function of pressure in Figure 3 for hydrogen at 65°, helium at 55°, and nitrogen at 68°. The compressibility thus shown is the quantity $\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$. This quantity is not exactly the same as that which I have previously called the compressibility, which I have usually defined as $\frac{1}{v_0}\left(\frac{\partial v}{\partial p}\right)_\tau$, where v_0 is the volume at atmospheric pressure. The change in the convention is necessary when dealing with gases because of the very large v_0 of a gas. In the same diagram are also shown the compressibility of two liquids, water and CS_2, defined in the same way as for the gas. It will be seen that under high pressures the compressibility of gases behaves in very much the same way as that of substances ordinarily liquid. At 5000 kg. the compressibility of nitrogen is the same as that of water at atmospheric pressure; hydrogen reaches this figure at 6000 kg., and helium at 7500.

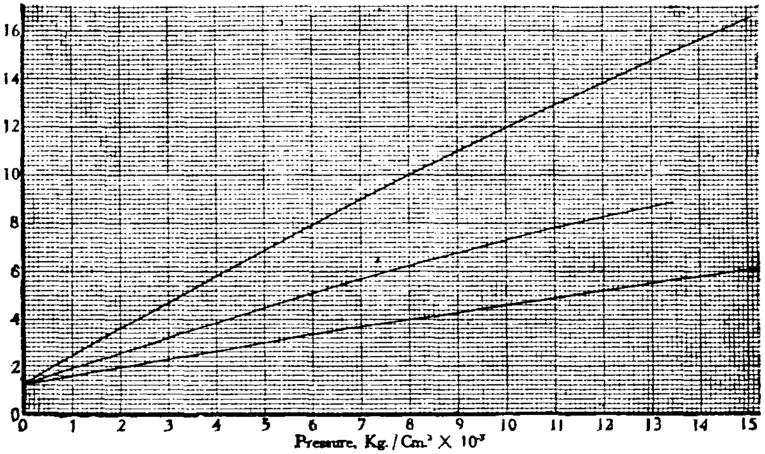

FIGURE 4. The product pv as a function of pressure for nitrogen, hydrogen, and helium (curves reading from top down). p is the pressure in kg/cm² and v is the volume of that weight of gas which under a pressure of 1 kg/cm² occupies 1 cc at 0° C. The temperatures are as in Figure 3.

From here on the decrease of compressibility is rapid, the high pressure gas behaving like an ordinary liquid at somewhat lower pressures. The diagram gives the impression that at pressures not very much in excess of 15000 the compressibility of the three gases and the two liquids will be very much the same. The order of sequence of the compressibilities of the three gases is what would be expected from the molecular volumes, the compressibility of helium being greatest and that of nitrogen least. (Lowering the fiducial volume at 3000 of helium would increase its compressibility by a factor becoming greater at the higher pressures.)

Carrying out the method of presentation of Amagat, I have shown in Figure 4 the product pv as a function of pressure. For a perfect gas, pv has the constant value 1.24 at 65°. The three gases are seen to behave quite differently in this regard; as before helium is most compressible (that is, it approaches most closely to the perfect gas), hydrogen is intermediate, and nitrogen is least. As a function of pressure, pv is nearly linear, but with a quite definite downward curvature. Possibly the simplest way to derive an equation of state would be to reproduce these curves. I have found that an ordinary

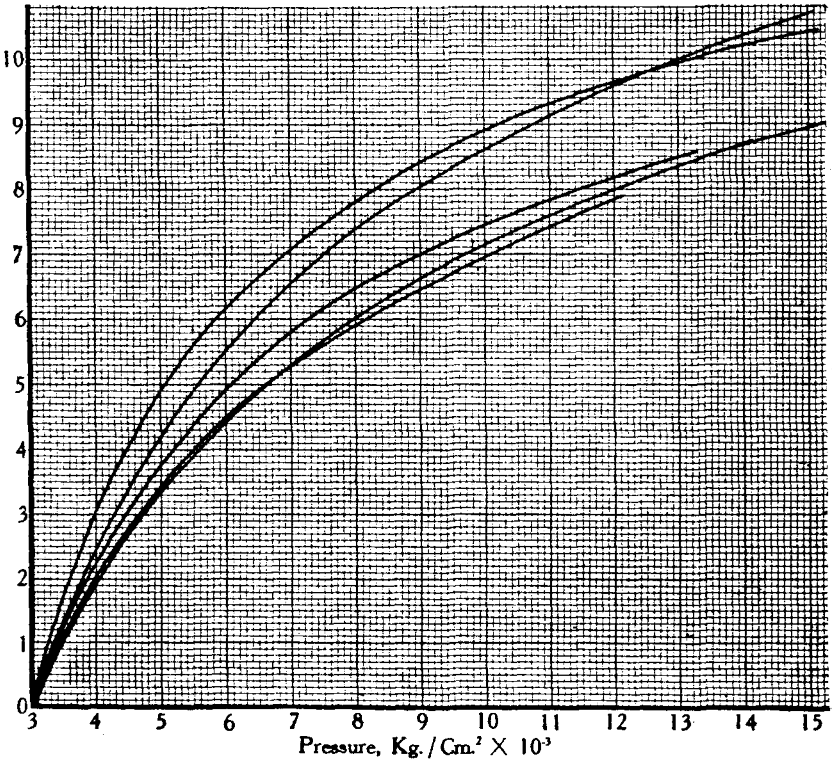

FIGURE 5. The change of volume in cc. per mol. as a function of pressure reckoned from 3000 kg/cm² as the fiducial pressure. At 10000 kg/cm² the order of the curves, reading from top down, is: helium at 55°, nitrogen at 68°, hydrogen at 65°, argon at 55°, and ammonia at 30°.

two constant power series in the pressure is not adequate within the experimental accuracy.

Turning now to the other two substances, ammonia and argon, whose fiducial volumes at 3000 I have not tried to estimate, a certain amount of information, although not as much as for the other substances, may be obtained from a study of the changes of volume above 3000. In Figure 5 are plotted the volume changes in cc. per mol. reckoned from 3000 as zero, including in the diagram also the volume changes of hydrogen, helium, and nitrogen. The volume changes of all five substances do not differ very much from each other. The changes for ammonia and argon are slightly less than for the others. There are two cases of crossing of these curves; the curve for nitrogen crosses that of helium near 13000 kg., and that of argon crosses that of ammonia near 5500 kg. It is also evident that argon would cross hydrogen not far beyond the pressures actually reached. It seems to me that the explanation of these crossings is to be sought in the compression of the structure of the atom or the molecule itself. Compare, for instance, nitrogen and helium. The nitrogen molecule is normally much larger than the helium atom; at comparatively low pressures this shows itself in the larger molecular volume of nitrogen. But as pressure increases toward the upper end of the range, there remain more possibilities in the way of decrease of volume in the nitrogen molecule than in the helium atom. The more complicated structure remains the more compressible at the high pressures. In the same way the argon atom is a more complicated structure than the ammonia molecule, consisting of 19 extra nuclear electrons against 10, so that it would be expected to remain more compressible, and the crossing of the curves becomes plausible. For the same reason the curve of argon crosses that of hydrogen. Of course the pressure of the crossing is without absolute significance (as the zero at 3000 is without absolute significance), but the relative shape of the curves, which makes the crossing possible, is of absolute significance.

EQUATIONS OF STATE FOR HIGH PRESSURES.

There have been proposed a number of equations of state for gases, mostly on the basis of the work of Amagat,[1] and it is of interest to find how these agree, over a pressure range much wider than Amagat's, with the new experimental facts. Of course if an equation is to be used to extrapolate to infinite pressures, a use which has frequently been made of the equations hitherto proposed, it must also be possible to extrapolate to 15000 with it.

In the first place, the classical equation of van der Waals,

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

which was constructed specially to take account of the size of the molecules and the forces between them, and which would therefore seem to be especially adapted to use at high pressures, is known not to work at all, if for the constants of the equation we use the values determined in the regular way from the critical data. Thus the value listed in the Smithsonian Tables for "b" of hydrogen is 880×10^{-6}; this is the theoretical volume at infinite pressure. But experimentally the volume assumes this value between 4000 and 5000 kg. For smaller volumes, corresponding to pressures greater than 5000, the formula demands negative pressures. Or stated in another way, van der Waal's equation predicts that under infinite pressure the volume will be that which we actually find under 4500 kg. This means that the actual gas under high pressures continues much more compressible than the equation indicates. It is, however, of interest to inquire whether the general form of van der Waal's equation cannot be maintained as an empirical equation with properly determined constants to give correctly the high pressure behavior. I have tried to do this for hydrogen. I have determined the a and the b of the equation above so as to agree with my experimental values at 3000 and 6000 kg. (this pressure range corresponds to more than half my volume range). With the constants so determined I have computed the volume to be expected at 13000 kg. This I find to be 8.06 cc. per gm. against 7.33 experimental, or expressed by differences, the decrease of volume between 3000 and 13000 demanded by the equations is 3.11 cc. per gm., whereas the experimental decrease is 3.76. The equation gives too large a volume (too small a compressibility) at high pressures, and is therefore not suitable, even when treated as a purely empirical equation with the constants so chosen as to fit the high pressure data. The fact that the "a" determined so as to fit the data at 3000 and 6000 is negative shows that the constants have no theoretical significance.

I have also tried an equation of simpler type than van der Waal's, which also represents the volume under infinite pressure as a finite quantity different from zero. This equation is

$$p(v - b) = CT.$$

The advantage of this equation is that it may be checked from a knowledge of only the changes of volume beyond a certain fiducial pressure, and does not require a knowledge of the actual volumes. This may be seen by writing it in the form $p\,(v_0 - \Delta v - b) = CT$, where v_0 is the volume (unknown) under the fiducial pressure. Δv is known experimentally, so that the equation is still a two constant equation, treating $v_0 - b$ as a new single constant, with p and Δv the experimental quantities to be substituted into it. I have used this as an interpolation equation, choosing the constants so as to reproduce the experimental volumes at 3000 and the maximum pressure, 13000 or 15000. The equation does not work for any of the gases; for hydrogen the experimental volume lies above that given by the equation over the entire intermediate range of pressure, for helium the experimental curve is below the equation for the first part of the pressure range and above it for the last, for ammonia the experimental curve is below that of the equation over the entire range, for nitrogen the experimental is also below the equation over the whole range, and for argon, the experimental is above the equation over the entire range. The discrepancies rise to as much as 25 or 30% of the volume change.

I have also tried to fit to the data for hydrogen a three constant formula, which at constant temperature is of the form:

$$(p + P)\,(v - b) = C.$$

A formula of this type has been applied by several investigators in the region of high pressures. Tumlirz[13] and Tammann[14] have applied it with considerable success to the data of Amagat for liquids to 3000 kg/cm², and recently Richards [15] has extensively applied it to my measurements of the compressibility of metals [16] over a pressure range of 12000 kg/cm². Tumlirz and Tammann differ as to the sort of function of temperature which they regard the "constants" to be, and Richards regards the formula as only a good approximation over a fairly wide pressure range, but recognizes that over a wider range both the P and the b of the formula are functions of pressure at constant temperature.

The three constants I determined so as to give the experimental volumes at 2000, 8000, and 13000 kg.; their values were: $P = 2204$ (so-called internal pressure), $b = 4.81$, and $C = 38200$. The values shown in Table VII were found. It is seen that the experimental values lie below the calculated ones from 2000 to 8000, above them from 8000 to 13000, and of course below them again beyond 13000.

That is, this formula again gives a too small compressibility at the high pressures.

I believe that one reason for the failure of equations of this type is to be found in the compressibility of the atoms or molecules. The atoms probably have a rather small compressibility, but one which stays fairly constant over a very wide range of pressure. It results that over a small pressure range the volume of the atoms will appear to be invariable within the limits of experimental error, and may be treated as a constant in the equations, but over a larger pressure range the change of volume of the atoms becomes apparent.

TABLE VII.

Comparison of Observed and Experimental Volumes of Hydrogen.

Pressure kg/cm^2	Volume cc. per gm. Calc.	Exp.
2000	13.90	13.90
3000	12.16	11.64
4000	10.98	10.12
5000	10.12	9.80
6000	9.48	9.29
7000	8.96	8.87
8000	8.55	8.55
9000	8.22	8.26
10000	7.94	8.01
11000	7.70	7.78
12000	7.50	7.55
13000	7.32	7.32

Recently Professor F. G. Keyes has developed a type of equation from a theoretical standpoint,[17] and has determined the constants from the best data hitherto available. It is of interest to compare how the results calculated by his equations for high pressures agree with experiment.

It is especially to be kept in mind that the pressure in all the following formulas of Keyes is in atmospheres, and not in kg/cm^2.

Hydrogen. Keyes' equation for hydrogen [10] is of the general type which he uses for all substances not monatomic, namely

$$P = \frac{R\tau}{v-\delta} - \frac{A}{(v-l)^2}$$

with the numerical values

$$\delta = 9.619e^{\frac{-2.898}{v}}, \quad A = 3.91 \times 10^4, \quad l = 1.18.$$

P is the pressure in atmospheres, and v is the volume in cc. per gm. At 30° I find the following numerical values of volume approximately.

Pressure	Volume cc. per gm.	
kg/cm²	Calc.	Exp.
2523	12	12.3 (12.0)
3116	11	11.6 (11.3)
3879	10	10.7 (10.4)
4713	9.5	10.1 (9.8)
5622	9	9.5 (9.2)
8907	8	8.3 (8.0)
14000	7.32	7.2 (6.9)

I have given two sets of figures in the experimental column. The first are those given in Table I and are based on Amagat's value at 3000 kg. as fiducial. I have already explained that Keyes' volume at 3000 is less than Amagat's. The experimental figures in parentheses are corrected by a subtractive constant so as to make the fiducial volumes agree, affording a juster basis of comparison. It will be seen that the equation makes the gas initially more compressible and finally less compressible: again the compressibility of the gas persists at high pressures. This may be seen in another way from the equation, which predicts a limiting volume at infinite pressure of about 6. I do not believe that the experimental data give the impression of a limiting volume (if there is one at all) so high.

Helium. This gas, being monatomic, has a somewhat simpler type of equation,[11] namely

$$P = \frac{R\tau}{v-\delta} - \frac{A}{(v+l)^2},$$

where $\delta = 3.17, \quad A = 3285.3, \quad l = 6.84.$

At 65° I find the following values:

Pressure kg/cm²	Volume cc. per mol. Calc.	Exp.
3314	21.2	21.1
3894	20.0	19.4
4660	18.8	17.8
5802	17.6	16.1
7681	16.4	14.5
9764	15.6	13.4
11351	15.2	12.8
13495	14.8	12.1

It will be seen that the calculated volumes become continually greater than the experimental ones as pressure increases. This is the same behavior as shown by hydrogen; the compressibility persists.

It should be said that the constants in this equation for helium were deduced from measurements of Onnes to a maximum of 70 atmospheres. The domain of extrapolation is very wide; it is surprising that the equation works as well as it does.

Ammonia. Keyes' equation for ammonia [18] is

$$P = \frac{4.8177}{v-\delta}\tau - \frac{34610}{(v+1.173)^2},$$

$$\log_{10}\delta = 0.9813 - \frac{3.08}{v}.$$

This is based on results obtained at the Mass. Institute of Technology in 1912, reaching to very low pressures. This equation, with its constants determined by such data, appears fundamentally incompetent to represent the facts at pressures even moderately high. Thus at a molecular volume of 50 cc. per mol. the pressure has already turned negative.

Nitrogen. Keyes gives two equations for nitrogen. The first [17] is based on the data of Amagat between 100 and 1000 atmospheres, and is

$$P = \frac{2.9138}{v-\delta}\tau - \frac{1587}{(v-0.007)^2}, \quad \text{where } \log_{10}\delta = 0.2200 - \frac{0.284}{v}.$$

At 68° I calculate the following values.

Pressure kg/cm²	Volume cc. per mol. Calc.	Exp.
2444	36.5	38.1 (36.0)
3091	33.7	35.8 (33.7)
4201	30.9	33.1 (31.0)
5831	28.0	30.7 (29.6)
8390	25.3	28.5 (26.4)
12670	22.5	26.2 (24.1)
15502	21.1	25.3 (23.2)

The experimental values in parenthesis have been corrected by a subtractive constant to make the volume at 3000 agree with that calculated. My value at 3000 is derived from Amagat's experimental value at 3000, whereas the calculated value at 3000 employs Amagat's results to 1000. The discrepancy is to be attributed in part to inconsistency between Amagat's high and low pressure results, as already explained.

It is to be noticed that the formula works differently in the case of nitrogen than for hydrogen and helium, for here it gives too small volumes (too high compressibility) at the high pressures.

The second equation of Keyes [19] is based on measurements made at the Mass. Institute of Technology in 1923 by Taylor and Smith on very pure nitrogen, but over a much smaller pressure range. The equation is:

$$P = \frac{2.9286}{v-\delta}\tau - \frac{1623.6}{(v+0.2954)^2},$$

$$\log_{10}\delta = 0.18683 - \frac{0.3113}{v}.$$

This equation is not adapted at all to the high pressure measurements. Thus it predicts that a volume of 21.1 cc. per mol. will be found at a pressure of 5000 kg/cm², whereas it will not be actually found at less than 25000, making a most conservative extrapolation. It is evident that the high pressure behavior contains features not sufficiently foreshadowed by the low pressure behavior.

Argon. Keyes' equation [11], based on Crommelin's low pressure results, is:

$$P = \frac{2.0578}{v-1.278}\tau - \frac{995.7}{(v-0.247)^2},$$

Applied at high pressures, this indicates a very much too rapid falling off of compressibility. The calculated change of volume between 3000 and 12500 kg/cm² is 5.42 cc. per gm. atom against 8.18 experimental.

The general conclusion seems to be that this type of equation does not give an adequate method of predicting the behavior at high pressures from that at low. It would without doubt be possible to redetermine the constants in the equations so that the equation could be used as an interpolation formula over the entire range of pressure, but I believe that in view of the experience above, there would still be a good deal of question as to the chances of using the equation thus determined as an extrapolation equation to very much higher pressures.

I believe that the method of procedure here is to find a formula of a type such that when its constants are determined to fit the experimental values from 3000 to 6000 it will give correct values by extrapolation to 15000. Then if the constants are readjusted so as to cover the range 3000 to 15000 with slightly greater accuracy, we may extrapolate over a much wider range with considerable confidence. Furthermore, it is always to be borne in mind when attempting to construct such equations, that physically a sufficiently high pressure is to be expected to freeze the amorphous fluid phase to the crystalline solid.

Becker's Equation for Nitrogen. Recently Becker [20] has constructed from Amagat's data for nitrogen, on the basis of some theoretical considerations, an equation expressly designed to give the volume at pressures much beyond those of Amagat's experiment. The equation is

$$p = \frac{R\tau}{v}\left(1 + \frac{\kappa}{v} e^{\frac{\kappa}{v}}\right) - \frac{a}{v^2} + \frac{\chi}{v^{\beta+2}}$$

Here p is in dynes/cm², and v is in cc. per mol. The constants are:

$R = 8.31 \times 10^7$, $\kappa = 40.3$, $a = 1.26 \times 10^{12}$, $\chi = 384 \times 10^{17}$, $\beta = 5$.

This equation gives at 3000 the same volume which I have given above (as it should, because both of us connect with Amagat's data at 3000), and at higher pressures gives volumes very slightly less than those which I have found experimentally, the agreement being much closer than that of any other formula which I have tried. Thus at 15000 kg/cm², the calculated volume is 25 against 25.4 experimental.

The formula contains four disposable constants, but perhaps this is not too many when it is considered that the ordinary critical behavior as well as the high pressure behavior is reproduced.

SUMMARY.

A simple modification of the apparatus with which the compressibility of liquids was measured is described with which the compressibility of hydrogen, helium, ammonia, nitrogen, and argon has been measured to pressures varying from 12000 to 15000 kg/cm^2. The changes of volume measured experimentally are from a fiducial pressure of 3000 as the zero. The results are shown in Tables and curves and various properties of the results are discussed. The compressibility under high pressures approaches that of ordinary liquids under the same pressures, and is much less than the compressibility of ordinary liquids at low pressures. The departure from a perfect gas is indicated by the behavior of pv; at 15000 kg. this is about 16 for nitrogen and 6 for helium.

The gaseous densities under the highest pressures are such that the molecules or atoms are very nearly in contact in the sense of kinetic theory. It is shown that the phenomena at high pressures probably depend on the compressibility of the atoms or molecules themselves; a complicated molecule like nitrogen retains more compressibility at high pressures than a simple atom like helium.

No general type of equation of state for high pressures has been yet proposed which agrees well with the facts. Becker's equation for nitrogen agrees much better than any of the others. The discrepancy between most of the equations and experiment is in such a direction that the actual volumes are smaller than the calculated volumes at high pressures. This is to be ascribed to the compression of the atoms.

Few satisfactory values were found for the thermal expansion of these gases, but it does seem highly probable that the expansion remains large under high pressure.

The data for gases enable a lower limit to be set on the compressibility of liquid and solid hydrogen and of liquid helium. It appears that these solids and liquids are very much more compressible than any hitherto measured.

It is a pleasure to acknowledge my indebtedness to my assistant Mr. H. B. Curry for many of the readings.

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SUMMARY OF A STUDY OF STELLAR DISTRIBUTION.

BY HARLOW SHAPLEY AND ANNIE J. CANNON.

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The study and classification of the spectra of stars during the past thirty years at Harvard has culminated in the preparation of the Henry Draper Catalogue, which contains, in nine volumes, the positions, magnitudes, and spectral classes of a little more than 225,000 stars. With this new survey, which covers the whole sky and extends in places to the tenth magnitude and fainter, a considerable advance can be made in the analysis of the distribution of stars in space and on the surface of the sky.

In the past, Seeliger, E. C. Pickering, Stratonow, Kapteyn, and many others have made extensive investigations of the distribution of stars, for the problem is recognized as fundamental in the study of galactic structure. Kapteyn and his associates in late years have examined, as far as possible, the spatial distribution, and for the brighter stars Pickering, among others, has considered the distribution with regard to spectral class as well as position and magnitude. It has long been recognized that the distribution differs greatly for the different classes, but up to the present time the study, for the whole sky, of the various spectral types separately has been possible for only the few thousand naked eye stars.

The discussion of the Henry Draper Catalogue has been in progress at the Harvard Observatory for the last three years. The report on the results given in the present paper is in the nature of a general summary of a number of investigations, most of which have already appeared in Harvard Observatory publications. The discussion is mainly based on the first manuscript of the catalogue, but the subsequent additions and revisions, incorporated in the printed volumes, are statistically unimportant.

I. GENERAL AND MISCELLANEOUS RESULTS.

1. In classifying a star, the spectrum has been assigned to one of about forty different subtypes, which are grouped into eleven classes. A few spectra appear in the less common classes P, O, R, N, S, Md, or

they were designated as "peculiar" or "continuous." More than ninety-nine per cent, however, have been assigned to the ordinary classes B, A, F, G, K, and M, indicating that the stars fall into a few typical groups.

2. Approximately twenty thousand stars are found to have spectra essentially identical with that of the Sun.

3. For statistical discussion the various subdivisions of the spectral classes are grouped into "divisions" as follows:

Division	Subdivisions
B	B0, B1, B2, B3, B5
A	B8, B9, A0, A2, A3
F	A5, F0, F2
G	F5, F8, G0
K	G5, K0, K2
M	K5, Ma, Mb, Mc

The divisions are listed in order of increasing redness, and therefore in order of decreasing surface brightness.

4. The distribution of 220,570 of the stars in the manuscript catalogue among these six principal divisions is as follows:

Division	B	A	F	G	K	M
Number	3,567	64,259	21,120	46,552	73,208	13,864

This tabulation, of course, does not differentiate the various intervals of apparent magnitude nor the positions with reference to the galactic plane. The relations of spectrum to apparent brightness and galactic coordinates are outlined later.

5. The photographs for the spectra of faint stars were made principally with 8-inch telescopes and objective prisms at Arequipa, Peru, for the southern sky and at Cambridge for the northern. The more favorable location at Arequipa makes possible the classification of fainter stars for the southern hemisphere. The limit in apparent visual magnitude to which the catalogue is essentially complete may be placed at 8.25 for northern stars, and possibly half a magnitude fainter for southern; but to avoid errors of selection the present investigation has been limited in most details to stars brighter than magnitude 8.25.

6. There are nearly sixty thousand stars in the manuscript cata-

logue brighter than this limit. Their distribution among the principal spectral classes is as follows (computed from Harvard Circular 226, 1921):

Spectrum	Number of Stars	Spectrum	Number of Stars
B0-5	2,061	F8	2,488
B8	1,604	G0	2,784
B9	2,752	G5	5,248
A0	6,320	K0	11,504
A2, 3	5,208	K2	4,008
A5	1,352	K5	2,832
F0	3,208	Ma	1,204
F2	1,976	Mb, c	455
F5	3,504	Total	58,508

7. If the stellar density for any spectral type were uniform throughout space, the number of stars visible should double with every increase of half a magnitude in brightness. This condition fails for all spectral divisions except G (H.C. 226). Apparently, then, the density falls off rapidly with increasing distance from the Sun, at least in directions away from the galactic plane. The anomalous behavior for the G stars of the catalogue indicates that they are predominantly dwarfs and are within the nearby region of fairly constant density. In low galactic latitudes, however, the decrease of space density with distance is very small for all classes except B, as Kapteyn and van Rhijn have already shown.

8. Naked eye stars are nearly twice as numerous in Class K0 as in any other subdivision. About sixty per cent of the brighter stars, however, are earlier in spectrum than the Sun.

9. The investigation of the distribution of stars of various spectral classes in different longitudes along the galactic equator brings out the following features:

a. Stars of division A brighter than magnitude 6.5 show remarkable uniformity of distribution in longitude. There is little or no evidence of the various groupings found for the bright B stars. The A stars are not distinctly more numerous in the direction of the center of either the local system or the Galaxy proper (*Cf.* H. C. 229, 1922).

b. Fainter stars of division A reveal through their distribution the relative nearness of the star clouds in Cygnus; and the Class B stars alone show the richness of the Carina region — the apparent center of the local cloud.

c. Both the A and K stars between magnitudes 7.0 and 8.25 clearly show a deficiency in the region of the Taurus nebulosity, in galactic latitude −10°, longitudes 130° to 170° (H.C. 240, 1922, Fig. 2). Classes F and G in this region are unaffected, indicating that the majority of those stars fainter than magnitude seven are dwarfs, and thus are located between the nebula and the Sun.

d. Assuming for the giant A and K stars an average absolute magnitude of +0.5, we compute that the distance of the obscuration in Taurus is not over three hundred parsecs, which agrees with estimates made by Dyson and Melotte, Pannekoek, and Seares.

e. A deficiency of stars of all spectral classes between longitudes 340° and 20° on the galactic equator indicates the great rift in the Milky Way, which has long been considered an effect of obscuring matter.

f. Giant stars brighter than the seventh magnitude appear to show the effect of the rift, suggesting that its nearer border may not be more than two hundred parsecs distant.

g. Similarly, the early B stars yield an estimate of between two hundred and three hundred parsecs for the distance to the distinct nebulosities in the Scorpio-Ophiuchus region.

10. The fainter stars of Classes Ma, Mb, and Mc show a strong preference for the region of Sagittarius, which is apparently an indication that they are highly luminous stars on the average, and that, like the novae, planetary nebulae, Cepheids, and O-type stars, they appear more frequently in the direction of the galactic center.

11. Away from the galactic center there is but one-half the average number of long period variable stars, per unit area, and in the direction of the center (in Sagittarius) there is twice the average number (H.C. 245, 1923).

II. THE LOCAL SYSTEM.

12. Figures 1 and 2, reproduced from Harvard Circular 239, show that the fainter B stars are confined closely to a narrow belt along the galactic circle, while the brighter stars of this division indicate the existence of a local star cloud, the central plane of which is not coincident with the corresponding plane of the general Galaxy. The local system appears to be distinct from the general field of the Milky Way, and contains nearly all the bright B stars and probably many stars of the later spectral classes. Barnard's dark nebulae also show a preference for this secondary galaxy. Hubble has shown that both the

luminous and obscuring extended nebulae appear to gather along the plane of the local system as well as along the galactic circle. Van de Linde has shown recently that all naked eye stars taken together indicate the existence of this preferential plane.[1]

13. Of the 346 early B stars brighter than visual magnitude 5.25 (Fig. 1), it is estimated that ninety per cent belong to the local cloud, the others to the general galactic field. Their distribution in successive intervals of forty degrees of galactic longitude is as follows, where the third column shows the inclination of ten to fifteen degrees between the planes of the local system and the Galaxy:

Median Longitude	Number	Median Latitude	Median Longitude	Number	Median Latitude
°		°	°		°
20	24	− 2.0	220	55	− 7.0
60	31	− 1.5	260	53	− 1.5
100	22	− 5.0	300	65	+10.5
140	28	−15.5	340	18	+14.5
180	50	−14.0			

14. Possibly eighty per cent of the early B stars in the magnitude interval 5.26 to 6.25 belong to the local system, and two-thirds of those in the interval 6.26 to 7.25.

15. The distribution in longitude of the 719 B stars in the magnitude interval 7.26 to 8.25 (Fig. 2) is tabulated below:

Median Longitude	Number	Median Latitude	Median Longitude	Number	Median Latitude
°		°	°		°
15	32	−3.5	195	50	−3.5
45	46	−0.5	225	79	−1.5
75	44	0.0	255	116	−0.5
105	46	−2.5	285	64	−1.5
135	44	+1.5	315	87	−1.0
165	65	−2.5	345	46	−1.5

The excess of stars in longitude 255° (near the center of the local system) indicates that possibly twenty per cent of all B's in this magnitude interval are local stars.

16. The average of the median galactic latitudes for B stars between 7.26 and 8.25 is −1°.4, which may be taken as the dip of the galactic circle for stars at the distance here concerned.

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[1] *Thesis*, Rotterdam, 1921.

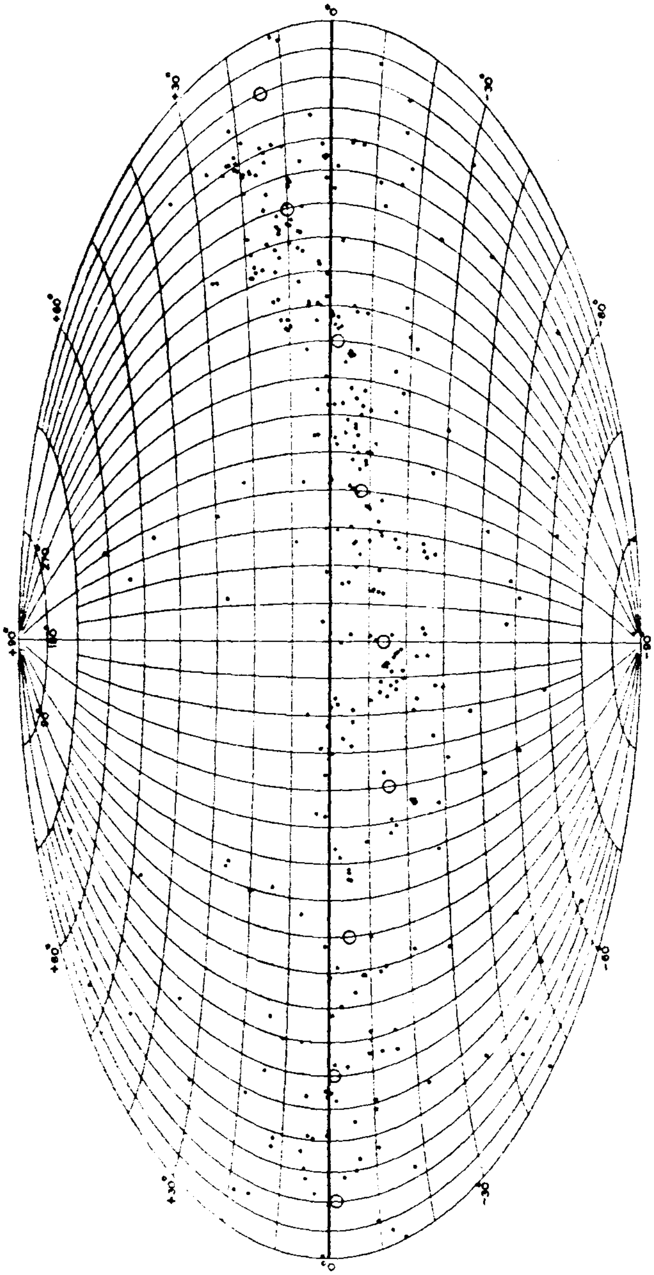

FIGURE 1. Galactic distribution of stars of division B, brighter than magnitude 5.26.

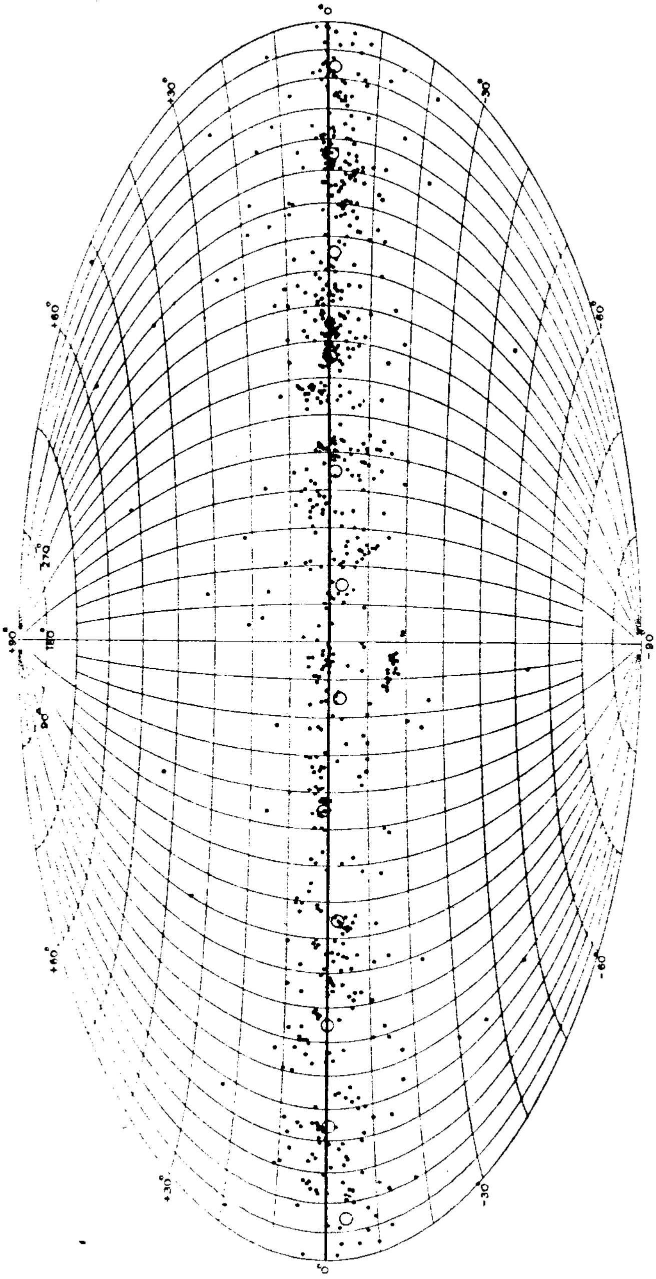

FIGURE 2. Galactic distribution of stars of division B, magnitudes 7.26 to 8.25.

17. The corresponding distance of the Sun to the north of the galactic plane, $\delta = 10^{0.2\,(m-M)+1}\sin 1^\circ.4$, is in excess of fifty light years. An accurate determination of this quantity is not possible from the present data because of uncertainties in the average absolute magnitude, M, of the distant B stars of apparent magnitude m, and also because of the unknown proportion of local and galactic objects.

18. The diameter of the local cloud, which is probably very irregular in form, is of the order of one thousand parsecs.

19. Stars of division A brighter than magnitude 6.5 show a deviation from the galactic plane of the same character as that shown by the B stars (H.C. 229). The inclination, however, is of the order of 5°, which may be considered as the resultant of the true inclination of the local cluster and of the zero inclination for the general galaxy of field stars. Possibly one-half of these bright A stars belong to the local cloud.

20. The faint stars of Class A0 show a high concentration to the primary galactic circle and no certain trace of the secondary.

21. From the distributional data now at hand, it is difficult to say whether the local system includes only the bright early B stars and a part of the bright early A's, or whether all spectral types are represented, but so proportioned and distributed on the surface of the sky that only the more highly concentrated first type stars show the existence of the local cloud.

III. THE GALACTIC CONCENTRATION.

22. To study the concentration of stars to the plane of the Galaxy and the frequency of the different spectral classes in all galactic latitudes and longitudes, eighty-four fields were selected, each containing one hundred square degrees. In every field all the stars of known spectrum brighter than 8.25 were grouped with respect to type and magnitude. The fields cover more than one-fifth of the whole sky. In the region with galactic latitude less than 15°, however, they cover nearly one-half of the total area. The fields are centered along the equator and on the latitude circles at 10°, 30°, 45°, 60°, 75°, and 90°, in both hemispheres.

23. The galactic concentration is shown by the diagrams in Figures 3a and 3b where the ordinates are numbers of stars per hundred square degrees and the abscissas are galactic latitudes. The diagrams in Figure 3a are for stars between magnitudes 7.0 and 8.25, while those in Figure 3b are for all stars to the fainter limit.

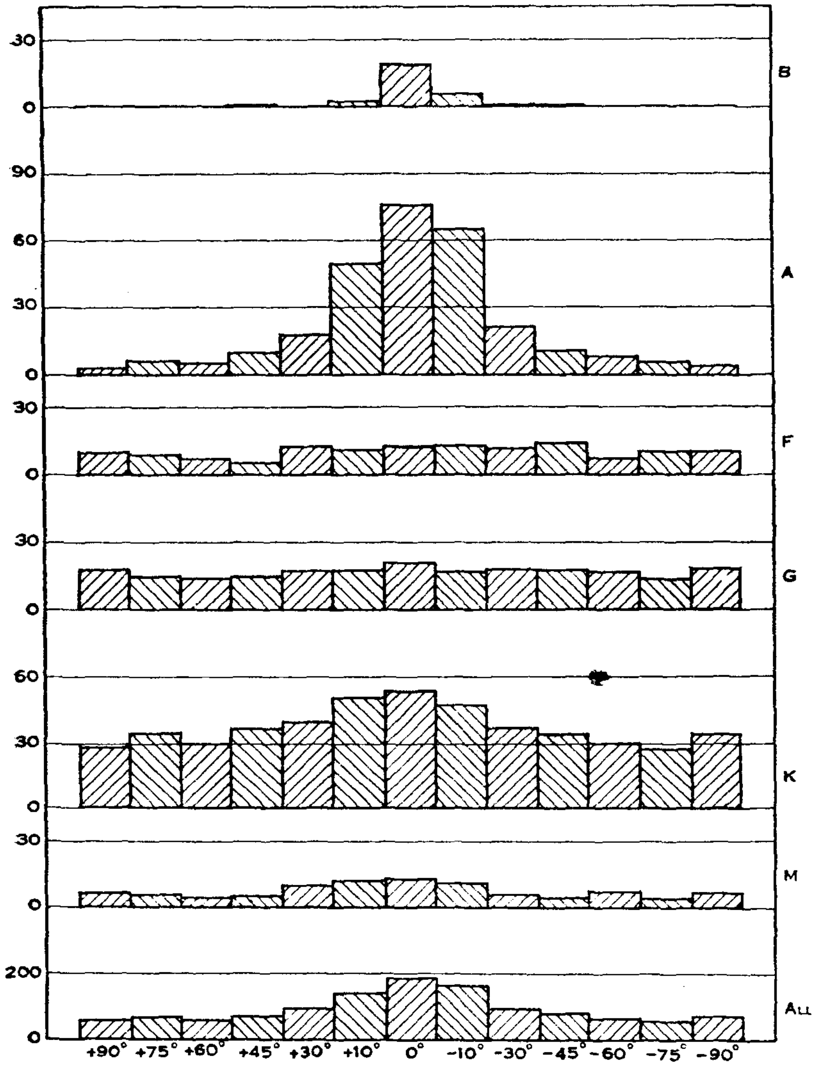

Figure 3a. The concentration of stars toward the galactic circle. Abscissas are galactic latitudes. Ordinates are average numbers of stars for each 100 square degrees. Magnitudes between 7.0 and 8.25.

24. High concentration is shown for all B and A stars, a phenomenon long recognized for the brighter magnitudes, and for faint stars of

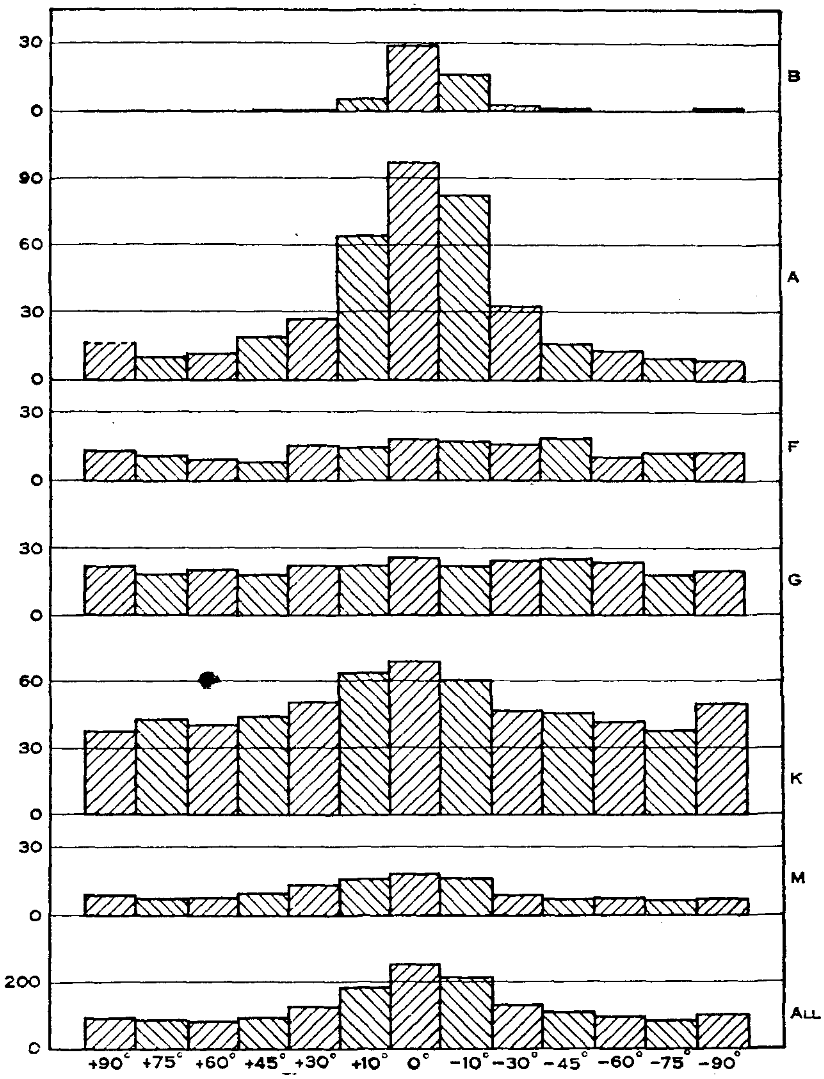

FIGURE 3b. Same as Figure 3a, but for all magnitudes brighter than 8.25.

all types together. It is well illustrated for the fainter B stars in Figure 2, where more than ninety per cent are seen to be within ten degrees of the galactic equator and only one-half of one per cent in higher galactic latitudes than 50°.

25. Little or no galactic concentration appears for F and G stars. They are largely dwarfs and a low concentration naturally results (*Cf.* No. 7 above).

26. A very sharp contrast occurs in Figure 3 between B8–A3 and A5–F2 stars. This has been interpreted to mean a rapid decrease of absolute magnitude with advance of spectral class from A0 to A5 (*Cf.* H.B. 796, 1923).

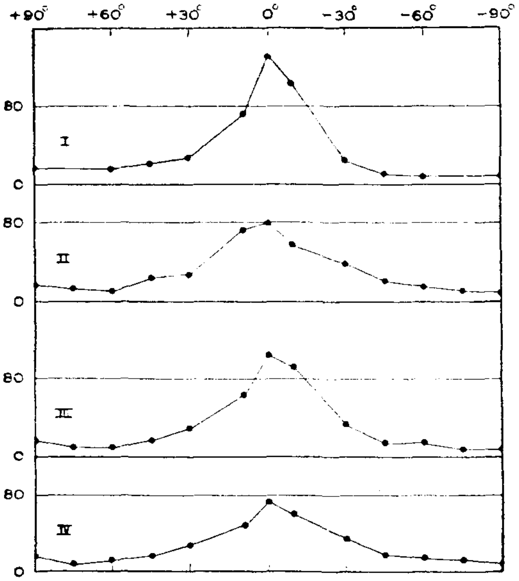

FIGURE 4. Galactic concentration of stars of Class A for four successive quarters of galactic longitude. The longitude interval for the first curve is 10° to 90°. Ordinates and abscissas as in Figure 3.

27. The superficial density is fairly uniform between 45° and 90° in both hemispheres for all spectral divisions except A. It does not fall off with increasing latitude, and, in fact, in Figure 3 a slightly greater density appears for F, G, and M stars at the poles than at latitudes 60° and 75°.

28. Longitude plays an important part in galactic concentration, largely because of the influences of the Cygnus star clouds, of the dark nebulosities in Taurus, Ophiuchus, Sagittarius, and elsewhere, and of the eccentric position of the Sun with respect to the local cloud and the general galaxy. This heterogeneity in the composition of the galactic

system will seriously disturb all attempts to make a closer analysis of galactic structure and concentration on the basis of general star counts.

29. The difference in galactic concentration in four quarters of longitude is illustrated by the curves in Figure 4 for stars of division A. Nort has previously given somewhat similar curves for fainter stars, but necessarily without discriminating the spectral classes.

30. An investigation of the distribution of M stars of all available magnitudes, reported in Harvard Circular 245, includes the following results:

a. Fainter than the eighth magnitude, the Ma and Mb stars are both moderately concentrated to the galactic plane.

b. Brighter than the eighth magnitude, stars of Class Mb show no galactic concentration, while the Ma stars are more numerous per unit area by thirty per cent between latitudes −10° and −30° than elsewhere.

c. Variable stars of Class Md appear to be concentrated to galactic latitude −20°, but the marked asymmetry in longitude is the more striking characteristic in the distribution of these stars (*Cf.* No. 11 above).

IV. THE APPARENT FREQUENCY OF VARIOUS SPECTRAL CLASSES.

31. The frequency of the spectral classes at different galactic latitudes is shown graphically in the twelve diagrams of Figure 5, where the ordinates are average numbers of stars (to magnitude 8.25) in a hundred square degrees and the abscissas are spectral divisions.

32. The preponderance of A stars in low latitudes and of K stars in high latitudes is a conspicuous feature to this magnitude limit. The diagrams indirectly indicate that the distant Milky Way regions are represented chiefly by their giant stars.

33. The excess of B and A stars in southern latitudes near the galactic equator presumably results from the dip of the local system. Elsewhere the differences between the two hemispheres are not significant, except for the bright group of A stars in Coma at the north galactic pole.

34. Of the stars within 15° of the galactic circle, thirty-eight per cent are of division A and twenty-nine per cent are of division K, thus indicating that these two classes definitely predominate in the nearer parts of the Milky Way (distance between one hundred and five hundred parsecs), in so far as stars of the ordinary catalogues are con-

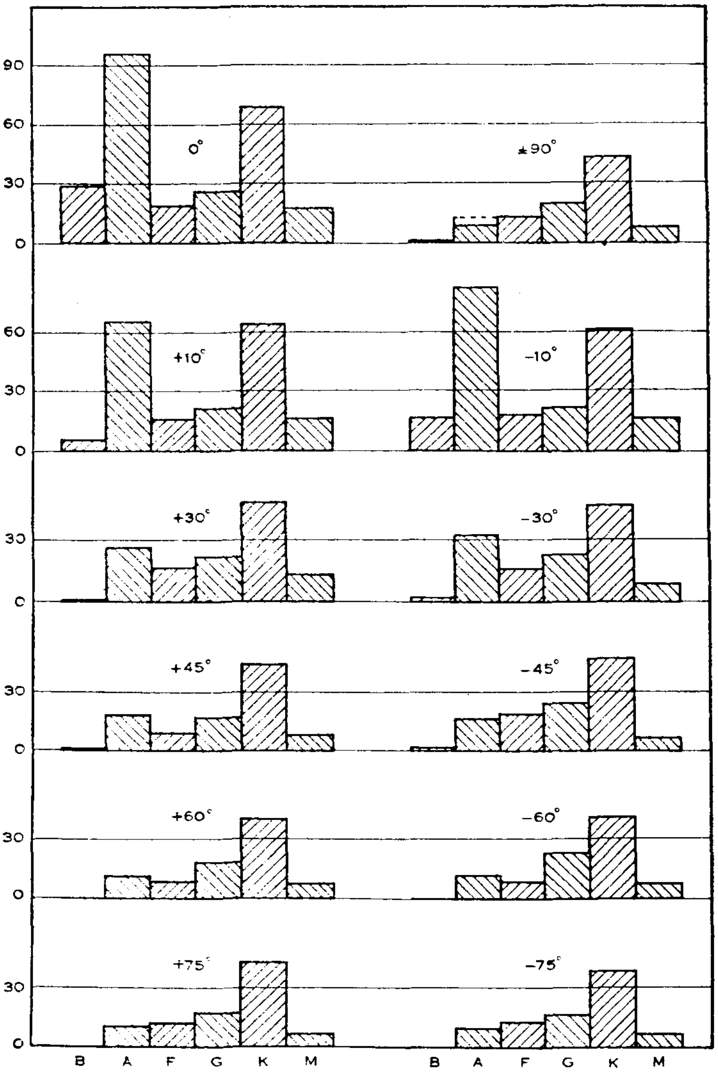

FIGURE 5. The relative frequency of spectral type in different galactic latitudes for all stars brighter than 8.25.

cerned. When we consider the stars in a given volume of space, however, the dwarf stars, of Class G and later, are found greatly to outnumber the giants of Classes A and K.

35. Taking the stars of all latitudes and magnitudes together, and also taking visual magnitude 8.75 as the fainter limit, we derive the distribution among the various spectral classes that is shown in Figure 6. Here the K stars are most numerous, as was also found for the naked eye stars.

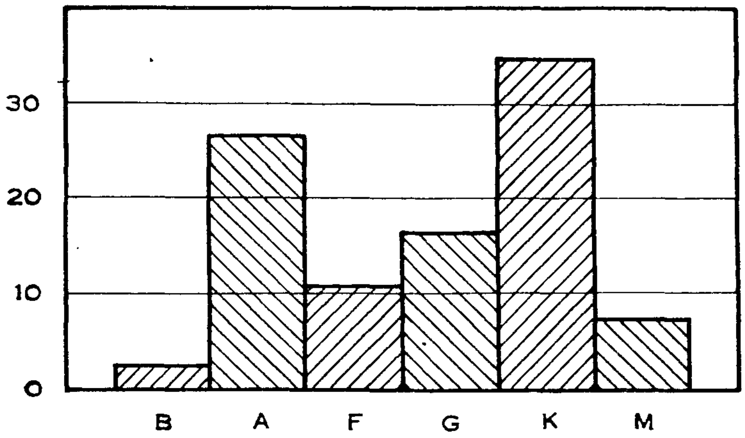

FIGURE 6. Frequency of spectral type without regard to galactic latitude.

V. THE DISTANCES OF THE STARS.

36. Increasing knowledge of the average intrinsic luminosity for most of the principal spectral types is transforming the problem of distances for faint objects into the task of sorting the stars into intervals of spectrum and apparent magnitude. The Henry Draper Catalogue thus becomes a fruitful source for the derivation of stellar parallaxes with an accuracy sufficient for many statistical purposes.

37. Brief consideration indicates that the B, A, K, and M stars of the Henry Draper Catalogue are very largely giants, at least to the magnitude 8.25. The F and G stars are chiefly dwarfs, although among them there is a scattering of giants and c-stars. If d denotes the distance in parsecs and m the apparent magnitude, we can compute the individual distances of about 150,000 faint stars with an average uncertainty of less than twenty-five per cent, by using the formula $d = b1.58^m$, where b is given by the relation $\log b = 1 - 0.2M$ and M is the mean absolute magnitude.

38. The following tabulation gives the value of *b* for the spectral classes where it can be safely inferred, and gives also the number of catalogue stars to which the formula can be applied:

Spectral Class	Absolute Magnitude	*b*	Number
B0-5	−1.5	20	3,567
B8, 9	−0.3±0.4	11.5	12,803
A0	+0.4±0.4	8.3	30,816
A2, 3	1.2±0.4	5.8	20,640
A5	1.7±0.45	4.6	5,064
F0	2.4±0.4	3.3	9,800
F2	2.8±0.4	2.8	6,256
F5	3.4±0.35	2.1	13,528
K0	0.6±0.4	7.6	22,100
K2	0.5±0.3	7.9	8,100
K5	0.2±0.3	9.1	10,048
M	0.0±0.3	10	3,816

For the red variables of Classes M and S, *b* is about 9.1 at maximum magnitude;[2] for the cluster type Cepheids it is 11.8 at median magnitude, and for the few R and N stars it may be provisionally taken as 21 and 30, respectively.

39. For Classes F8, G0, and G5 the dispersion in absolute brightness is so great that spectrum and apparent magnitude are not sufficient criteria of parallax. For Classes K0 and K2 the increasing admixture of dwarfs makes it unsafe to apply the tabulated mean absolute magnitude to stars fainter than 9.0. For the other classes the method applies to all the stars in the catalogue, with the probability that now and then a bad error will be made by unknowingly applying the above formula to occasional unrecorded c-stars or reddish dwarfs. Fainter than magnitude eight the accidental and systematic errors in the apparent brightness are probably the source of greatest uncertainty in the deduced parallaxes.

40. Since the mean absolute magnitude of M stars can be taken as zero, the new data on the frequency of spectral classes, and the relation $\log d = 0.2m+1$, show that there are fifteen hundred giant M stars within five hundred parsecs of the Sun, and that there are three thousand within eight hundred parsecs.

---

[2] *Cf.* recent work on proper motions by R. E. Wilson. *Astronomical Journal*, **35**, 125, 1923.

41. The total number of early B stars in the Henry Draper Catalogue is about 3600, of which at least twenty per cent are farther away than one thousand parsecs (H.B. 787, 1923). The faintest are of visual magnitude 11.6, and are probably more than three thousand parsecs distant.

42. A great majority, probably more than ninety-five per cent, of the stars of all classes and magnitudes in the catalogue, are within one thousand parsecs of the Sun.

43. Figure 2 shows a group of twenty faint stars in the region of Orion, which are presumably members of the Orion cluster. Accepting Kapteyn's value of the distance of the stars in Orion, we deduce that these early B's have the somewhat anomalous absolute magnitudes of +1 to +2. An alternative hypothesis is that their apparent magnitudes have been diminished by the dark obscuring nebulosities of that region, their absolute luminosities being normal.

VI. NUMBER OF STARS IN UNIT VOLUME.

44. From the average number of stars in one hundred square degrees, we can determine for each spectral class the average number of stars in a given volume of space, if, through knowledge of the absolute luminosity, we first determine the limit of distance for that class and assume uniform stellar density out to the limit. By using star counts in low galactic latitude only, the assumption of uniform density is qualitatively safe, except possibly for B stars. Using the values of the average absolute magnitude given in Section V, and the data of star counts represented by the diagrams in Figure 3, we readily compute the results shown by the following tabulation:

Spectral Division	Surface Number	Distance Limit	Space Number
B	29.7	880	4.4
A	96.9	340	250
F	18.7	140	680
G	26.0	70	7600
Giant K	69.0	350	160
Giant M	17.5	430	22

The second column gives the average number of stars in a hundred square degrees, the third gives the distance in parsecs to which the stars extend when we go down to apparent magnitude 8.25, and the last column gives the number of stars in a million cubic parsecs.

45. The space frequency of the various spectral classes has previously been considered by Eddington, Hertzsprung, Luyten, and others for the region within a few parsecs of the Sun, which, however, is not sufficiently large to represent the relative numbers of giant stars. The same problem has also been investigated for globular clusters, where only giant stars could be reached. The present survey covers a much greater region in space. For example, the volume explored for the B stars is about 1.5×10^8 cubic parsecs.

46. From the third column it is seen that the space throughout which the B stars lie is two thousand times as great as that for the dwarf stars of Class G.

47. The tabulation does not take account of the Cepheids, the giant stars of Class G, the abnormally faint A and F stars, nor the dwarfs of Classes K and M. Stars of these last two classes are probably much more numerous per unit volume than dwarf stars of Class G. The giant G stars, as a first approximation, may be taken as one-half as numerous as the giants of Class M.

48. The counts used for the tabulated space frequencies above were made for 2300 square degrees in all longitudes along the Milky Way. Local aggregations therefore probably have not affected the results appreciably. Although the deduced space numbers for various reasons cannot be considered highly accurate, they are believed to be of the right order of magnitude, since the material is extensive and the absolute magnitudes are fairly well known.

49. The most important deduction from the table above is that the great majority of the stars are extreme dwarfs, apparently indicating that a star spends most of its life in the later stages of the spectral series. There are nearly two thousand times as many dwarf stars like the Sun as early B stars, and three hundred and fifty times as many solar stars as giants of Class M. These numbers are of obvious significance in considering the course and speed of stellar evolution.

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A REVISION OF THE ATOMIC WEIGHT OF GERMANIUM.

I. THE ANALYSIS OF GERMANIUM TETRACHLORIDE.

By Gregory Paul Baxter and William Charles Cooper, Jr.

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CONTENTS

Introduction 235
Purification of Materials
    Reagents 236
    Extraction of Germanium from Zinc Oxide Residues 237
    Preparation and Distillation of Germanium Tetrachloride . . 238
Purity of Germanium Tetrachloride 242
Analysis of Germanium Tetrachloride 244
Results 250
Discussion of Results 252
Summary 254

Introduction.

The atomic weight of germanium has been determined by two investigators. Winkler,[1] the discoverer of germanium, analyzed the tetrachloride and obtained the value 72.5. Quite recently, in a very concordant series of experiments, Müller [2] converted weighed amounts of potassium fluorgermanate into potassium chloride, and found in this way the atomic weight of germanium to be 72.42.

Because germanium has recently become available in moderate quantities and because the tetrachloride may be readily prepared and purified by fractional distillation, by methods perfected in this laboratory for handling other volatile mineral halides, notably the chlorides and bromides of silicon,[3] titanium,[4] and boron,[5] we have undertaken the analysis of germanium tetrachloride. The value which we have obtained is not at all in agreement with that of Müller and is considerably higher even than that of Winkler.

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[1] Winkler, Jour. prakt. Chem. (2), **34**, 177 (1886).

[2] Müller, Jour. Amer. Chem. Soc., **43**, 1085 (1921).

[3] Baxter, Weatherill and Scripture, These Proceedings, **58**, 245 (1923).

[4] Baxter and Fertig, Jour. Amer. Chem. Soc., **40**, 1228 (1923).

[5] Baxter and Scott, Science, N. S., **54**, 524 (1921); These Proceedings, **59**, 21 (1923).

PURIFICATION OF MATERIALS.

Reagents.

Water. Ordinary distilled water was twice redistilled, once from alkaline permanganate, and once from very dilute sulfuric acid. The condensers were block tin tubes, fitted to Pyrex flasks with constricted necks which served as stills. The connection was made by a water seal, no cork or rubber being used. The water was collected in Pyrex flasks, generally just previous to use.

Nitric Acid. Concentrated C. P. nitric acid was distilled through a quartz condenser, the first two-thirds, as well as the last tenth, being rejected. If necessary, the fractional distillation was continued until the product was free from chloride.

Silver. This substance was prepared by standard methods. These consisted in brief of the following processes: double precipitation as chloride, followed by reduction with alkaline sugar solution, fusion of the metal on charcoal, solution in nitric acid and repeated crystallization of silver nitrate, precipitation with ammonium formate, fusion on pure lime, electrolytic transport, fusion on pure lime in hydrogen, etching, drying in a vacuum at 500°. Details of these processes may be found in earlier papers from the Harvard Chemical Laboratories.[6] Recent work by Baxter and Parsons [7] and Baxter [8] as well as earlier investigations have shown that these processes yield a product of adequate purity.

Sodium Hydroxide. This substance was prepared by metathesis from barium hydroxide and sodium carbonate, both of which had been recrystallized until practically free from chloride. The solution was freed from precipitate by centrifugal settling and after evaporation in platinum dishes was allowed to crystallize. Since the product was found to contain a small amount of chloride it was recrystallized several times in platinum dishes until the chloride had been completely removed.

Nitrogen. Nitrogen was prepared by the Wanklyn process. Air was charged with ammonia and passed over a hot copper catalyst. The excess of ammonia was removed in wash bottles containing dilute sulfuric acid, and the gas was further purified and dried by passing through towers filled with beads moistened with silver nitrate, potassium hydroxide and concentrated sulfuric acid and through a tube containing resublimed phosphorus pentoxide. The hydrogen formed by decomposition of part of the excess of ammonia was not removed. The apparatus was constructed wholly of glass, with either fused or ground connections, the latter being lubricated with concentrated sulfuric acid.

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[6] See especially Richards and Wells, Pub. Car. Inst., No. 28, 16 (1905).
[7] Baxter and Parsons, Jour. Amer. Chem. Soc., **44**, 577 (1922).
[8] Baxter, ibid., **44**, 591 (1922).

Chlorine. This gas was drawn in part from a tank of liquid material, in part it was prepared from concentrated hydrochloric acid and manganese dioxide. The chlorine was scrubbed with water and dried with concentrated sulfuric acid and finally phosphorus pentoxide, in an apparatus constructed wholly of glass.

Extraction of Germanium from Zinc Oxide Residues.

We are very greatly indebted to the New Jersey Zinc Company for the generous gift of one hundred and twenty-five pounds of germaniferous zinc oxide, containing approximately 0.25 per cent of germanium dioxide. In extracting the germanium we followed essentially the procedure described by Dennis and Papish,[9] by which the germanium and arsenic are first separated together. At the time the improved method of Dennis and Johnson [10] had not been described. On the other hand the final separation of the germanium from the arsenic was effected by fractional distillation of the chlorides in vacuum, instead of by distillation from hydrochloric acid solution in a current of chlorine, as practised by Dennis and his collaborators. Because of the relatively large difference in boiling points, 43°, this separation is comparatively rapid.

The crude zinc oxide was dissolved in a large flask in a considerable excess of hydrochloric acid, and the solution was distilled until the volume of the distillate was about twice the volume of the residue. After the acidity of the distillates had been brought to 6 normal by the addition of sulfuric acid, they were saturated with hydrogen sulfide. The precipitated sulfides of arsenic and germanium were washed until nearly free from chloride, with 5 normal sulfuric acid which had been saturated with hydrogen sulfide. Drying at 110° followed.

A large portion of the arsenic was next removed by roasting the sulfides in a current of air at about 500°. Dennis and Papish [11] state that about 90 per cent of the arsenic is eliminated in this way. In the earlier experiments the sulfides were contained in a copper boat placed inside a glass tube which could be heated electrically. During the roasting a fairly rapid current of dry air was passed through the tube. Owing to extensive contamination of the product by copper oxide, the copper boat was later replaced by one of nickel steel.

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[9] Dennis and Papish, Jour. Amer. Chem. Soc., **43**, 2131 (1921). See also Buchanan, J. Ind. Eng. Chem., **8**, 585 (1916); **9**, 661 (1917).

[10] Dennis and Johnson, Jour. Amer. Chem. Soc., **45**, 1380 (1923).

[11] Dennis and Papish, ibid., **43**, 2139 (1921).

The crude germanium dioxide was next finely powdered and subjected to a second roasting in a current of dry air at about 650°, while contained in an alundum boat inside a quartz tube. Material evaporating during this treatment and the following one was collected in traps connected with the outlet of the silica tube. After the roasting the air was replaced by a current of pure dry electrolytic hydrogen and the temperature, after being maintained at redness for some time, was ultimately raised to approximately 1000°. Most of the reduced material melted and coalesced at this temperature and at the same time a very considerable black sublimate consisting chiefly of arsenic was formed. About one hundred and thirty grams of crude germanium metal were thus secured.

Preparation and Distillation of Germanium Tetrachloride.

Germanium tetrachloride was next prepared by heating the metal in a current of chlorine, purified and dried as described on page 237. The chlorine train was connected through a ground joint with a hard glass tube in which the metal was loosely packed between asbestos plugs. This tube was also connected with a supply of dry atmospheric nitrogen prepared as described on page 236. The tube containing the germanium was inclined downward and connected at its lower end by means of a ground joint with two condensing bulbs in series, one cooled with ice and salt, the second with alcohol-carbon dioxide mixture.

The nitrogen train and germanium tube were swept out with nitrogen for many hours. At the same time the chlorine train was swept out through a side tube. Chlorine was then admitted to the reaction tube and the germanium was heated electrically by means of a sleeve wound with resistance wire. After a considerable quantity of the chloride had been formed, the apparatus became choked so badly that the experiment was discontinued. The contents of the hard glass tube were washed with water, dried and ignited in hydrogen and the experiment was repeated with similar apparatus. In all about one hundred cubic centimeters of chloride were prepared.

The chloride at this point contained free chlorine. It also was turbid owing in part at any rate to suspended copper salts, for the original metal was contaminated with copper as stated on page 238. It presumably contained hydrogen chloride and either germanic acid or oxychloride owing to some contact with the moisture of the air. Subsequent examination showed it to contain arsenic trichloride in considerable proportions, and traces of tin and antimony. Silicon might have been introduced in the reaction with chlorine in the hard glass tube but we were unable to discover this impurity. In the formation of boron halides [12] this is a serious difficulty. The boiling points of the chlorides mentioned above and some others are given in the following table, together with the effect on the observed atomic weight of germanium of a proportion of 0.1 per cent of each substance alone.

TABLE I.

Boiling Points of Inorganic Chlorides.

	Boiling point degrees	Effect of 0.1% on atomic weight of Ge
HCl	−83	−0.070
CCl_4	76.5	−0.061
$SiCl_4$	58	−0.045
$TiCl_4$	136	−0.025
$GeCl_4$	86.5	
$SnCl_4$	114	+0.048
VCl_4	154	−0.021
PCl_3	76	−0.032
$AsCl_3$	130	+0.028
$SbCl_3$	223	+0.049

The germanium tetrachloride was now purified by fractional distillation in a fashion very similar to that used in purifying the halides of silicon, titanium and boron in this laboratory.[3, 4, 5] The two lots of germanium tetrachloride were combined in a single glass bulb (*A*, Figure 1) in which a few cubic centimeters of mercury had been placed. The bulb was connected with a side tube *a* for exhausting and with a special joint *b* by means of which the bulb could be connected later with additional systems, without breaking the vacuum. In the vertical view of the special joint [13] the sealed-in capillary *m* is closed at the end and is scratched with a file at several points to facilitate breaking.

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[12] Wöhler and Deville, Ann. chim. phys. (7), 6, 296 (1895).

[13] Baxter, Weatherill & Scripture, These Proceedings, **58**, 250 (1923).

To break the joint the closed tube of glass weighted with mercury *n* is allowed to strike the capillary with some force. The bulb was exhausted by means of an efficient water pump through a tube containing sodium hydroxide, while at the same time it was cooled with ice and water to lower the vapor pressure of the germanium tetrachloride. Even under these conditions the tetrachloride boiled gently, thus flushing out the air in the bulbs, together with a part of the chlorine and the hydrochloric acid formed by contact with moisture in the air. After the boiling had proceeded for a short time the tube *a* leading to the pump was sealed off. In order to remove the remainder of the chlorine the tetrachloride and the mercury were emulsified by shaking, and allowed to stand for some time.

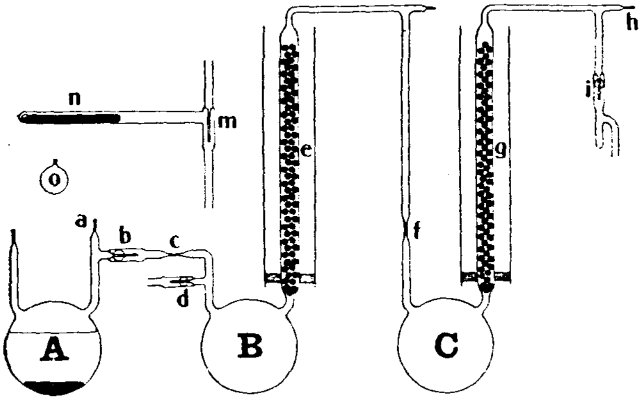

Fig. 1.

Next the bulb *A*, containing the tetrachloride, was connected through the special joint with another bulb *B* attached through a vertical tube to a fractionating column *e* filled with glass pearls and jacketed with a large glass tube. Beyond the fractionating column were another similar bulb *C* and fractionating column *g* terminating in a special joint *i*. The bulbs *B* and *C* and the fractionating columns were exhausted by a Gaede mercury pump through the capillary *h* which was sealed while the pump was operating. Communication between *A* and *B* was now made by breaking the special joint *b* and the tetrachloride was largely distilled from *A* and *B* by chilling *B* with salt and ice. After three-quarters of the liquid had distilled the volatility of the remainder became so much less that only by surrounding *A* with hot water could the distillation be continued. As subsequently found, this was due to the concentration of the less volatile arsenic trichloride. The bulb *A* containing a small amount of liquid together with a residue of mercury and mercurous chloride was detached by sealing the capillary *c*. In the next distillation from *B* to *C* the fractionating column *e* was first chilled with ice water. The still *B* was then surrounded with warm water and the bulb *C* was chilled with salt and ice. Under these conditions a very efficient fractionation took place with refluxing throughout the fractionating column but without choking. When the residue in *B* had been reduced to about ten cubic centimeters the volatility was obviously so much less than that of the main portion that the apparatus was divided by sealing the capillary *f*.

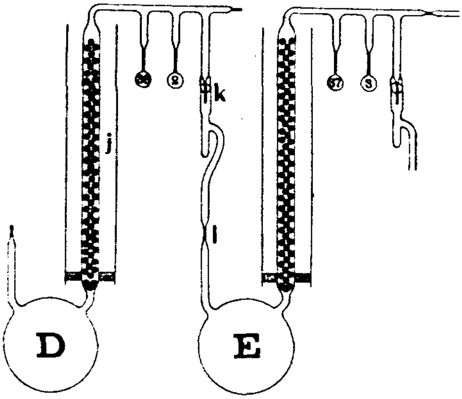

FIG. 2.

The less volatile residue in *B* was now fractionated into five exhausted bulbs attached through the special joint *d*. These fractions, 40, 41, 42, 43, 44 (Figure 3), were hydrolyzed and found by treatment with hydrogen sulfide to consist largely of arsenic. Spectroscopic examination of the sulfide fractions on graphite electrodes with a Féry quartz spectrograph indicated also the presence of traces of tin and antimony.

In the next distillation from *C* to a similar system attached through the special joint *i*, difficulty was experienced in condensing a light fraction in a small bulb even with the use of alcohol-carbon dioxide mixture, owing apparently to the presence of non-condensible gas. Therefore, a one-liter exhausted globe was inserted between two special joints, and, the bulb containing the tetrachloride having been warmed and the globe chilled with salt and ice, connection was made between the two and the globe was sealed off. Before the seal could be made several cubic centimeters of the tetrachloride distilled into the globe, carrying with it the bulk of the troublesome gas. This most volatile fraction, 1 (Figure 3), was hydrolyzed and the solid hydrolytic product when tested spectroscopically, except for a trace of arsenic, seemed to contain nothing but germanium.

In all twenty fractional distillations with Hempel columns were carried out in apparatus of the general type shown in Figure 2. The tetrachloride being in the exhausted bulb *D* and the fractionating column *j* being chilled with ice and water, a less volatile fraction was condensed in the small bulb 2 by means of alcohol and carbon dioxide, and was sealed off at the capillary. The special joint *k* was then broken and the main bulk of the liquid was condensed in *E* with great care to have the fractionating column operating with greatest efficiency. When the residue in the still had been reduced to a few cubic centimeters the capillary *l* was sealed. The residue in *D* was finally condensed in bulb 38 by chilling with alcohol-carbon dioxide, the rest of the apparatus being warmed at the same time.

PURITY OF GERMANIUM TETRACHLORIDE.

In this way all the tetrachloride was separated into fractions without accident, so that the material was not exposed to the air at any time after the introduction into the distilling apparatus. This fractional distillation is shown diagrammatically in Figure 3, in which the fractions are numbered in the order of decreasing volatility. The light and heavy fractions from the early distillations were tested for impurities as follows. The least volatile fractions from the first seven distillations were hydrolyzed with a considerable amount of water and treated with hydrogen sulfide. These showed rapidly diminishing proportions of arsenic. In fact, Fraction 34 seemed to be practically free from this impurity. The heavy residue from the eighth distillation, Fraction 33, was hydrolyzed with ammonia and the solution evaporated to dryness. This residue was tested spectrographically as described above and no evidence whatever of the presence of arsenic, tin or antimony could be detected. Dennis and Johnson [10] were able to detect 0.0005 per cent of arsenic in germanium by photographing the arc spectrum. We have no reason to believe that our spectroscopic procedure was any less sensitive.

Although in our experiments seven distillations were thus required for removing arsenic there is no question that, if a somewhat larger residue had been rejected in each distillation, fewer steps would have been necessary. Our somewhat limited quantity of material led us to proceed in a more cautious fashion. Although we have assumed that the arsenic was completely removed at this point, the residue of the eleventh distillation, Fraction 29, as well as Fractions 18 and 19, was tested spectrographically. No impurities of any sort could be detected.

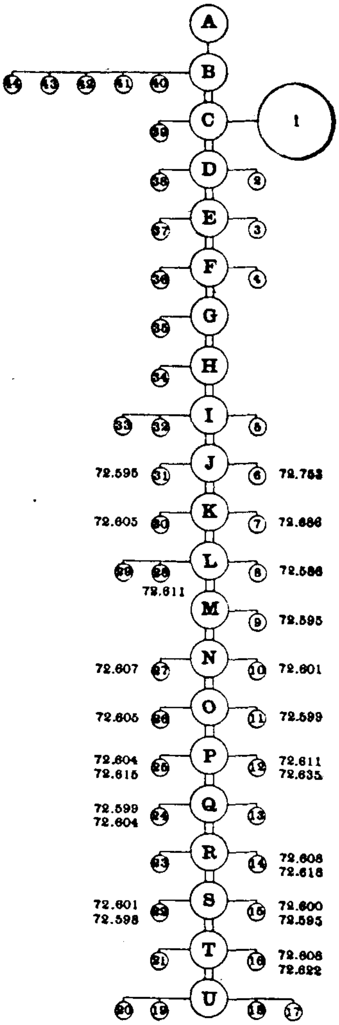
A
B
44
43
42
41
40
C
1
39
D
38
2
E
37
3
F
36
4
G
35
H
34
I
33
32
5
J
72.595
31
6
72.763
K
72.605
30
7
72.686
L
29
28
72.611
8
72.586
M
9
72.595
N
72.607
27
10
72.601
O
72.605
26
11
72.599
P
72.604
72.615
25
12
72.611
72.635
Q
72.599
72.604
24
13
R
23
14
72.608
72.618
S
72.601
72.598
22
15
72.600
72.595
T
21
16
72.608
72.622
U
20
19
18
17

FIG. 3.

Since the germanium tetrachloride was synthesized in a glass tube the possibility of the formation and presence of silicon tetrachloride existed. The boiling point of this substance, 58°, is so far below that of the germanium compound that rapid separation, but in the more volatile fractions, might reasonably be expected. Only one test for silicon was made. The contents of the bulb containing the most volatile fraction from the sixteenth distillation, Fraction 13, were added to a small amount of water in a clean platinum crucible and the crucible was nearly filled with constant boiling hydrochloric acid which had been distilled through a quartz condenser. Evaporation on an electric hot plate followed. Twice more in succession the crucible was filled with constant boiling hydrochloric acid and this acid was eliminated by evaporation. During these three evaporations the germanium was vaporized as chloride, while any silicon tetrachloride must have been hydrolyzed to silicic acid. The crucible was next ignited and carefully weighed by substitution. There was no evidence of any residue in the crucible except a small amount of platinum which had been dissolved by the acid. In order to expel any silica the crucible was next filled with hydrofluoric acid which had been freshly distilled through a platinum condenser, and after evaporation of the acid and ignition, the crucible was reweighed. The observed loss in weight, 0.03 mg., is insignificant, and is as likely to be due to experimental error as to the presence of silicon tetrachloride. Even if this silica really represents impurity in the germanium tetrachloride, the corresponding amount of silicon tetrachloride, 0.09 mg., in a fraction weighing approximately five grams would lower the apparent atomic weight of germanium by only 0.001 unit.

ANALYSIS OF GERMANIUM TETRACHLORIDE.

The analysis of germanium tetrachloride followed closely the procedure used in the analysis of silicon tetrachloride.[3] After being weighed the bulb containing germanium tetrachloride was broken under an excess of sodium hydroxide and the glass was collected and weighed. The solution was diluted to considerable volume and made acid with nitric acid, and then was precipitated with a solution of a weighed, very nearly equivalent amount of pure silver. The point of exact equivalence between chloride and silver was then found with the assistance of a nephelometer. In several experiments the silver chloride was collected and weighed.

The bulb containing the material for analysis was first scrubbed with soap solution and then was allowed to stand for some time in sulfuric acid — dichromate solution. After thorough rinsing it was soaked for at least twenty-four hours in the purest water. From this point the bulb was not touched with the fingers. Next the bulb was placed in a basket of platinum wire which had already been weighed under water with an accuracy of one milligram, and the bulb and basket together were weighed under water of known temperature. Rinsing with pure water and draining were followed by drying, first over sulfuric acid, finally over potassium hydroxide which had been fused with permanganate to destroy organic matter. The bulb was then weighed by substitution and the density of the air in the balance case at the time of weighing was found by weighing a sealed standardized glass globe.[14]

In order to hydrolyze the germanium tetrachloride without forming a semipermanent precipitate of germanium hydroxide, it was found advisable to dissolve the liquid in a solution of at least one hundred per cent excess of the sodium hydroxide necessary to form sodium chloride and sodium germanate. Furthermore, if the solution of sodium hydroxide was less concentrated than 4 normal at the time the reaction occurred germanium hydroxide was precipitated, although subsequent dilution of the solution did not result in precipitation.

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[14] Baxter, Jour. Amer. Chem. Soc., **43**, 1317 (1921).

A 4 normal solution of the sodium hydroxide was prepared in a heavy walled, two-liter Erlenmeyer Pyrex flask provided with a carefully ground glass stopper, the weight of glass in the bulb containing the tetrachloride being estimated in computing the amount of sodium hydroxide necessary. In the later analyses the sodium hydroxide solution was first filtered through a platinum sponge crucible. The weighed bulb was carefully introduced, and, before the stopper was inserted the flask was warmed gently so that ultimately the pressure should be slightly inward. After cooling the flask with tap water the inner walls of the flask were wetted with sodium hydroxide solution by inclining the flask and rotating, and then the bulb was broken by shaking the flask. A dense fog immediately appeared in the flask. Again the flask was cooled and although the fog disappeared in the course of fifteen minutes the flask was allowed to stand for three hours longer. In the first analysis a 2.5 normal solution of only fifty per cent excess of sodium hydroxide was used in the initial decomposition of the tetrachloride, and the precipitated germanium hydroxide could be dissolved only by long standing after adding more sodium hydroxide. In the third analysis the capillary of the bulb was first broken off and the precipitate which was formed by slow hydrolysis through the opening could be only imperfectly dissolved even by adding a large additional quantity of sodium hydroxide and long standing. In the sixth analysis, although one hundred per cent excess of sodium hydroxide was used, the solution was only 2.5 normal when the bulb was broken. In this experiment also germanium hydroxide was precipitated and the precipitate could be ultimately dissolved only by the addition of more sodium hydroxide. In all the other analyses no difficulties of this sort were experienced.

The fragments of the glass bulb were collected on a weighed, platinum sponge crucible. The solution was filtered directly into the glass-stoppered precipitating bottle or Erlenmeyer flask, and the glass was washed by decantation ten or twelve times with about one hundred cubic centimeters of boiling hot water and once with cold water before being transferred to the crucible. Drying at 300° for eighteen hours in an electrically heated porcelain air bath preceded the weighing of the crucible both empty and with the glass. In order to make sure that no germanium hydroxide was retained by the crucible and glass, the two were further treated with hot 4 normal nitric acid by slowly pouring from 400 to 800 cc. through the crucible. This treatment usually produced a slight loss in weight of a few hundredths of a milligram, but a second similar treatment produced no further change of appreciable magnitude.

Since it seemed likely that during the treatment of the glass with sodium hydroxide and with acid, solution would occur to some extent, experiments were carried out to test this point. A solution was prepared by dissolving germanium tetrachloride in sodium hydroxide solution as in an analysis, and was filtered through platinum sponge. The glass fragments of several bulbs were weighed in a platinum sponge crucible and then were allowed to stand in the above solution for some time. The glass was collected in the crucible in which it had been initially weighed and then was washed thoroughly with hot water and finally with hot 4 normal nitric acid. Drying at 300° followed. After being weighed a second time the glass was washed with hot nitric acid, dried and weighed until the weight was constant. In the first four experiments the glass consisted of a mixture of fragments of different sizes and shapes. In Experiment 5 coarse fragments, in Experiment 6 fine fragments were used. The weights given in the table represent differences between the crucibles and counterpoises.

The slight gain in Experiment 2 is apparently due to an undetected accident. In the other five experiments the losses are consistent with the conditions and indicate an average loss in weight of 0.007 mg. per gram of glass per hour of treatment with the alkaline germanate solution, combined with the nitric acid treatment. A correction of this magnitude was therefore applied to the weight of glass obtained in each analysis. Except in Analyses 1, 3 and 6 (Table III) this correction did not exceed 0.03 mg.

From the observations with the bulb before breaking and the weight of glass corrected as above and for the buoyant effect of the air, the weight of germanium tetrachloride was found. The weight of silver necessary to precipitate the chloride was computed, and was weighed out, chiefly in the form of a very few large buttons, the final adjustment being made with small electrolytic crystals. After careful solution of the silver in chloride-free nitric acid and elimination of nitrous acid by heating the solution, in a flask provided

TABLE II.

SOLUTION OF GLASS BY ALKALINE SODIUM GERMANATE.

Experiment	Wt. of glass gm.	Crucible + glass before treatment with alkali gm.	After treatment with alkali gm.	After treatment with HNO_3 gm.	After treatment with HNO_3 gm.	Glass dissolved gm.	Period of treatment with alkali hours	Glass dissolved per hour per gram
1	2.212	0.25764	0.25659	0.25644	0.25648	0.00120	69	0.000008
2	3.888	0.52810	0.52844	0.52827	0.52826		5	
3	2.212	0.25648	0.25639	0.25637		0.00011	7	0.000007
4	2.206	0.82372	0.82344	0.82350		0.00028	16	0.000008
5	3.616	4.61864	4.61836	4.61834		0.00030	20	0.000004
6	3.839	4.13618	4.13588	4.13585		0.00033	6	0.000010

with a spray trap in the form of a column of bulbs ground into the neck of the flask, fifty cubic centimeters additional of concentrated nitric acid were added, together with sufficient water to make the solution nearly normal in silver. The acid solution was then added slowly with continual agitation to the germanium solution which had been diluted to less than normal concentration during the washing of the glass and also had been made acid by adding 4 normal nitric acid through a funnel tube extending to the bottom of the solution. The mixture was allowed to stand for several days with occasional shaking, and the solution was tested for excess of chloride or silver in a nephelometer. If an excess of either was found, the deficiency of the other was made up by adding hundredth normal silver nitrate or potassium chloride until the endpoint had been reached. In the earlier analyses it was necessary to add a considerable amount of chloride in this way because the atomic weight of germanium assumed at the beginning was considerably lower than that actually found. In the later analyses the correction to produce equivalence amounted to only a few tenths of a milligram of silver. Even after the apparent endpoint had been reached the solutions were allowed to stand for some weeks longer with occasional shaking in order to allow included or occluded material to be extracted from the precipitate of silver chloride. Although at the beginning of a comparison of this sort the endpoint is likely to alter somewhat with time, after two or three weeks have elapsed only slight changes are usually produced.

The manipulation of precipitation and testing of the solutions were always carried out in ruby light. In using the nephelometer all the precautions noted by Richards and Wells[15] were observed, such as preparing the comparison tubes under as nearly as possible identical conditions of temperature, concentration and time, allowing the tubes to come to constant ratio by standing for an hour or more, and taking the average of several readings of each ratio.

In the last seven analyses the silver chloride was collected on a platinum sponge crucible and weighed. First the solubility of the silver chloride was reduced by adding an excess of silver nitrate and cooling to 0° for twenty hours with occasional shaking. In Analyses 19–22 one-half gram of silver nitrate was added, in Analyses 23–25 one gram. This relatively large excess of silver nitrate seemed advisable because of the comparatively large amount of nitric acid and sodium nitrate in the solution which increase the solubility of the silver chloride materially. Filtration of the supernatant liquid through a large weighed platinum sponge crucible was followed by thorough washing of the precipitate by decantation with a chilled solution of silver nitrate containing 0.05 gram per liter. The precipitate was transferred to the crucible with chilled water, and after being dried in an electrically heated porcelain oven for about eighteen hours at 300° it was weighed. Moisture retained by the precipitate was determined by fusing the main bulk and finding the loss in weight. In the first two experiments, Analyses 19 and 20, faulty technique may have occasioned too large a loss in weight. The later experiments show a water content of the precipitate more nearly in accord with other experiments of the same sort.

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[15] Richards and Wells, Amer. Chem. Jour., **31**, 235 (1904); **35**, 510 (1906).

No correction is applied for silver chloride dissolved in the mother liquor. The solubility of silver chloride in the chilled silver nitrate washings was assumed to be 0.03 mg. per liter. This correction was never larger than 0.05 mg. The precipitating flask was rinsed with ammonia and the rinsings were added to the aqueous washings. The resulting solution was diluted to a definite volume, either 500 cc. or 1000 cc., and its chloride content was found by comparison with standard chloride solutions in a nephelometer after adding nitric acid and silver nitrate.

In the tables the original weight of silver chloride has been corrected for chloride introduced in the comparison of the germanium chloride with silver. The results of all the analyses undertaken are given.

Weighings were made on a No. 10 Troemner balance, sensitive to 0.02 mg. with a load of fifty grams. A 5 milligram rider was used to determine quantities less than this and more than 0.05 mg. Interpolation from zero points was employed only for amounts less than 0.05 mg.

The weights were of gold-plated brass, except the fractional weights which were of platinum, and were compared by the Richards substitution method.[16]

All weighings were by substitution. In the case of the bulb and silver, the weights were substituted for the object weighed. In the case of the glass and the silver chloride the crucibles were substituted for similar counterpoises. A small quantity of impure radium bromide was kept in the balance case to prevent electrostatic effects.

Vacuum corrections were applied as in Table V, page 251.

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[16] Richards, Jour. Amer. Chem. Soc., **22**, 144 (1900).

RESULTS.

TABLE III.

ATOMIC WEIGHT OF GERMANIUM.

Ag = 107.880 $GeCl_4 : 4 Ag$ Cl = 35.458

Number of analysis	Fraction of $GeCl_4$	Weight of $GeCl_4$ in vacuum	Weight of Ag in vacuum	Weight of Ag added or subtracted in solution	Corrected weight of Ag in vacuum	Ratio $GeCl_4 : 4 Ag$	Atomic weight of germanium
		gm.	gm.	gm.	gm.		
1	6	4.32030	8.69804	−0.01010	8.68794	(0.497276)	(72.753)
2	31	4.86658	9.80158	−0.00790	9.79368	0.496910	72.595
3	7	5.16272	10.38923	−0.00400	10.38523	(0.497121)	(72.686)
4	30	3.85268	7.75236	+0.00055	7.75291	0.496933	72.605
5	8	4.67090	9.39916	+0.00110	9.40026	0.496891	72.586
6	28	4.27610	8.60360	+0.00110	8.60470	0.496949	72.611
7	9	5.96671	12.00707	+0.00050	12.00757	0.496912	72.595
8	10	4.53734	9.13063	+0.00020	9.13083	0.496925	72.601
9	27	3.11571	6.26925	+0.00055	6.26980	0.496939	72.607
10	11	3.97498	7.99912	+0.00010	7.99922	0.496921	72.599
11	26	5.07110	10.20445	+0.00030	10.20475	0.496935	72.605
12	12	4.91397	9.88796	+0.00035	9.88831	0.496947	72.611
13	25	6.05244	12.17973	−0.00010	12.17963	0.496931	72.604
14	24	5.21384	10.49237	−0.00005	10.49232	0.496920	72.599
15	14	5.20164	10.46750	−0.00020	10.46730	0.496942	72.608
16	15	4.34510	8.74401	0.00000	8.74401	0.496923	72.600
17	22	4.18788	8.42750	+0.00010	8.42760	0.496924	72.601
18	16	4.24281	9.53775	+0.00010	8.53785	0.496941	72.608
				Average, omitting Analyses 1 and 3		0.496928	72.602

TABLE IV.

$GeCl_4 : 4\ AgCl$

Number of analysis	Fraction of $GeCl_4$	Weight of $GeCl_4$ in vacuum	Weight of AgCl in vacuum	Loss on fusion	Dissolved AgCl	Corrected weight of $AgCl_4$ in vacuum	Ratio $GeCl_4 : 4\ AgCl$	Atomic weight of germanium
		gm.	gm.	gm.	gm.	gm.		
19	12	4.91397	13.13679	0.00054	0.00068	13.13693	0.374058	72.635
20	25	6.05244	16.18131	0.00059	0.00130	16.18202	0.374024	72.615
21	24	5.21384	13.94013	0.00019	0.00066	13.94060	0.374004	72.604
22	14	5.20164	13.90641	0.00021	0.00090	13.90710	0.374028	72.618
23	15	4.34510	11.61745	0.00016	0.00097	11.61826	0.373989	72.595
24	22	4.18788	11.19722	0.00013	0.00064	11.19773	0.373994	72.598
25	16	4.24281	11.34233	0.00016	0.00114	11.34331	0.374036	72.622
						Average	0.374019	72.612
						Average, omitting Analyses 19 and 20	0.374010	72.607
						Average of both series		72.605

TABLE V.

VACUUM CORRECTIONS.

	Density	Vacuum correction per gram
Weights	8.3	
Air	0.001293 at 0° and 760 mm.	
Glass	2.5	+0.000335
Silver	10.49	−0.000031
Silver chloride	5.56	+0.000071

Discussion of Results.

In the following table the results are arranged in the order of decreasing volatility of the fractions analyzed.

TABLE VI.

Atomic Weight of Germanium.

Fraction	Atomic weight $GeCl_4 : 4\ Ag$	Atomic weight $GeCl_4 : 4\ AgCl$
6	72.753	
7	72.686	
8	72.586	
9	72.595	
10	72.601	
11	72.599	
12	72.611	72.635
14	72.608	72.618
15	72.600	72.595
16	72.608	72.622
22	72.601	72.598
24	72.599	72.604
25	72.604	72.615
26	72.605	
27	72.607	
28	72.611	
30	72.605	
31	72.595	

Aside from the first two fractions, no systematic trend is apparent, and in view of the uniformity of the material over a wide range of fractions, the conclusion may reasonably be drawn that the purification of the germanium tetrachloride had been carried as far as it is possible to carry it by the method of fractional distillation. Furthermore, direct evidence of the absence of arsenic and silicon in the fractions analyzed was secured. So far as hydrogen chloride is concerned, the uniformity of the material is the only evidence which can be adduced that this substance had been eliminated. However, aside from the possibility of the existence of a constant boiling distillate of germanium tetrachloride and hydrogen chloride, the wide difference in boiling points of the two substances would lead to the expectation of rapid separation at the start.

The high results of the analyses of Fractions 6 and 7 cannot be ascribed to the presence of arsenic since these fractions were the *most* volatile of those analyzed. These two analyses were the first and third undertaken and as explained on page 245 considerable difficulty was experienced in the hydrolysis of these fractions. Since it seems unlikely in view of the uniformity of the rest of the material that these fractions were really different from the rest, the more complicated and protracted manipulation and technical inexperience with the method seem to us more probable reasons for the divergence.

Aston[17] has recently found by means of the mass spectrograph that germanium consists of at least three isotopes of masses 74, 72, and 70, the relative abundance being that of the order listed. He also states that the atomic weight 72.5 is not incompatible with the apparent proportions of the three isotopes. A difference in observed atomic weight of 0.1 unit is not likely to introduce inconsistency in this situation. With two isotopes of chlorine, there are evidently possible fifteen tetrachlorides of germanium with molecular weights ranging, with intervals of two units, from 210 to 222. The unlikelihood of appreciable separation of such isotopic molecules during fractional distillation has already been emphasized in the case of silicon and boron halides. Furthermore, as pointed out in the case of the boron halides, experimental evidence that isotopic separation is inappreciable is furnished by the ratios between the silver used in a comparison with the germanium chloride and the silver chloride obtained in the second step in the same analysis, since separation of the chlorine isotopes is as probable as that of the germanium isotopes. In the following table these ratios are given.

TABLE VII.

Analyses	Fraction of BCl_3	Ratio Ag : AgCl
12 and 19	12	0.752711
15 " 22	14	0.752659
16 " 23	15	0.752609
18 " 25	16	0.752677
17 " 24	22	0.752617
14 " 21	24	0.752645
13 " 20	25	0.752664
	Average	0.752655
	Average, excluding Fractions 12 and 14	0.752642
	Richards and Wells found	0.752627

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[17] Aston, Nature, **111**, 771 (1923).

As explained on page 249 the silver chloride determinations in Analyses 19 and 20 are subject to some uncertainty in the estimation of the moisture. Furthermore it is certain that silver chloride cannot be made wholly insoluble by the addition of silver nitrate in excess. In fact we found that by adding more silver nitrate to the filtrate of Analysis 22 a very slight but unmistakable opalescence was produced. Winkler[1] found that silver chloride precipitated in the presence of germanium carried down germanium hydroxide. Although the results of our silver chloride determinations show little indication of such an effect, the silver chloride from Analyses 24 and 25 was tested for germanium spectrographically on graphite electrodes. In both cases a trace of germanium was present, for the line λ 2651 was faintly visible in both cases, although the two lines λ 3039 and λ 3270 could not be seen in the photographs. From previous experience with other elements we estimate the proportion of germanium in the silver chloride as less than 0.001 per cent. This proportion would affect the atomic weight of germanium by 0.002 unit.

There is no reason to believe that this slight contamination of the silver chloride is unique in the case of germanium. It is a question whether it is possible by a single precipitation to prepare absolutely pure silver chloride, owing to the phenomena of inclusion and occlusion. Furthermore, experience in the past has been that in this type of analysis the results of a comparison with silver is in general slightly higher than that of a silver halide determination, a fact which is in accord with the idea that slight contamination of the silver halide invariably occurs.

Because of these two uncertainties, solubility and contamination, a silver chloride determination is sure to be somewhat less reliable than a comparison with silver.

It is difficult to reconcile the result of this investigation with that of Müller, 72.418. From the standpoint of simplicity in the formula of the substance analyzed, however, much can be said in favor of the tetrachloride over the potassium fluorgermanate. Further investigation is obviously necessary and we propose to prepare and analyze the tetrabromide of germanium as the next step.

SUMMARY.

1. A method for the purification of germanium tetrachloride from arsenic trichloride by fractional distillation in exhausted vessels is described.

2. The atomic weight of germanium has been found to be 72.60 by analysis of germanium tetrachloride.

We are especially indebted to the Bache Fund of the National Academy of Sciences for generous assistance in providing much of the necessary apparatus and materials, and to the New Jersey Zinc Company for the gift of the germaniferous zinc oxide.

CAMBRIDGE 38, MASSACHUSETTS

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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. No. 142.

MARINE BORERS IN BERMUDA.

By E. L. Mark

With Four Plates.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. NO. 142.

MARINE BORERS IN BERMUDA.

By E. L. Mark.

Received March 29, 1924. Presented November 14, 1923.

The sudden collapse of certain docks in San Francisco Bay in 1919 and 1920, due to the boring of the piles by "shipworms," sharply called the attention of the shipping interests of the region to the necessity of finding out more than was then known about the life-history and habits of marine "borers," and of devising, if possible, better means of protection from them than had been hitherto used, for the loss due to such destruction, actual and threatened, amounted to several million dollars. The importance of the matter naturally stimulated the corresponding interests of the Atlantic ports to institute inquiries regarding conditions on the eastern coast of the United States.

Several agencies, chiefly engineering, carried on investigations in one or more directions, but early in 1922 the National Research Council appointed a "Committee on Marine Piling Investigations," which was organized under the joint control of two of its divisions, those of Engineering and of Biology and Agriculture. The purpose of the Council in appointing this Committee was to secure the coöperation of a large number of scientific and commercial organizations in support of a comprehensive plan involving the study of existing conditions, and experimentation as to possible means of controlling the pests, or protecting piling from their destructive work, along the whole United States coastline.

Knowing that these investigations into the life-history, distribution, depredations, etc. of Teredo and other marine "borers" were already being carried on under the auspices of the Council, it seemed to me in the summer of 1922 that the study of the conditions in Bermuda might be of some value for comparison with those on our own Atlantic seaboard. Accordingly, I got into communication with Colonel William G. Atwood, in New York, Chairman of the Committee having these investigations in hand, with a view to aiding in the work.

The present brief statement may serve as an introduction to the more important results which, it is hoped, will follow from the study of Bermudian material.

There are not many places in Bermuda where considerable wooden structures accessible to borers are still left, the former wooden piers and landings having been mostly replaced by concrete. There is left, however, some evidence of earlier activities, one or two instances of which are shown in the accompanying illustrations (Figs. 1–8, pls. I–IV). At Pitt's Bay, near Hamilton, a dock supported by wooden piles was built by Mr. H. C. Masters in 1914. Mr. Masters, to whom I am indebted for helpful suggestions in this whole inquiry, has furnished the following information about the piling.

"The wood of the piling is South African black oak; it came out of an old British wooden warship, built in Lord Nelson's time. This wood had been under water for about twenty years before I used it as piling, so that it has now (1923) been in the water a total of about twenty-nine years. It was not protected when used as piling. The piles are now pretty well bored through by the Teredo. I judge that about two thirds of the wood has been destroyed. I do not see any signs of the Limnoria making any attack at all."

The construction of the dock is such that it is rather difficult to get satisfactory pictures of the piling. The view (Fig. 1, pl. I), however, shows something of the external evidence of destruction, caused in this case by a "*shipworm.*"

At Agar's Island, on the other hand, the attacks on wooden structures have been due chiefly to the minute crustacean *Limnoria lignorum* (Rathke). This is the only species of this genus at present known in the western Atlantic. The stone-and-concrete wharf at Agar's Island (Fig. 2, pl. I) is protected by heavy beams; two of these, as "stringers," are bolted to the masonry, into which they are sunk a few inches; a third one, the top of which is level with the top of the wharf, is also securely anchored to the masonry and projects six or eight inches beyond the outer edge of the top stringer. This cap, together with vertical posts about ten feet apart bolted to both cap and stringers, serves as a protection, or fender, to the wharf. The posts, about eight inches square, originally had their bottom ends set in foot-plates of concrete. The wharf with its woodwork was built about 1895. The lower portions of the posts, from the bottom up to the level of high water, have now entirely disappeared. The destruction has undoubtedly been caused by Limnoria, rather than any of the several species of Teredo found in Bermuda.

But the most satisfactory evidence secured regarding the rate of destruction by Limnoria was that got from the log-float at the mooring used for the boats of the Biological Station, situated about 150 feet east of the main landing steps at Agar's Island. This cylindrical float was spruce somewhat more than seven feet long and about eighteen inches in diameter, and was shackled to a chain running down to the bight of the bridle connecting the two anchors. The shackle attachment was such as to allow the float considerable freedom of motion around its axis, somewhat more than half of the float being submerged. As a result of this rolling motion the erosion due to the borers (Limnoria), working chiefly at or near the water line, caused the float to take on the form of an hourglass. The mooring was put down in the summer of 1907 by Mr. L. L. Mowbray, and the float was well tarred before being put overboard. Notwithstanding this protective covering, it had become reduced in diameter at the narrowest region of the hourglass to about two inches, so that an accidental collision with a steam tug caused it to break in two early in the summer of 1922. A portion of the float, suspended by the shackle, is shown in Figure 3 (pl. II). During the fifteen years that it had been uninterruptedly exposed to the action of the borers and the sea, its diameter had been reduced about sixteen inches, and its radius, of course, half that amount, so that the actual rate of erosion was a little more than half an inch per year. A somewhat greater rate of destruction has been independently inferred by Mr. Clapp (1924, These Proceedings, vol. 59, p. 280) from a study of blocks attached to four test-boards — to be described a little further on — exposed at Bermuda during the twelve months ending about August 1, 1923. His conclusion was reached before he was aware of the results shown by the mooring float.

A study of the remnant of this float (Figs. 3–8) shows that the direction of the boring by Limnoria is influenced to some extent by the alternating arrangement of the harder and softer parts of the year's growth of the wood. The softer parts are more completely tunnelled than the harder parts, the harder layers being often bored through in a direction nearly perpendicular to the surface of the "growth rings." The result is that the substance of the softer layers is so weakened by the passages bored in them that the harder layers become detached from each other by the action of the waves and hang as thin sheets of woody substance so perforated with holes as to give them a lace-like appearance (Figs. 5, 6). These perforated sheets are in time torn away, little by little, through the influence of wind and water; the animals meanwhile have bored their way into the deeper layers of the wood, which in turn undergo the same fate.

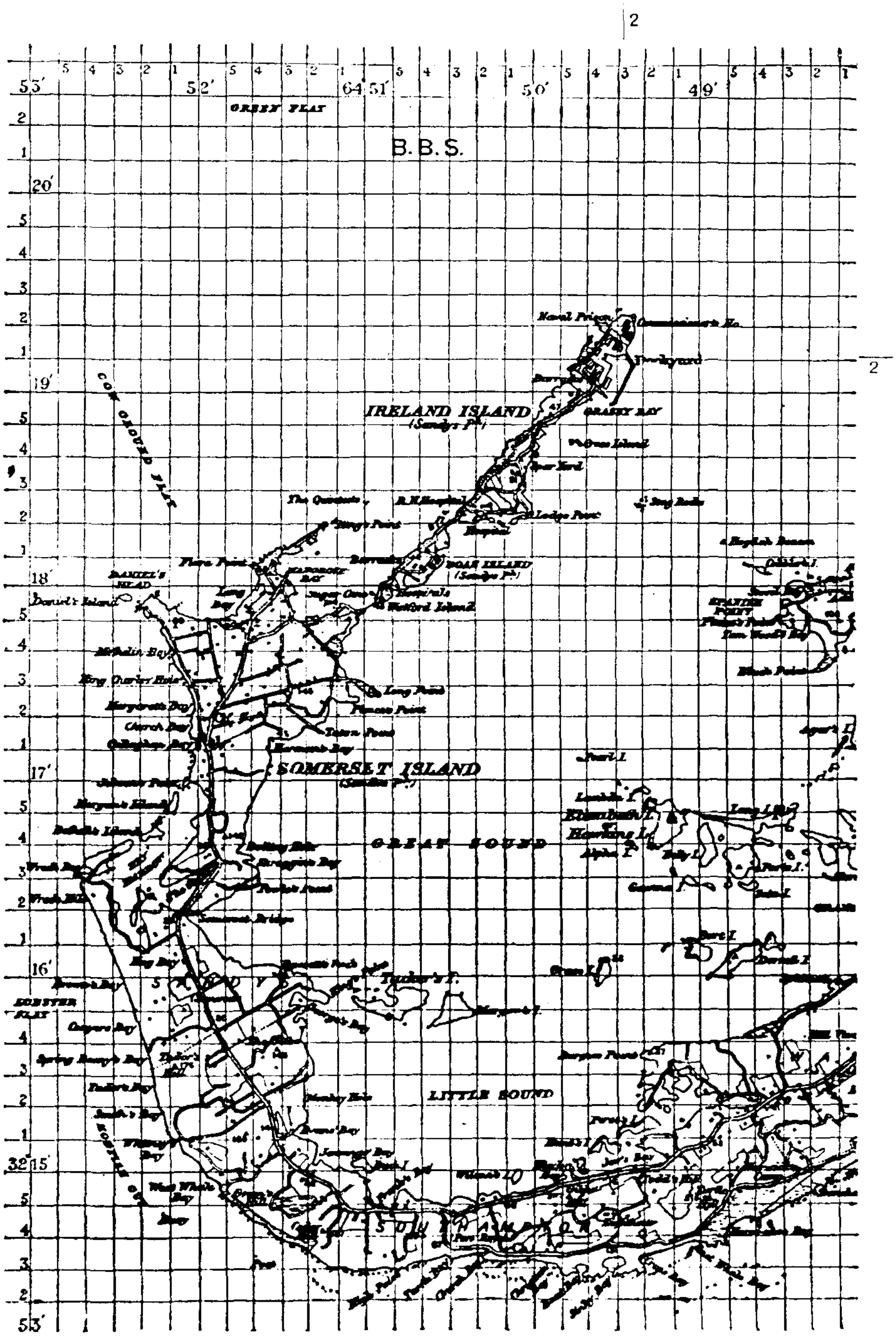
B.B.S.
GREY FLAT
COW GROUND FLAT
IRELAND ISLAND
(Sandys P.)
Naval Prison
Commissioner's Ho.
Dockyard
GRASSY BAY
BOAZ ISLAND
(Sandys P.)
Watford Island
DANIEL'S HEAD
Daniel's Island
Long Bay
Mangrove Bay
Ely's Harbour
SOMERSET ISLAND
(Sandys P.)
GREAT SOUND
Tucker's I.
LITTLE SOUND
LOBSTER FLAT
SPANISH POINT
Pearl I.
53'
52'
64 51'
50'
49'
20'
19'
18'
17'
16'
32 15'
2

Fig. A. (Explanation on page 269.)

Thus the devastation advances slowly but steadily during active boring. The borers, however, rarely penetrate more than half or three-quarters of an inch below the temporary surface of the wood (Fig. 4, pl. II). The rate at which the destruction takes place doubtless varies somewhat with the kind of wood and the abundance of the borers, but, as already stated, is probably from half an inch to an inch in the course of a year. That the temperature of the water also influences to a considerable extent the boring activities of Limnoria, is probable, though it is not so much restricted in its work by cold water as are the shipworms. The subject of temperatures and their influence on "borers" will be discussed in a future paper.

As a result of my correspondence with Colonel Atwood, I had made four test-boards, constructed after the pattern adopted by the New York office, and put each of them in charge of a responsible person. The boards were located at places fairly remote from one another, as indicated on the accompanying maps (Figs. A–C) by the numerals 0, 1, 2, 3.

The locations, surrounding conditions, names of the persons in charge of the boards, etc., were as follows:

No. 0. (Fig. B.) At *Agar's Island*, near the south landing steps of the east side of the island, in latitude 32° 17′ 12″ N., longitude 64° 48′ 02″ W.[1] The board was submerged on July 16, 1922, in water about 12 feet deep at low tide, the bottom of the board resting on the sea bottom. Mr. Antony Cabral had charge of it. There was a gentle flow of water past the board with each turn of the tide.

No. 1. In *Bar's Bay*, Hamilton Harbor, near Mr. Wainwright's boat landing. Lat. 32° 17′ 02″ N., long. 64° 46′ 52″ W. (Fig. B.) Board installed August 1, 1922, by Mr. Harley G. Spurling — who was in charge of it — in water about 12 feet deep at low tide, in a protected spot where there was not a strong current.

No. 2. At the *Navy Yard*, Ireland Island, "at the northwest corner of the North Basin, close to one of the open-sea inlets, where there was a good flow of changing water." Lat. 32° 19′ 12″ N., long. 64° 49′ 28″ W. (Fig. A.) The board was placed in water 8 feet deep at low spring

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[1] The latitude and longitude here given for each of the four stations correspond with the latitude and longitude of the maps accompanying this paper (Figs. A–C), which are based on the map of Bermuda published by the Ordnance Survey, Southampton, 1901; but the true latitude is approximately 16″ greater than that of the maps. For the explanation of the discrepancy, see J. F. Cole: "Magnetic Declination and Latitude Observations in the Bermudas." Terr. Magn. and Atmosph. Electr., Vol. XIII, No. 2, pp. 49–56. June, 1908.

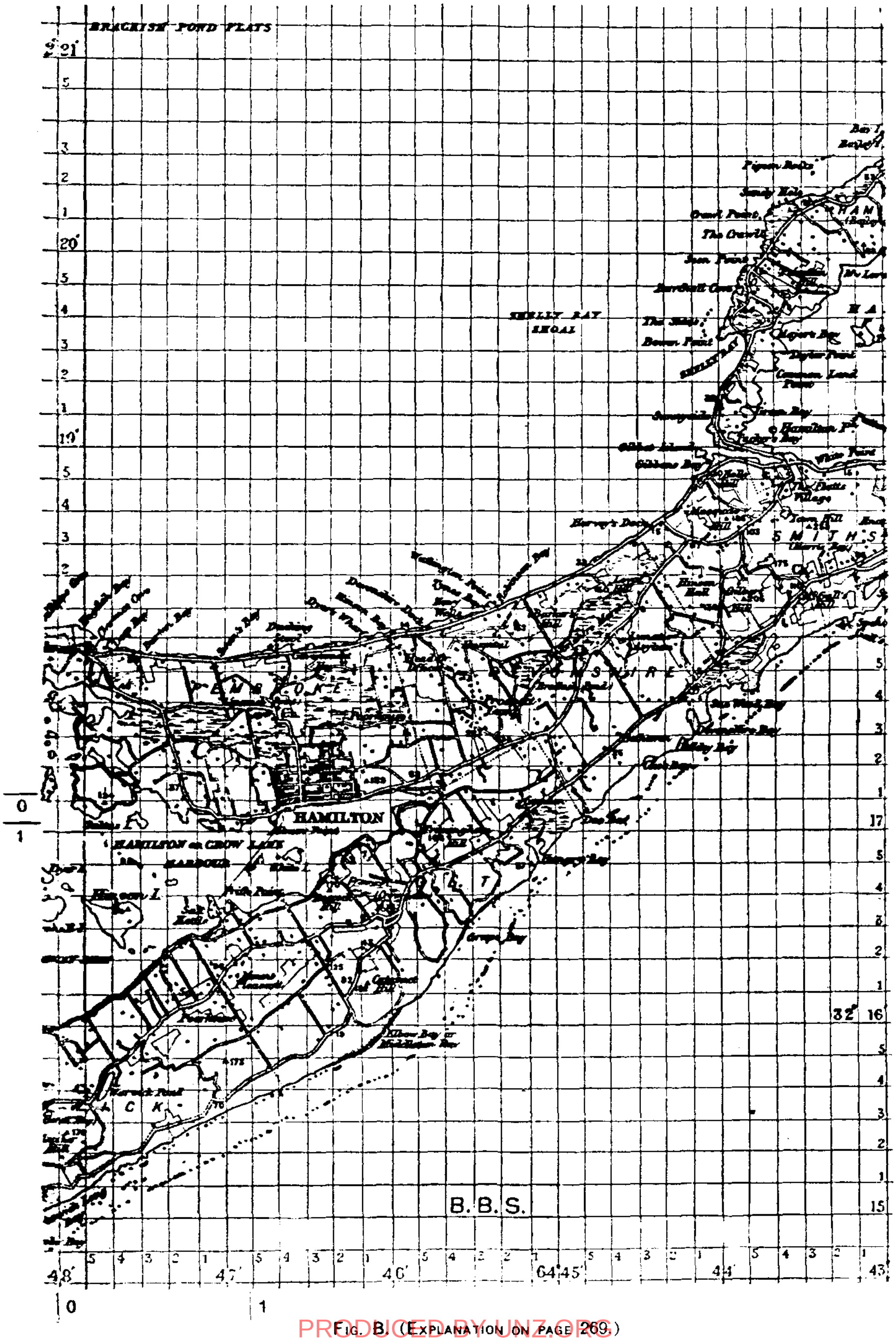
BRACKISH POND FLATS
SHELLY BAY SHOAL
HAMILTON
HAMILTON or CROW LANE HARBOUR
Bailey's Bay
Pigeon Rocks
Sandy Hole
Gravel Point
The Crawl
Iron Point
Barrbell Cove
Bowen Point
Abbot's Cliff
Sunnyside
Green Bay
Hamilton P.
Flatts Village
Gibbons Bay
Harvey's Dock
Town Hill
SMITH'S
DEVONSHIRE
PEMBROKE
Spanish Point
Fairylands
Point Shares
Hinson I.
Salt Kettle
Mount Pleasant
Paget Marsh
Gibbet Hill
Grape Bay
Elbow Bay or Middleton Bay
Warwick Pond
WARWICK
Devil's Hole
B.B.S.
21'
20'
19'
17
32° 16'
15
48'
47'
46'
64° 45'
44'
43'
0
1

FIG. B. (EXPLANATION ON PAGE 269.)

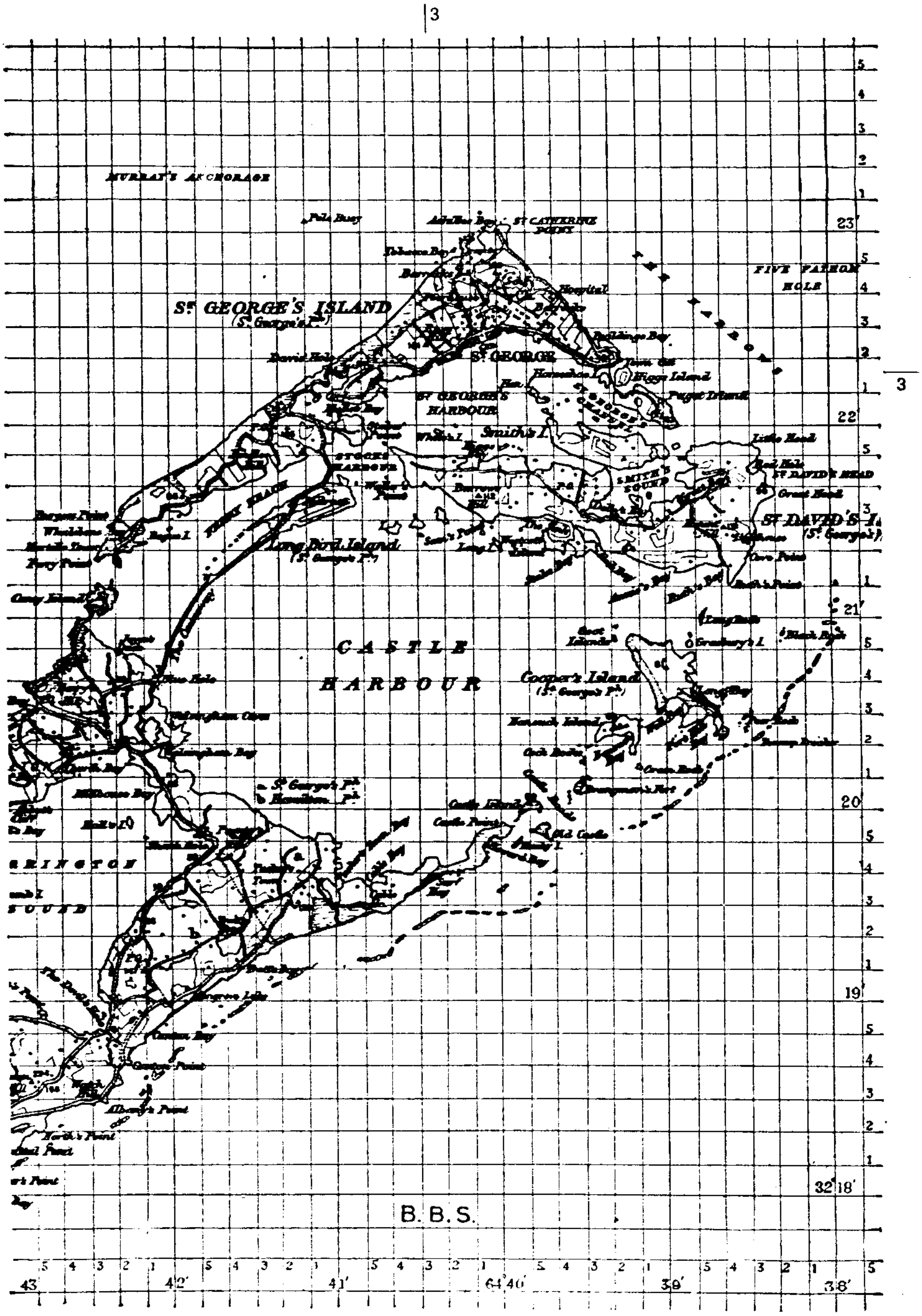
3
MURRAY'S ANCHORAGE
ST CATHERINE POINT
FIVE FATHOM HOLE
ST GEORGE'S ISLAND (St George's Pᵃ)
ST GEORGE
Higgs Island
Paget Island
ST GEORGE'S HARBOUR
Smith's I.
STOCKS HARBOUR
SMITH'S SOUND
ST DAVID'S HEAD
ST DAVID'S I. (St George's P)
Ferry Point
Coney Island
Long Bird Island (St George's Pᵃ)
CASTLE HARBOUR
Cooper's Island (St George's Pᵃ)
Castle Island
Castle Point
Old Castle
HARRINGTON SOUND
B. B. S.
23'
22'
21'
20'
19'
32°18'
43
42'
41'
64°40'
39'
38'

FIG. C. (EXPLANATION ON PAGE 269.)

tide, on August 2, 1922, under direction of the "Captain in charge," at the Navy Yard, Lieut. A. B. Baker, who kindly consented to look after the matter.

No. 3. In *St. George's Harbor,* at the boat landing of R. S. McCallan, Esq., who generously took charge of this board. Lat. 32° 22′ 16″ N., long. 64° 40′ 25″ W. (Fig. C.) The board was submerged July 31, 1922, in water 8 feet deep at low tide.

To each of those who took charge of a test-board I am much indebted, and especially to Mr. McCallan and to the officers of the Navy Yard,— Lieutenant Baker and his successor, Lieutenant F. H. Bishop,— who gave their services gratuitously.

The construction of the test-board, a fuller account of which is given by Mr. Clapp in the article immediately following this, is seen in Figure D, which shows, at the left, a face view of the completed board, and at the right an edgewise view. The wood used in the Bermuda test-boards was spruce, the most satisfactory available material. The twenty-four blocks were numbered by chiseling on the exposed (4″x 5″) face Roman numerals from I to XXIV,[2] and they were attached to the board in sequence (beginning at the top of the board, twelve on a side), each by means of two countersunk brass screws. Experience showed that there was some danger of the numbers being destroyed or obscured by borings and incrustations, so that the replacement blocks, those used after the first set of 24, were numbered on the face in contact with the board as well as on the exposed face.[3]

The procedure with each of the four boards after installment was substantially the same. At approximately regular intervals of about two weeks, varying somewhat according to the sailing dates of the mail steamers, the numbered blocks were removed from the board in sequence, one at a time, beginning with number one, and a new block screwed on in place of the one removed. The detached block was

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[2] The numbered coppered disks at first employed by the Committee proved to be unsatisfactory.

[3] As considerable difficulty was experienced in getting the Teredos out of the thick "blocks" in an uninjured condition, it was suggested by Mr. Clapp that the use of a bundle of shingles firmly bound together would probably solve the difficulty. In place of shingles, however, I used pine slats a quarter of an inch thick, three inches wide, and fifteen inches long. Thirteen or fourteen of these slats were bolted together in a bundle by means of two 5-inch eye-bolts made of quarter-inch galvanized iron. The bundle was lashed to the edge of the original test-board by boring holes near the edge of the board and passing the lashings through these holes and the bolt-eyes, as shown in Figure E.

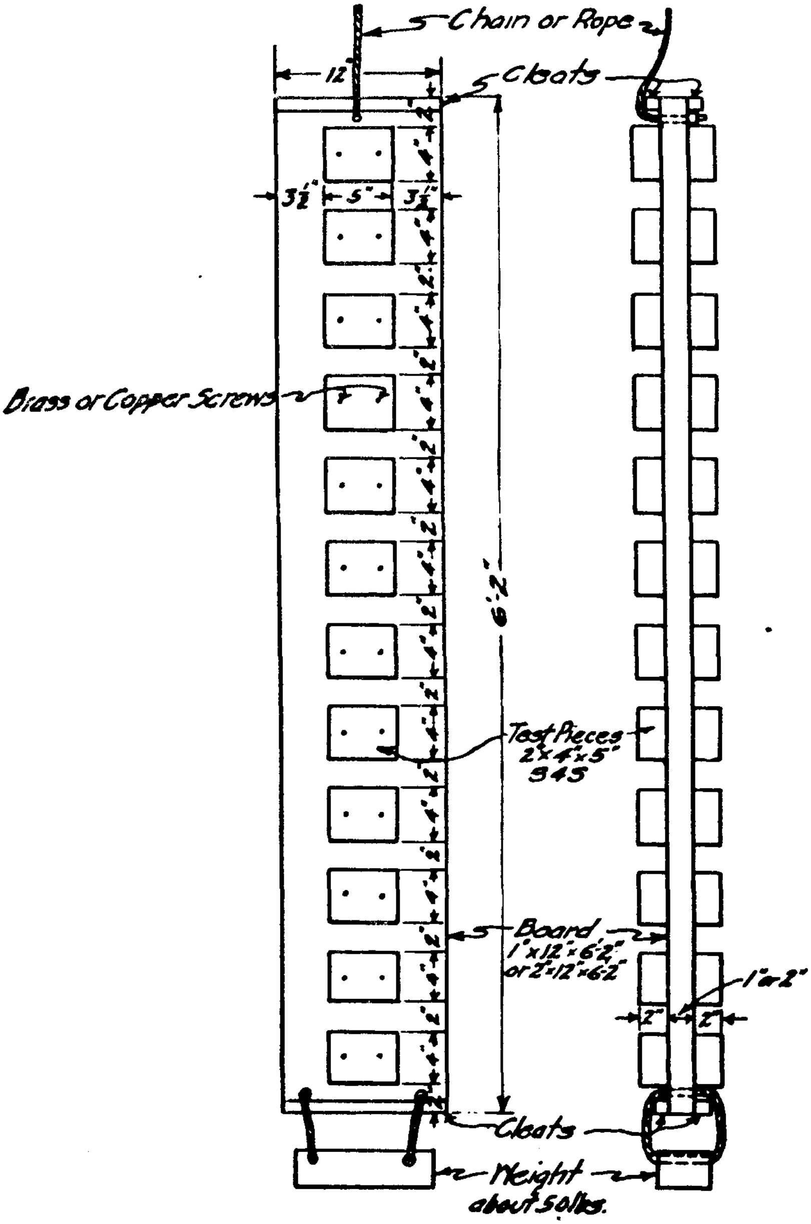

Chain or Rope
12"
Cleats
3½"
5"
3½"
2"
4"
Brass or Copper Screws
6'2"
Test Pieces
2"x4"x5"
S4S
Board
1"x12"x6'2"
or 2"x12"x6'2"
1" or 2"
2"
2"
Cleats
Weight
about 50lbs.

FIG. D. (EXPLANATION ON PAGE 269.)

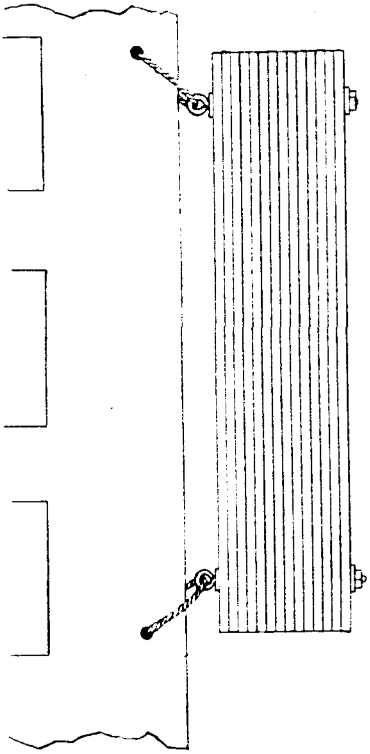


FIG. E. (EXPLANATION ON PAGE 269.)

wrapped in many layers of paper and sent by parcels post to Cambridge for study by Mr. W. F. Clapp.

The plan contemplated the shipment of the original twenty-four blocks during the first twelve-months' exposure of the board, and if the board should last for another twelve months, the shipment of the twenty-four replacement blocks.

One of the results of Mr. Clapp's study of the Bermuda blocks has been the discovery of two species of "shipworms" which are new to science, and a third which is possibly so. His descriptions of these three species are given in the number of these Proceedings immediately following the present one.

EXPLANATION OF TEXT FIGURES.

FIGURE A. One of three maps which together include the whole of the Bermuda Islands. The location of test-board No. 2 is indicated on this map by the place where the two short lines outside the map, marked by the numeral 2, would intersect, if continued.

FIGURE B. Map of the middle portion of the Bermuda Islands, to show the location of test-boards No. 0 and No. 1, indicated as in Figure A.

FIGURE C. Map of the northeastern portion of the Islands, to show the place of test-board No. 3.

FIGURE D. Face view (left) and edgewise view of test-board, giving dimensions of parts.

FIGURE E. Face view of a part of a test-board with a bundle of slats, seen edgewise, lashed to it.

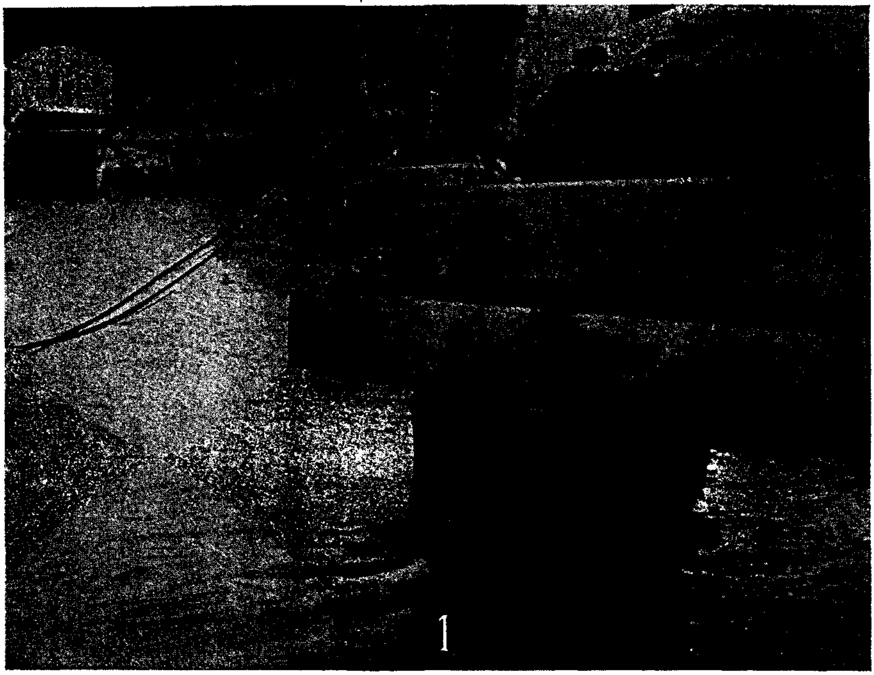
EXPLANATION OF PLATES.

(The photographs from which Figures 1–6 were reproduced were taken by Mr. A. J. Lanchner.)

PLATE I.

Figure 1. View of a corner of Mr. H. C. Master's dock in Pitt's Bay, showing that the inner one of two piles is half eaten away near the surface of the water. Photograph taken at about half tide. See text.

Figure 2. View of a part of the landing dock at Agar's Island, showing that the lower ends of all the vertical posts have been destroyed (by Limnoria) up to the level of high water.



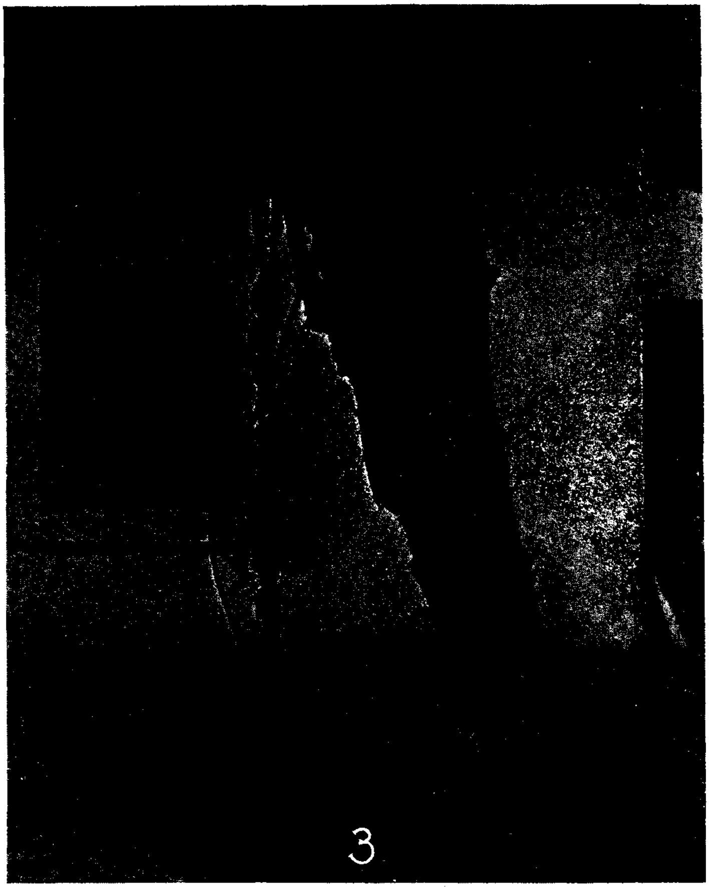
1

2

PLATE II.

FIGURE 3. Portion of log-float of mooring at Agar's Island, suspended by the shackle, to which a part of the heavily encrusted anchoring chain is still attached. The wire cable was attached to the shackle in 1921, and secured to a temporary float, to prevent the possible loss of the original log-float.

FIGURE 4. Freshly cut surfaces of the log-float exposed by sawing the float in two about 18 inches from the broken-off end. The D-shaped cross section measured 7 inches by 11 inches, and shows the slight depth to which the Limnoria bores. A knot, protruding in the middle of the flattened face of the right-hand portion, shows that it has resisted the attack of Limnoria.



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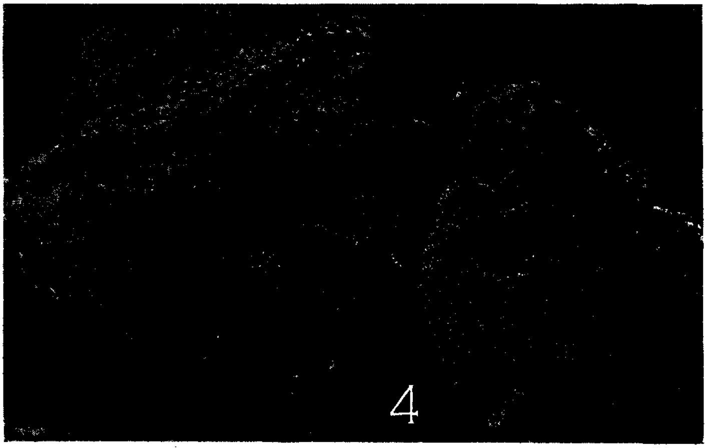

4

PLATE III.

Figure 5. Profile view of one side of log-float to show the lace-like appearance of the detached hard-wood laminae.

Figure 6. Surface view of a portion of the float, showing the richly perforated overlapping sheets of the harder wood.

5

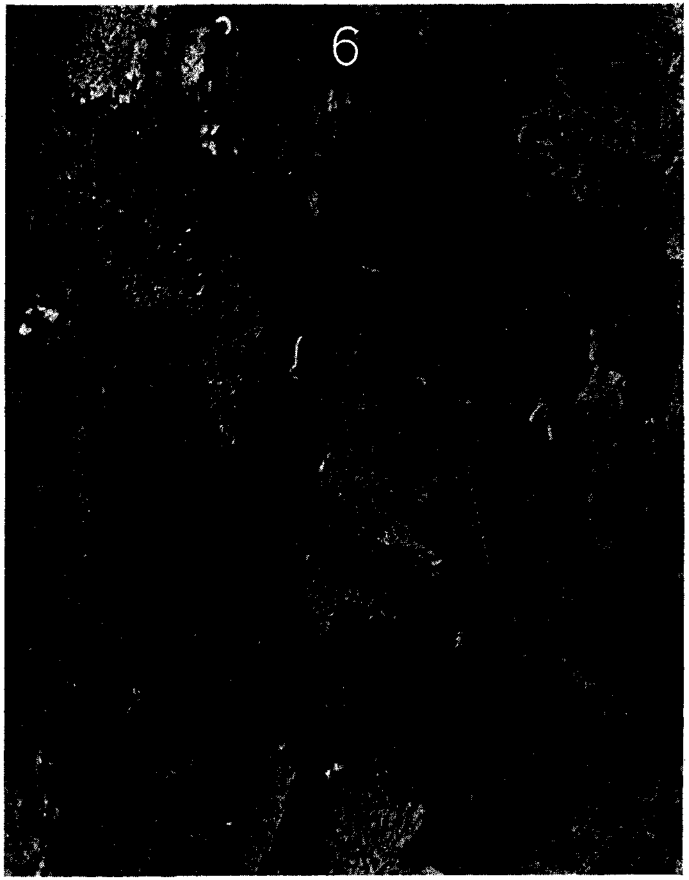

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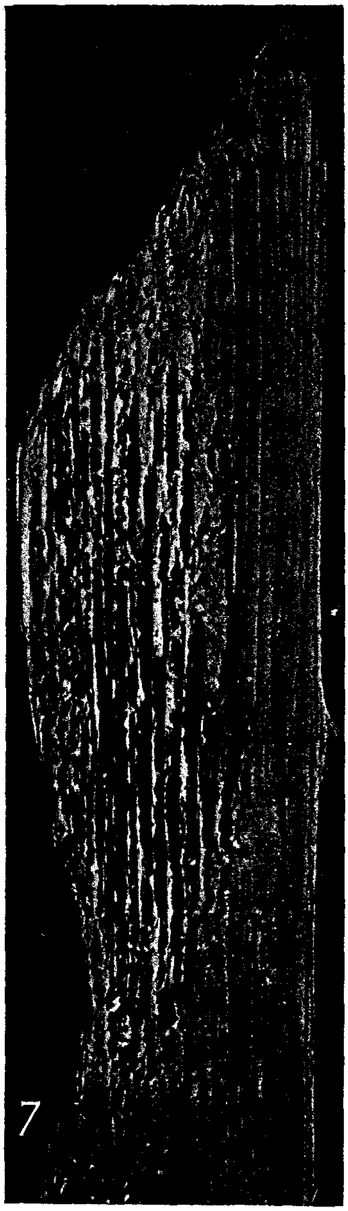
PLATE IV.

Figure 7. A split-off portion of the log-float seen edgewise, to show the lamination of the bored region and its relation to the annual rings of growth. In a few places in the deeper portions of the wood, next to the unbored parts, a dried-up borer, as a whitish object, may be seen in its burrow.

Figure 8. Face view of the same piece of wood as shown in Figure 7, seen from the face that is at the left in Figure 7. A few laminae, in the upper half of the figure, have been removed so as to show the tortuous courses of the borings in the softer wood.



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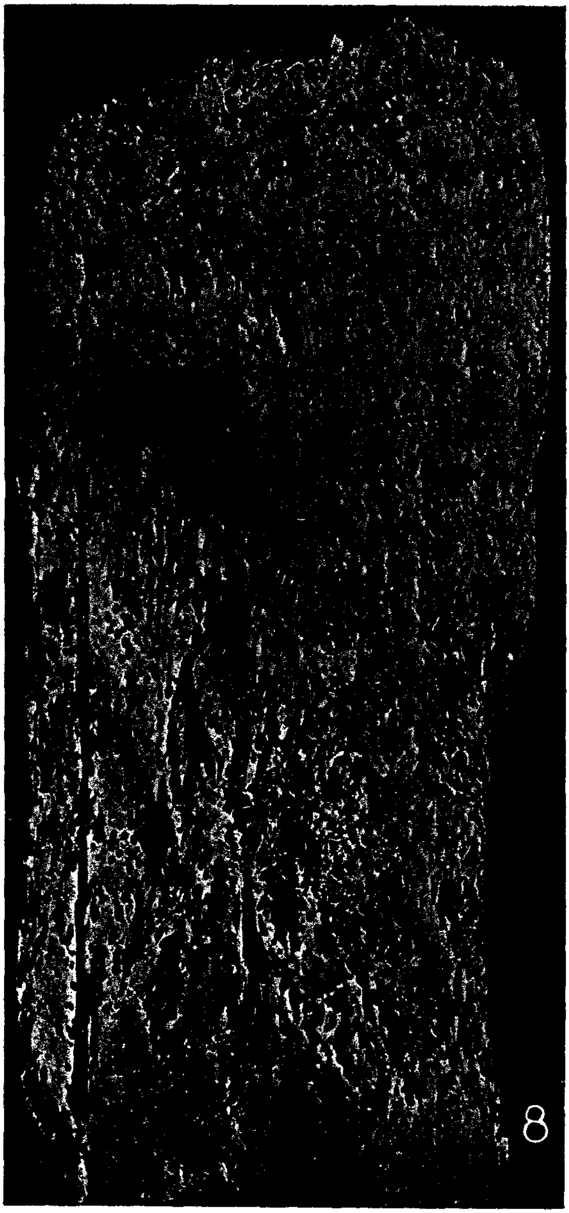

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Proceedings of the American Academy of Arts and Sciences.

Vol. 59. No. 12.— May, 1924.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION
FOR RESEARCH. No. 143.

NEW SPECIES OF SHIPWORMS IN BERMUDA.

By W. F. Clapp.

With Three Plates.

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. NO. 143.

NEW SPECIES OF SHIPWORMS IN BERMUDA.

BY W. F. CLAPP.

Received March 29, 1924. Presented by E. L. Mark, November 14, 1923.

A PRELIMINARY investigation of the activities of shipworms and other borers at Bermuda was begun in the summer of 1922 by Dr. Mark, Director of the Bermuda Biological Station for Research.

A study of the destruction caused by wood-boring marine organisms in the coastal waters of the United States, under the direction of the Committee on Marine Piling Investigations of the National Research Council, had already been under way for some months. In order that precise information regarding the destructive organisms might be obtained systematically, a special test-board had been designed by the Committee (cf. Mark, 1924, p. 267, Fig. D). Each board was 6 feet long, 12 inches wide and 1 inch thick. Twenty-four numbered blocks, each 2 by 4 by 5 inches, were attached consecutively 2 inches apart on the two faces of the board, 12 on a side. Every two weeks after the test-board was submerged, it was taken from the water and one block after another in regular sequence was removed and sent to the author at Cambridge for examination; a new numbered block being installed in place of the one removed. For a further description of test-boards used and the method of handling the blocks, consult Clapp (1923) and Mark (1924). Early in the year 1922 many such test-boards were submerged by the agents of the Committee at various stations along the Atlantic and Pacific coasts, and they were already furnishing important information.

Dr. Mark, desiring to coöperate with the Committee, installed similarly constructed test-boards at four stations in Bermudan waters, and has given an account of them, and of the places of their installment, etc. (cf. Mark, 1924).

At none of the four stations could the attack by shipworms be considered severe when compared with the destruction caused by the same species at Key West, Florida, and in the West Indies. Few of the one hundred or more blocks received contained more than thirty or forty specimens of Teredo. The attack on the wood by Limnoria during the same period of time was of such severity that the posterior ends of the shipworm tubes were soon exposed to the action of the water. This resulted in the premature death of many of the shipworms; possibly this may account for the fact that specimens of large size were rare. The destruction caused by Limnoria at the four stations was found to be approximately at the rate of one inch in twelve months (cf. Mark, 1924, p. 261). The blocks exposed at Bermuda presented conditions very similar in appearance to those submerged for the same length of time at Key West, Florida, not only as regards the severity of the Limnoria attack and the light destruction caused by the shipworms, but also in respect to the associated organisms found on the surface of the wood, particularly the bryozoans and serpulas. In each of the four cases the boards were exposed under somewhat similar conditions, due especially to the fact that they were in more or less sheltered places. Judging from an examination of driftwood from Bermuda — the exact locality, of course, unknown — there is much reason to believe that at more exposed locations the shipworm attack would be found to be much more severe.

Among the shipworms received in the test-blocks were three species of Teredo which have not been previously described. The description of two of these species, *Teredo* (*Teredo*) *batilliformis* and *Teredo* (*Zopoteredo*) *somersi*, will be found in the following pages. *T. batilliformis* is the only species which has not been found elsewhere than at Bermuda. *Teredo somersi* has occurred in test-blocks at Santo Domingo, Haiti, and the Virgin Islands, and also, in considerable numbers, in southern Florida at Key West and Channel Five. The third species, one belonging to Teredo *s. s.*, and still to be described, has even a wider distribution, being found at the localities mentioned for *T. somersi* and in addition, at Guantanamo, Cuba; in Porto Rico; and at Coco Solo, Panama, at all three of which regions it is plentiful. It is closely related to, if not identical with, a species which we have found in test-blocks submerged at Pearl Harbor, Hawaiian Islands, and also in blocks from Cavite in the Philippine Islands. Bartsch (1921, p. 28) had already described as *Teredo parksi* specimens collected in Pearl Harbor, Hawaii. Desiring Dr. Bartsch's opinion regarding the relationship between the Pacific and the Atlantic forms, I sent to him material from all our sources and he has very kindly compared the undescribed Atlantic form with the type of *Teredo parksi*. From a study of the limited amount of material at hand, he thinks that the two cannot be considered cospecific. I have, therefore, refrained for the present from considering this widely distributed

Atlantic form as identical with *T. parksi*. Not until it is possible to obtain more material and to devote more time to a thorough study of the two forms can the question be answered satisfactorily.

In addition to the undescribed species mentioned above, there also occurred in the test-blocks from Bermuda the following previously described species.

TEREDO (TEREDORA) THOMSONI Tryon.

Large specimens of this species occurred in many of the test-blocks from Ireland Island. Since they were found only in the blocks near the top of the board, it seems probable that the embryo of this species is more of a surface swimmer than are the embryos of the other species found in Bermuda. A careful comparison of the specimens of this species from Bermuda with specimens from the text-blocks at Key West, Florida, shows them to be identical.

TEREDO (ZOPOTEREDO) CLAPPI Bartsch (1923^a, p. 96).

This species, described by Bartsch from Key West, is one of those most commonly found in Bermuda, occurring plentifully at Bar's Bay and at St. George's. It was also common in the test-blocks from Key West, Florida; Cuba; Haiti; Santo Domingo; Porto Rico; and Coco Solo, Panama.

In addition to the above, one test-block from Agar's Island contained forty or more specimens of a species of shipworm belonging to the genus *Lyrodus*. Careful examination failed to show any specimens of this species in any of the other blocks from Agar's Island, and no specimens were found in the test-blocks from any of the other Bermudan stations. Specimens identical with these were found, however, in test-blocks from three different boards at Key West, Florida. A larger series might possibly prove that this species is the same as that described by Jeffreys (1860, p. 123) as *Teredo* (*Lyrodus*) *bipartita*.

It is remarkable that all of the six species of shipworms found at Bermuda belong to the genus Teredo, and that the genus Bankia (Xylotria) should not be represented, either in the collections of the present investigation or in any previous records. Of the six species at Bermuda, five are also found in southern Florida, and of these five, at least three, and possibly four, are also found at numerous localities in the West Indies.

To aid in following the technical descriptions of Teredos, Figure A is here introduced.

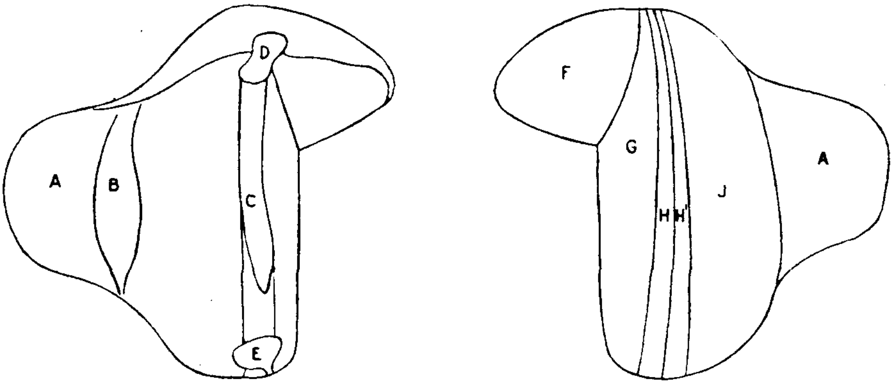

Figure A. A, Auricle or Posterior part; B, Shelf; C, Apophysis or Blade; D, Dorsal articulation or Dorsal knob; E, Ventral articulation or Ventral knob; F, Anterior area; G, Anterior-median area; H, Middle-median area, anterior part; H′, Middle-median area, posterior part; J, Posterior-median area.

Teredo (Teredo) batilliformis,[1] sp. nov.

(Pl. I, figs. 1–6; Pl. III, figs. 13, 14.)

Shell subglobular, white, covered with a thin horn-colored periostracum. The anterior area joining the anterior-median area in a slightly curved incised line. The ventral edge of the anterior area curving downward and backward, but forming with the anterior edge of the anterior-median area an angle of only slightly more than 90°.

Externally, the anterior area with the usual denticulate ridges. These ridges are one half as wide as the intervening spaces. Beginning at the ventral posterior edge, there are ten of these denticulate ridges to the millimeter, each ridge bearing eighty denticles to the millimeter (Pl. III, fig. 13). The width of the anterior-median area, at a point opposite the ventral edge of the anterior area, slightly less than one third that of the entire median area. The denticulate ridges on this area average twenty-six to the millimeter, counted along a line extending from the ventral edge of the anterior area perpendicular to the ridges. In the type specimen there are fifteen of these ridges, each bearing about twenty-eight denticles to the millimeter (Pl. III, fig. 14). The denticles on the ventral ends of these ridges end abruptly where the ridges, as faint growth lines, bend backward over the anterior portion of the middle-median area. The posterior portion of the middle-median area, thin, transparent, slightly concave. The posterior-median area opaque, milk-white, considerably more than half as wide as the entire median area; its posterior half marked by narrow deeply incised growth lines, dividing this part of the shell into regularly spaced broad ridges. The auricle small and semi-transparent, excepting the outer edge, which is milk-white. Where the anterior margin of the auricle meets the posterior-median area, the latter presents a narrow shoulder, but this does not prevent tracing, on the auricle, the growth lines, which there appear as narrow, transparent, concentric threads.

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[1] From *batillum*, a shovel, referring to the shovel-shaped pallet, a name suggested by Dr. Mark.

Internally, the left valve with a very large broad hinge-plate. In both valves the blade two thirds of the length of the shell, thin, the dorsal part narrow, the middle and ventral portions broadened and slightly reflected posteriorly. The ventral knob large. The juncture of the anterior area with the anterior-median area marked by a broad, thick, transparent callous. The auricle extending over the posterior-median portion in a broad, thick, opaque callous, ending in an abrupt ridge, but lacking any trace of a shelf or overhang.

Pallets (Pl. I, figs. 5, 6) with a broad, nearly square blade, which envelops with a thin sheath a long, very slender stalk. The proximal portion of the blade white, arising from the stalk in an abrupt curve. The calcareous portion extends to the extreme distal end of the blade, the distal two thirds of which is covered with a yellowish horn-colored periostracum. The inner face of the blade flat, the distal edge nearly straight. The outer face slightly convex, its distal portion with a broad shallow depression.

The posterior end of the shelly tube with two long, narrow ridges, arising from opposite (lateral) sides of the internal surface; the wall greatly thickened at the base of these ridges and continuing beyond the rest of the shelly portion of the tube as sharp points.

The type specimen (Mus. Comp. Zoöl. 45305) is from St. George's, Bermuda.

The measurements of the type are:

Shell: height 4 mm.; length 3.75 mm.

Pallets: entire length 3.7 mm.; length of stalk 2.1 mm., of blade 1.6 mm.; width of blade 1.3 mm.

It is very difficult to separate the *shell* of *Teredo batilliformis* from that of other closely related species of Teredo. From *T. portoricensis* it may usually be distinguished because possessing a proportionately larger anterior area. The ventral edge of the auricle forms a less abrupt angle with the posterior-median area than in *T. portoricensis.* The broad flat hinge-plate is much larger, and the blade proportionately longer. The juncture of the auricle with the posterior-median area is less well marked. The internal ridges of the posterior end of the tube are considerably longer than is usual in *T. portoricensis.*

The *pallets* are very different from those of any previously described species, the delicate stalk and the broad, squarish blade being so dissimilar to those of other species of the subgenus Teredo that I place this species here with some doubt.

T. batilliformis has been found only at Bermuda. It has occurred at frequent intervals in test-blocks at all four of the stations (Agar's Island, Bar's Bay, Ireland Island and St. George's) established there. A test-block submerged at Agar's Island in August, 1922, and removed January 1, 1923, contained, among others, one specimen with a tube 35 mm. in length and holding in the gills about one hundred fully developed embryos. The first specimens to be found appeared in the test-blocks at all four stations simultaneously in the latter part of December. None attained a total length of more than 60 mm., and no specimens contained more than one hundred embryos in the gills.

TEREDO (ZOPOTEREDO) SOMERSI,[1] sp. nov.

(Pl. II, figs. 7–12; Pl. III, figs. 15, 16).

Shell subglobular, white, covered with a transparent, colorless or very light horn-colored periostracum. The anterior area with the usual denticulate ridges. The juncture of the anterior with the anterior-median area marked by a curved, slightly incised line. The ventral edge of the anterior area forming an angle of 100° or more with the anterior edge of the anterior-median area. On the ventral posterior portion of the anterior area the denticulate ridges of approximately the same width as the intervening spaces. Dorsally, the intervening spaces only a very little wider than the denticulate ridges. At the ventral portion there are twelve of these ridges to the millimeter, each ridge bearing one hundred and ten minute denticles to the millimeter (Pl. III, fig. 15). The anterior-median area occupies slightly more than one third of the entire median area. The broadest portion of this area — at the level of the ventral edge of the anterior area — bears seventeen of the usual denticulate ridges, which average thirty-five to the millimeter; the anterior ridge bears thirty-two denticles to the millimeter (Pl. III, fig. 16). The posterior-ventral ends of these ridges can be faintly seen as growth lines passing backward over the middle-median area, quickly disappearing on the posterior-median area, which is nearly smooth. The auricle is small, its outline nearly semicircular and its juncture with the posterior-median area perceived only with difficulty.

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[1] Named from "Sir George Somers, Knight, Admirall of the seas," who has been called the "Father of Bermuda." This name also was suggested by Dr. Mark.

Interiorly, there is the usual broad, flat hinge-plate in the left valve directly beneath the umbone. The apophysis, curving downward and backward for slightly more than one half the length of the shell, narrow and thin, excepting at its extreme ventral portion, where it is considerably broadened. The concave surface of the blade reflected posteriorly. The juncture of the auricle with the posterior-median area having no trace of a shelf, but marked by a broad, thickened area, with a narrow groove at its anterior edge.

Pallets (Pl. II, figs. 11, 12) very strong and solid. The stalk short, thick, opaque-white, equal in length to the blade. The juncture of the stalk with the sheath of the blade abrupt and well marked, owing to the much larger diameter of the sheath. The blade calcareous, white, expanding from the stalk by a broad regular curve, the distal half covered by a nearly opaque, heavy, smooth, closely fitting periostracum of light horn-color. Only the chitinous horns at the extreme distal end of the blade lacking the calcareous substance. The sides of the blade converging slightly distally. Inner surface of the blade very slightly concave, outer surface convex, deeply cupped at the distal end, the outer face somewhat less deeply cupped than the inner.

The posterior end of the tube with a very short, thin, horizontal partition, dividing the tube into equal parts. The tube greatly thickened on each side at the base of the partition.

The type specimen (Mus. Comp. Zoöl. 45304) is from Ireland Island, Bermuda. There are also in the U. S. National Museum additional specimens from the type locality.

The measurements of the type are:

Shell: height 3.2 mm.; length 2.8 mm.

Pallets: length 2.9 mm., divided equally between blade and stalk; width of blade at widest portion 1 mm.

The shell of *Teredo somersi* can be separated from that of *Teredo* (*Zopoteredo*) *clappi* Bartsch (1923^a, p. 96) only with great difficulty. I have placed this species in the subgenus Zopoteredo because it has at the juncture of the auricle with the posterior-median area the structure typical of that subgenus. From *Teredo clappi*, however, it may usually be distinguished by the fact that the angle made by the ventral edge of the anterior area with the anterior edge of the anterior-median area is more obtuse. On the anterior area of *T. clappi* there are eighteen denticulate ridges to the millimeter, whereas in *T. somersi* there are usually only twelve. On the anterior-median area, the denticulate ridges are narrower in *T. somersi* than in *T. clappi* (*T. somersi* thirty-five, *T. clappi* twenty-eight to the millimeter), but the denticles on these ridges are considerably broader (*T. somersi* thirty-two, *T. clappi* forty to the millimeter).

The pallets of *T. somersi* are very different from those of *T. clappi*, the most important difference being that those of mature specimens show no trace of the sinus, or double cup, characteristic of *T. clappi*. Pallets of very young specimens, however, clearly show this sinus.

The entire length of the tube rarely exceeds fifty millimeters, and specimens less than fifteen millimeters in length frequently contain well developed embryos in the gills. The number of embryos in an individual is small; no specimen examined contained more than seventy-five, and small specimens had, in an apparently full litter, not more than twenty to twenty-five. Possibly because of this fact, the species is rare and the damage caused by it practically negligible.

Specimens of this species have been found not only at Bermuda, but also in the test-blocks placed by the Committee on Marine Piling Investigations at the following locations: Channel 5 and two stations at Key West, Florida; San Pedro de Macrois, Santo Domingo; Port au Prince, Haiti; and Christiansted, St. Croix, Virgin Islands.

In Bermuda, test-blocks placed at Agar's Island, Bar's Bay, Ireland Island, and St. George's, all contained specimens of this species at irregular intervals. The first specimens found occurred in test-blocks placed in the water August, 1922, and removed January 15, 1923. No blocks removed before that date contained any specimens of this species.

Specimens from Florida and the West Indies are generally smaller than those from Bermuda; however, one specimen in wood from the Virgin Islands had a tube length of one hundred millimeters. Stenomorphic (Bartsch, 1923^b, p. 330) specimens are frequent in all localities.

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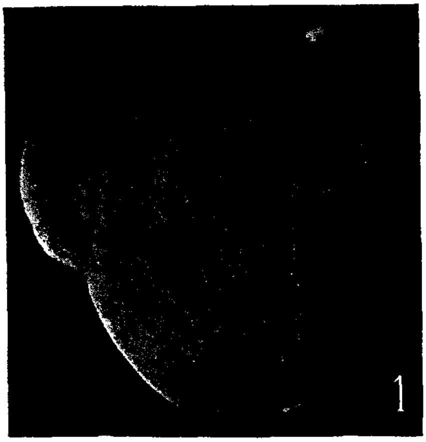
EXPLANATION OF PLATES

Figures 1-12 (Plates I and II) are each magnified 14 diameters; those of Plate III (Figures 13-16) 130 diameters.

PLATE I.

Teredo batilliformis, sp. nov. Type.

Figure 1. Exterior of right valve.
2. “ “ left “
3. Interior “ left “
4. “ “ right “
5. Pallet, outer face.
6. “ inner face.



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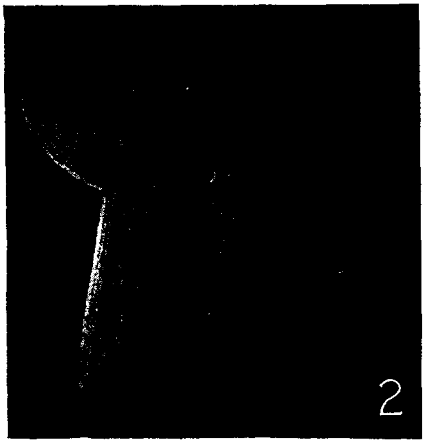

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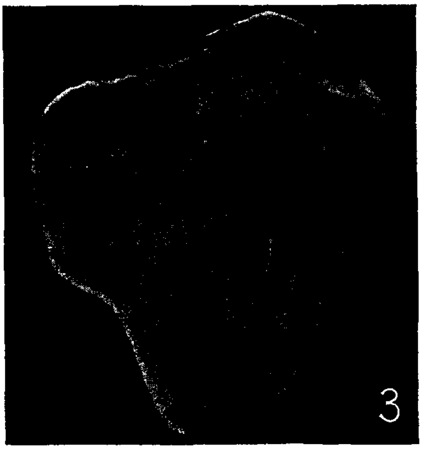

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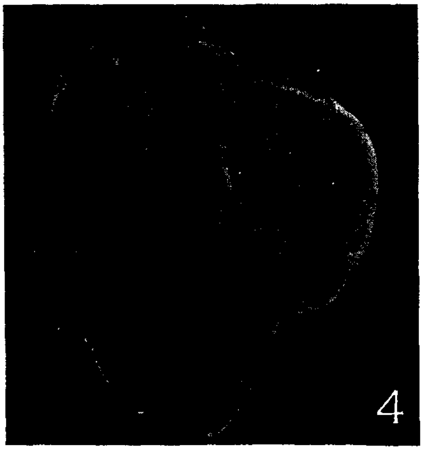

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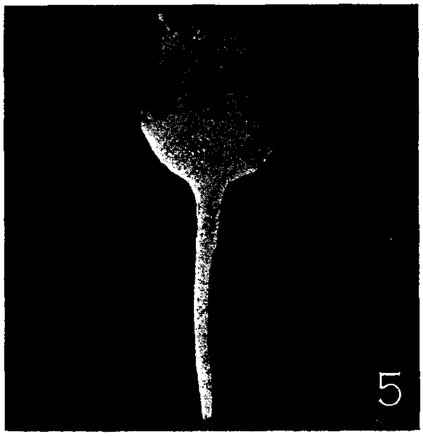

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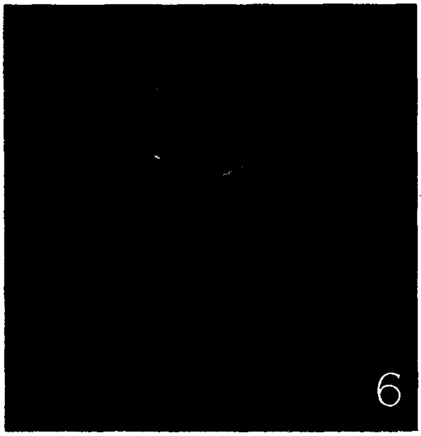

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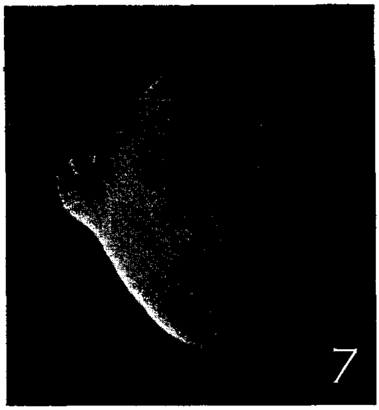
PLATE II.

TEREDO SOMERSI, sp. nov. Type.

FIGURE 7. Exterior of right valve.
8. " " left valve.
9. Interior " left "
10. " " right "
11. Pallet, outer face.
12. " inner "



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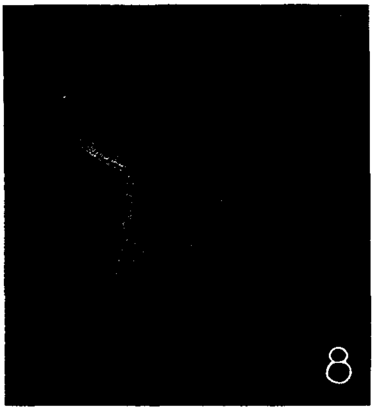

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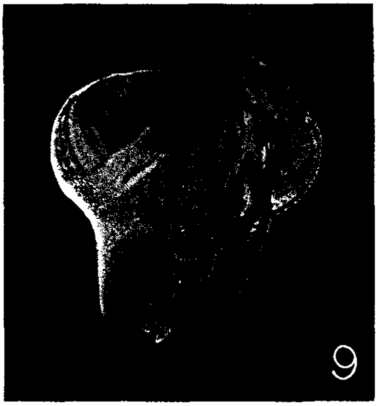

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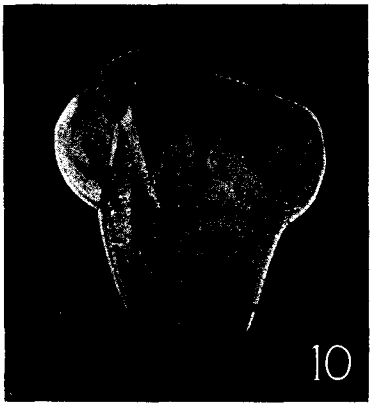

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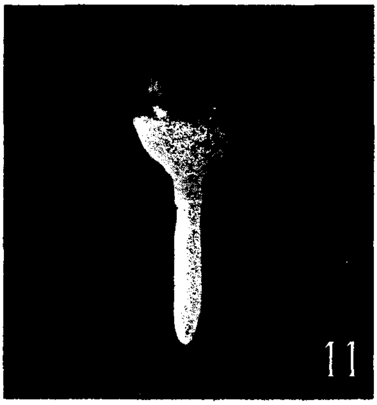

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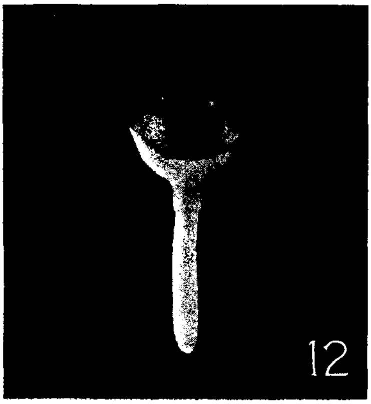

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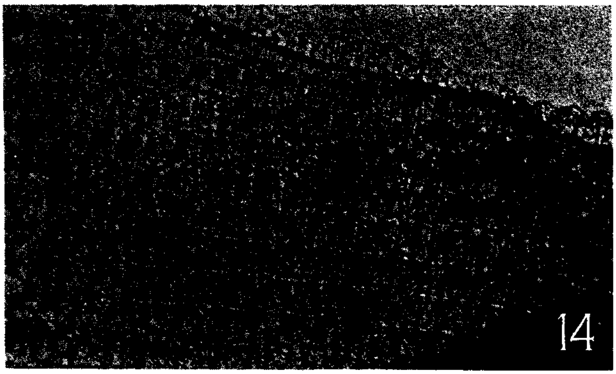
PLATE III.

FIGURE 13. Teredo batilliformis, denticulate ridges, anterior area.
14. Teredo " " " anterior-median area.
15. Teredo somersi, denticulate ridges, anterior area.
16. Teredo " " " anterior-median area.



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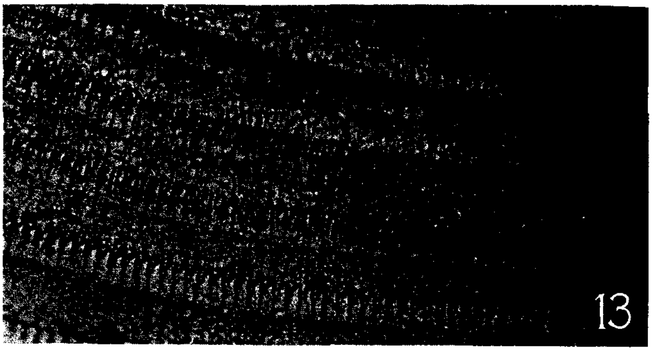

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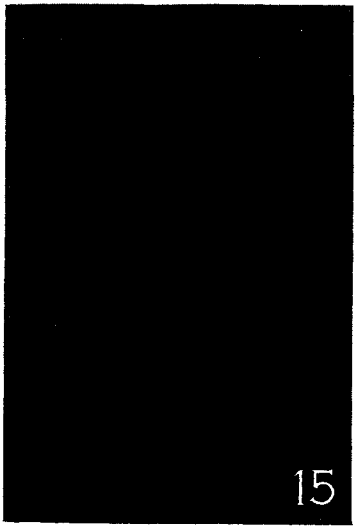

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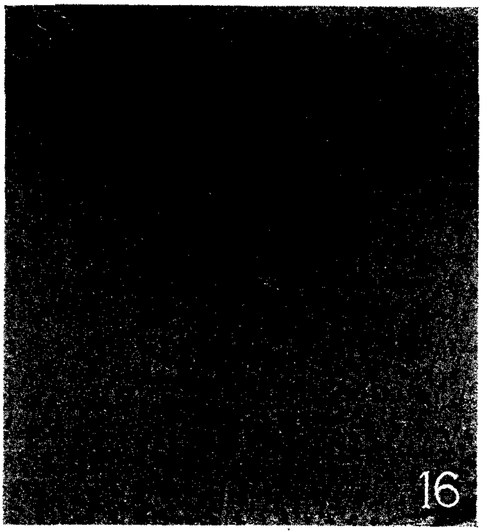

16

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SOME PROPERTIES OF THREE-TERMINAL ELECTRICAL CONDUCTING NETWORKS.

By A. E. Kennelly.

SOME PROPERTIES OF THREE-TERMINAL ELECTRICAL CONDUCTING NETWORKS.

BY A. E. KENNELLY.

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IT is known that a network of electrically conducting elements, such as that indicated in Figure 1, with any two pairs of terminals *a*, *g* and *b*, *h*, behaves with respect to those terminals, at any single alternating-current frequency, like a certain *T* as in Figure 2, or like a certain Π as in Figure 3. If two of these terminals *b* and *h* are selected at one point, say No. 8 of the network, as a common terminal, the system becomes a "three-terminal network" and reduces to either a *T* or a Δ. A particular delta is indicated in Figure 4. In general it is dissymmetrical, or no two sides have the same impedance.

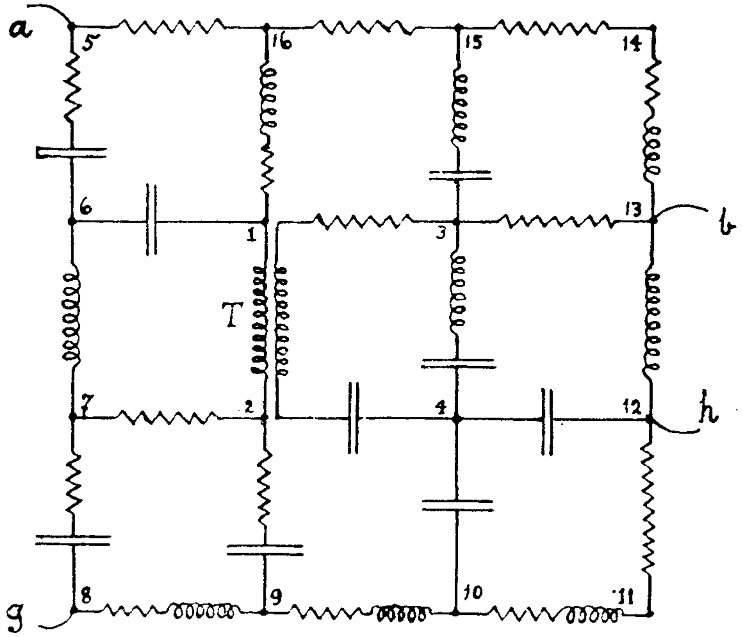


FIGURE 1. Example of a conducting network with two pairs of terminals *a*, *g* and *b*, *h*.

Let ρ be the impedance between a and b, R_1 that between a and g, R_2 that between b and h; all expressed in complex numbers of ohms; then we may denote their vector sum by Σ, or

$$\Sigma = \rho + R_1 + R_2 \qquad \text{ohms} \angle \quad (1)$$

and their planevector product Π will be

$$\Pi = \rho R_1 R_2 \qquad \text{ohms}^3 \angle \quad (2)$$

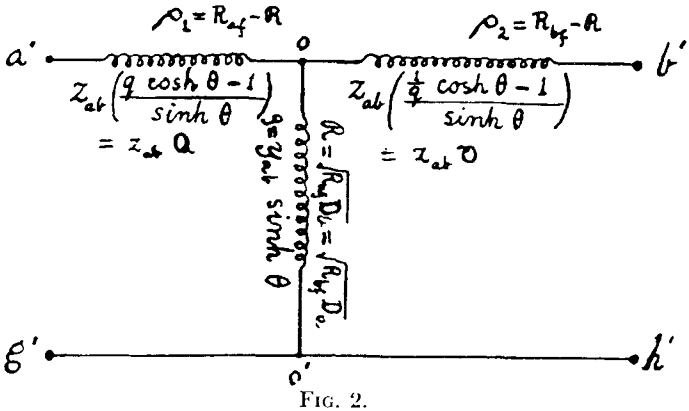

ρ₁ = R_af − R
a'
o
ρ₂ = R_bf − R
b'
Z_ab ((q cosh θ − 1)/sinh θ) = z_ab a
Z_ab (((1/q) cosh θ − 1)/sinh θ) = z_ab σ
g = y_ab sinh θ
R = √(R_af D_b) = √(R_bf D_a)
g'
h'
o'

FIG. 2.

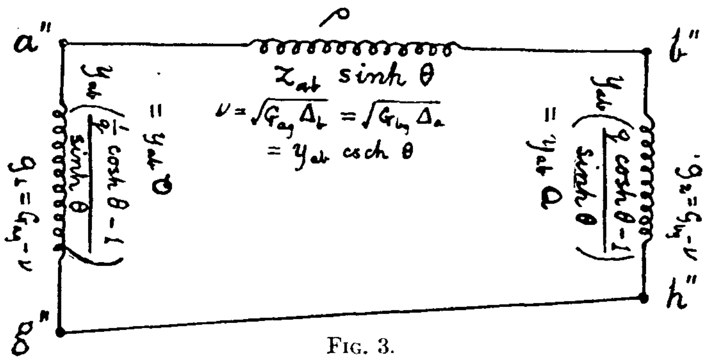

ρ
a''
b''
z_ab sinh θ
ν = √(G_ag Δ_b) = √(G_bh Δ_a) = y_ab csch θ
y_ab (((1/q) cosh θ − 1)/sinh θ) = y_ab σ
g₁ = G_ag − ν
y_ab ((q cosh θ − 1)/sinh θ) = y_ab a
g₂ = G_bh − ν
g''
h''

FIG. 3.

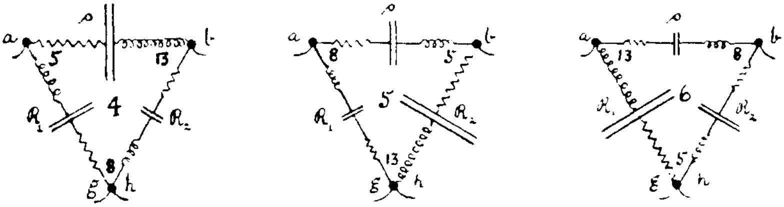
FIGURES 2 and 3. Dissymmetrical T and Π corresponding to Figure 1.

Constancy of z_{ab} in the three different aspects of a three-terminal network.

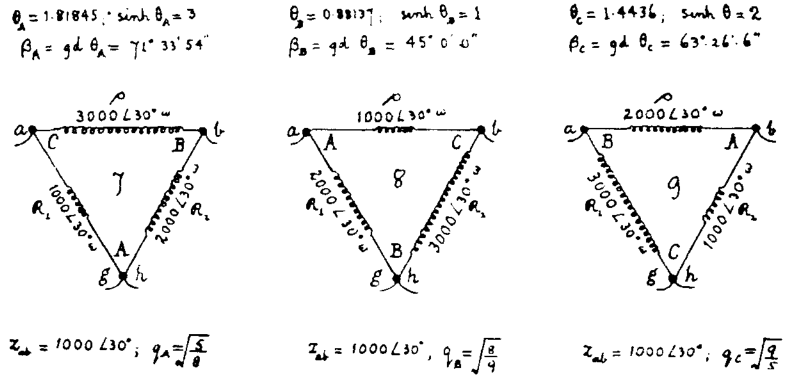
It is easily shown that the geomean surge impedance z_{ab} of the system; i. e. the geometrical mean of the apparent surge impedance z_{oa} at the $a\ g$ terminals, and that z_{ob} at the $b\ h$ terminals, is:

$$z_{ab} = \sqrt{z_{oa} \cdot z_{ob}} = \sqrt{\frac{\rho R_1 R_2}{\rho + R_1 + R_2}} = \sqrt{\frac{\Pi}{\Sigma}} \qquad \text{ohms } \angle. \quad (3)$$

If then we shift the three terminals around the network of Figure 1; from the aspect shown in Figures 1 and 4 with a on point 5, b on 13, and g, h on 8, or what may be called the 5–13 aspect, to the 8–5 aspect of Figure 5, with a on point 8 and b on 5, it is evident that the same triangle or delta of impedances will exist between the three terminals a, b and gh; but with ρ, R_1 and R_2 mutually interchanged. Their vector sum Σ, and their vector product Π, will, however, be the same as in Figure 4; so that the geomean surge impedance z_{ab} of the network in the new aspect will, by (3), remain the same. Similarly, if the terminals be again changed; so that a is on point 13, b on 8, and gh on 5, the network will assume the 13–8 aspect of Figure 6; but its geomean surge impedance will again remain the same. Consequently, *in any*


ρ
a
b
5
13
4
R₁
R₂
8
g
h
ρ
a
b
8
5
5
R₁
R₂
13
g
h
ρ
a
b
13
8
6
R₁
R₂
5
g
h

FIGURES 4, 5 and 6. Three aspects of equivalent delta of a network at terminals a, b and g h, for a given frequency.


θ_A = 1·81845 ∴ sinh θ_A = 3
β_A = gd θ_A = 71° 33′ 54″
θ_B = 0·88137; sinh θ_B = 1
β_B = gd θ_B = 45° 0′ 0″
θ_C = 1·4436; sinh θ = 2
β_C = gd θ_C = 63° 26′ 6″
ρ
3000∠30° ω
a
b
C
B
7
R₁
1000∠30° ω
R₂
2000∠30° ω
A
g
h
ρ
1000∠30° ω
a
b
A
C
8
R₁
2000∠30° ω
R₂
3000∠30° ω
B
g
h
ρ
2000∠30° ω
a
b
B
A
9
R₁
3000∠30° ω
R₂
1000∠30° ω
C
g
h
z_ab = 1000∠30°; q_A = √(5/8)
z_ab = 1000∠30°, q_B = √(8/9)
z_ab = 1000∠30°; q_C = √(9/5)

FIGURES 7, 8 and 9. Three aspects of the equivalent delta of network.

three-terminal network of conductors, with the three terminals disposed around three selected points in any of the three possible aspects, the geomean surge impedance of the network, or of its T and Π, *with respect to the three terminals, will be the same.*

As an example, we may consider the simple case of an equivalent Π of a network, such as is shown in Figure 8. Here $AC = \rho = 1000 \angle 30^\circ$ ohms, $AB = R_1 = 2000 \angle 30^\circ$ ohms, $CB = R_2 = 3000 \angle 30^\circ$ ohms. Hence $\Pi = 6 \times 10^9 \angle 90^\circ$, and $\Sigma = 6 \times 10^3 \angle 30^\circ$; so that $z_{ab} = \sqrt{\Pi/\Sigma} = 1000 \angle 30^\circ$ ohms. Here we have the network in its AC aspect. Figures 7 and 9 represent the same network in its CB, and BA aspects, respectively. The geomean surge impedance $1000 \angle 30^\circ$ ohms is the same, in all three cases.

Values of sinh θ in the three aspects of a three-terminal network.

It may be readily shown that the sine of the complex hyperbolic angle θ subtended by a network, with respect to three terminals, is

$$\sinh \theta = \frac{\rho}{z_{ab}} = \rho \sqrt{\frac{\Sigma}{\Pi}} \qquad \text{numeric } \angle. \quad (4)$$

so that in the three different aspects of Figures 4, 5 and 6, with z_{ab} constant, $\sinh \theta$ is directly proportional to the architrave impedance ρ. In the cases of Figures 7, 8, and 9, $\sinh \theta$ is respectively 3.0, 1.0 and 2.0; from which the angle of the network in these three aspects is $\theta_A = 1.81845$, $\theta_B = 0.88137$ and $\theta_C = 1.4436$ hyperbolic radians, respectively.

Consequently, in any three-terminal network, the sine of its angle θ in each of the three different aspects of the terminals, is directly proportional to the impedance between the terminals a, b.

Relations between the values of the inequality ratio q of a three-terminal network.

The value of the inequality ratio q for a dissymmetrical Π or Δ is known to be

$$q = \sqrt{\frac{\rho + R_2}{\rho + R_1} \times \frac{R_1}{R_2}} = \sqrt{\frac{g_2 + \nu}{g_1 + \nu}} \qquad \text{numeric } \angle \quad (5)$$

and this is in general different in each aspect of a three-terminal network, but if we denote the three different values of q for these aspects by q_A, q_B and q_C respectively; then we find from (5) that

$$q_A \cdot q_B \cdot q_C = 1 \angle 0^\circ \qquad \text{numeric} \quad (6)$$

In the case of Figures 7, 8, and 9, $q_A = \sqrt{\frac{5}{8}}$, $q_B = \sqrt{\frac{8}{9}}$, and $q_C = \sqrt{\frac{9}{5}}$.

Consequently, the continued product of the three inequality ratios of a three-terminal network, in its three aspects, is always equal to unity.

Equality between the planevector sum and planevector product of the sines of the angles subtended by a three-terminal network in its three different aspects.

We have already seen, by (4) that $\sinh\theta = \rho/z_{ab}$, in each aspect of the network. Consequently, if θ_A, θ_B and θ_C are the three angles of the network in its three different aspects, we have:

$$\sinh\theta_A \cdot \sinh\theta_B \cdot \sinh\theta_C = \frac{\Pi}{z^3_{ab}} = \Pi \cdot \frac{\Sigma}{\Pi} \cdot \sqrt{\frac{\Sigma}{\Pi}} = \Sigma\sqrt{\frac{\Sigma}{\Pi}} \quad \text{numeric} \angle \ (7)$$

Moreover,

$$\sinh\theta_A + \sinh\theta_B + \sinh\theta_C = \frac{\Sigma}{z_{ab}} = \Sigma\sqrt{\frac{\Sigma}{\Pi}} \qquad \text{“} \quad \angle \ (8)$$

Consequently, in any three-terminal network,

$$\sinh\theta_A + \sinh\theta_B + \sinh\theta_C = \sinh\theta_A \cdot \sinh\theta_B \cdot \sinh\theta_C \qquad \text{“} \quad \angle \ (9)$$

or the sum of the sines of the three hyperbolic angles is equal to their product. Thus, in the simple case of Figures 7, 8, and 9, with $\sinh\theta_A = 3$, $\sinh\theta_B = 1$, and $\sinh\theta_C = 2$, both the sum and product are equal to 6.

Relations between the gudermannian angles of a three-terminal network represented by a Π of equislope impedances.

Formula (9) connecting the sines of the three hyperbolic angles of a three-terminal network, in its three successive aspects, is of general application, whatever the impedances in the network may be. In the more limited case, however, when the three impedances of the equivalent delta of the network, at the three terminals, have the same slope (same powerfactor and same reactance factor), the angles θ_A, θ_B and θ_C will have no imaginary components, or will be real hyperbolic angles. Under these conditions, they are connected by an additional relation. It is well known that $\sinh\theta$, the sine of any real hyperbolic angle θ, is numerically equal to the tangent of a related circular angle, commonly called the "gudermannian angle." Thus

$$\sinh\theta = \tan\beta \qquad \text{numeric (10)}$$

where

$$\beta = gd\,\theta \qquad \text{circular radians or degrees (11)}$$

Consequently, when θ_A, θ_B, and θ_C are all imaginaryless hyperbolic angles, we may substitute (10) in (9), and obtain

$$\tan\beta_A + \tan\beta_B + \tan\beta_C = \tan\beta_A \cdot \tan\beta_B \cdot \tan\beta_C \qquad \text{numeric (12)}$$

or the three circular gudermannian angles of the three-terminal network have the sum of their tangents equal to the product of their tangents. This condition of equality between the product and sum of three tangents in circular trigonometry, is a well known property of the interior angles of each and every plane triangle.

Consequently, *in any three-terminal network for which* θ_A, θ_B and θ_C *are real, there exists a corresponding characteristic type of plane triangle, with interior angles* β_A, β_B *and* β_C, *these interior angles being respectively the gudermannians of* θ_A, θ_B *and* θ_C.

Thus, in the simple case of Figures 7, 8 and 9, and in which θ_A, θ_B and θ_C have no imaginary components, $\beta_B = 45°$, $\beta_C = 63° .26' .6''$ and $\beta_A = 71° .33' .54''$. The sum of these circular angles is π radians, or 180°; i. e.

$$\beta_A + \beta_B + \beta_C = gd\,\theta_A + gd\,\theta_B + gd\,\theta_C = \pi \qquad \text{circular radians (13)}$$

The plane triangle corresponding to such a network is defined only by its internal angles β_A, β_B, β_C, and its sides are indeterminate. An infinite series of similar plane triangles have the same interior angles. The ratios of the sides a, b, c of all these plane triangles are defined, however, by the relation:

$$a : b : c :: \sin\beta_A : \sin\beta_B : \sin\beta_C :: \tanh\theta_A : \tanh\theta_B : \tanh\theta_C \qquad (14)$$

The sides of the characteristic plane triangle corresponding to a three-terminal network, having real hyperbolic angles θ, are thus respectively proportional to the tangents of the hyperbolic angles, or to the sines of their gudermannians.

For the particular network of Figures 7, 8 and 9, the plane triangle A', B', C', of Figure 10 is of the characteristic type, the interior angles of this triangle are respectively the gudermannians of the network hyperbolic angles θ_A, θ_B and θ_C.

In order that the angles θ_A, θ_B and θ_C of a three-terminal network shall each and all be imaginaryless, it is necessary and sufficient that the three impedances of the delta in Figures 4, 5 and 6 shall all be either reactanceless (pure resistances) or have the same slope, as in Figures 7, 8 and 9. Thus, let ρ be the impedance AC, Figure 8, having a size $|\rho|$ and a slope $\bar{\rho}$. In Figure 8, $|\rho| = 1000$ and $\bar{\rho} = 30°$. Then if $R_1 = m\rho$ and $R_2 = n\rho$, m and n will be real numbers, if $\bar{R}_1 = \bar{R}_2 = \bar{\rho}$, or all three impedances have the same slope. Then, by (4),

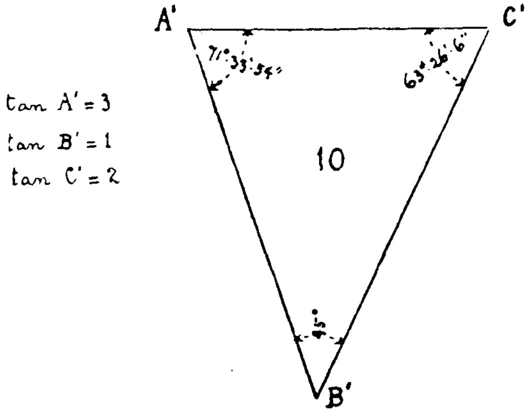
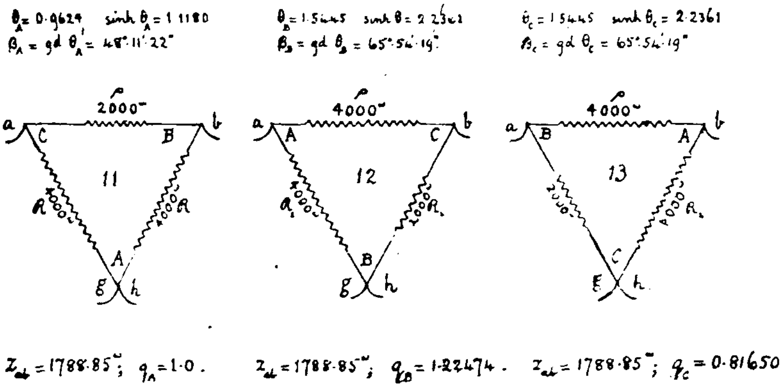

A'
C'
B'
tan A' = 3
tan B' = 1
tan C' = 2
71°33'54"
63°26'6"
45°
10

FIGURE 10. Plane triangle corresponding to network of Figures 7, 8 and 9.


θ_A = 0·9624 sinh θ_A = 1·1180
β_A = gd θ'_A = 48°11'22"
θ_B = 1·5445 sinh θ = 2·2361
β_B = gd θ_B = 65°54'19"
θ_C = 1·5445 sinh θ_C = 2·2361
β_C = gd θ_C = 65°54'19"
ρ 2000°
ρ 4000°
ρ 4000°
a
b
g
h
A
B
C
R
R₁
4000°
2000°
11
12
13
Z_ab = 1788·85°; q_A = 1·0.
Z_ab = 1788·85°; q_B = 1·22474.
Z_ab = 1788·85°; q_C = 0·81650

FIGURES 11, 12 and 13. Equivalent delta of network, with two sides equal.

$$\sinh \theta_B = \rho \sqrt{\frac{(m+n+1)\,\rho}{mn\,\rho^3}} = \sqrt{\frac{m+n+1}{mn}} \qquad \text{numeric}$$

This is a real number; because m and n are reals. The same reasoning applies to $\sinh\theta_A$ and $\sinh\theta_B$. Hence θ_A, θ_B and θ_C are all real, if ρ, R_1, R_2 have the same slope.

Particular case of two equal impedances in a Three-Terminal Network Delta.

If two of the impedances representing a network at the terminals a, b amd gh are equal, as in the simple case of Figures 11, 12 and 13; then in the aspect (Figure 11) where the two pillars R_1 and R_2 are equal, suppose $\rho = m R_1 = m R_2 = m R$ ohms, or $q = 1$.

Then $$\Sigma = \rho \frac{(m+2)}{m} \quad \text{ohms} \angle \tag{16}$$

and $$\Pi = \frac{\rho^3}{m^2} \quad \text{ohms}^3 \angle \tag{17}$$

so that by (3) $$z_{ab} = z_o = \frac{\rho}{\sqrt{m(m+2)}} \quad \text{ohms} \angle \tag{18}$$

and by (4) $$\sinh\theta = \sqrt{m(m+2)} \quad \text{numeric} \angle \tag{19}$$

or $$\cosh\theta = m+1 \quad \text{numeric} \angle \tag{20}$$

also $$\text{versh}\,\theta = \cosh\theta - 1 = m \quad \text{numeric} \angle \tag{21}$$

Consequently, the angle θ of the network, in this aspect, will have m for its versed sine. In general, m will be complex.

In the case of Figures 11, 12 and 13, in which the delta sides are all reactanceless, or simple resistances, there is a corresponding gudermannian triangle and it is isoceles. Here $\cosh\theta_A = 1.5$, or $m = 0.5$.

Particular case of three equal impedances in a Three-Terminal Network Delta.

When the equivalent delta of a network, with respect to the terminals a, b, and gh happens to contain three equal impedances; i. e. equal both as to size and slope; then $q = 1$, $m = 1$, $\Sigma = 3\rho$, and $\pi = \rho^3$.

$$z_{ab} = z_o = \frac{\rho}{\sqrt{3}} \quad \text{ohms} \angle \tag{22}$$

$$\sinh\theta = \sqrt{3} \quad \text{numeric} \tag{23}$$

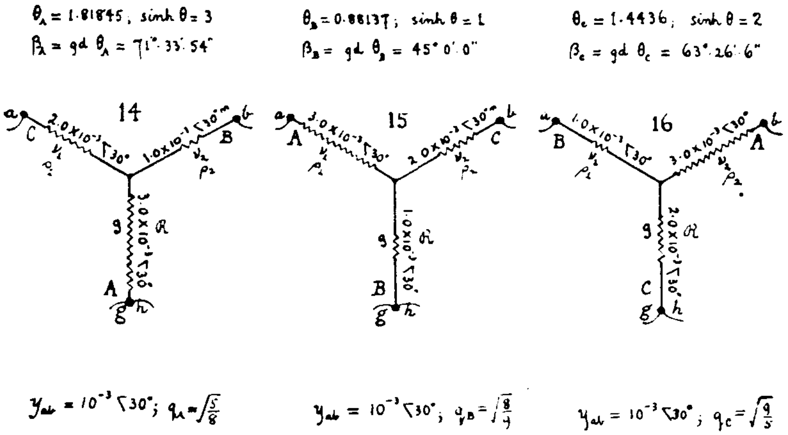
$$\cosh \theta = 2 \qquad \text{numeric (24)}$$

$$\operatorname{versh} \theta = 1 \qquad \text{numeric (25)}$$

The three angles θ_A, θ_B and θ_C will be equal, and each will have unit versed sine. This angle is 1.31696 $\angle 0°$ hyps. There is thus a corresponding gudermannian triangle and it is equilateral, each interior angle being 60°, or

$$gd\,\theta = \frac{\pi}{3} \qquad \text{circular radians (26)}$$

Particulars concerning the Equivalent T *of a Three-Terminal Network.*
We have hitherto confined attention to a delta connection of three impedances as representing a conducting network, at a given single frequency, with respect to three terminals a, b and gh. We may, however, replace such a delta of impedances by an equivalent star or T, as in Figures 14, 15 and 16, which correspond to Figures 7, 8 and 9. Then it will be found that if ν_1, ν_2 and g are the three planevector admittances of the three star branches, and if


θ_A = 1·81845; sinh θ = 3
β_A = gd θ_A = 71°·33'·54"
θ_B = 0·88137; sinh θ = 1
β_B = gd θ_B = 45° 0' 0"
θ_C = 1·4436; sinh θ = 2
β_C = gd θ_C = 63°·26'·6"
14
a
b
C
B
A
g
h
2·0×10⁻³ ⌈30°
1·0×10⁻³ ⌈30°
3·0×10⁻³ ⌈30°
ν₁
ν₂
ρ₁
ρ₂
g
R
15
a
b
A
C
B
g
h
3·0×10⁻³ ⌈30°
2·0×10⁻³ ⌈30°
1·0×10⁻³ ⌈30°
ν₁
ν₂
ρ₁
ρ₂
g
R
16
a
b
B
A
C
g
h
1·0×10⁻³ ⌈30°
3·0×10⁻³ ⌈30°
2·0×10⁻³ ⌈30°
ν₁
ν₂
ρ₁
ρ₂
g
R
y_ab = 10⁻³ ⌈30°; q_A = √(5/8)
y_ab = 10⁻³ ⌈30°; q_B = √(8/4)
y_ab = 10⁻³ ⌈30°; q_C = √(9/5)

FIGURES 14, 15 and 16. Equivalent star of network in three aspects.

$$\Sigma' = \nu_1 + \nu_2 + g \qquad \text{mhos } \angle \text{ (27)}$$

$$\Pi' = \nu_1 \nu_2 g \qquad \text{mhos}^3 \angle \text{ (28)}$$

the geomean surge admittance $y_{ab} = 1/z_{ab}$ of the system, or of its equivalent T, is

$$y_{ab} = \sqrt{\frac{\Pi'}{\Sigma'}} \qquad \text{mhos } \angle \quad (29)$$

$$q = \sqrt{\frac{\rho_1 + R}{\rho_2 + R}} \qquad \text{numeric } \angle \quad (30)$$

and

$$\sinh \theta = \frac{q}{y_{ab}} = q\sqrt{\frac{\Sigma'}{\Pi'}} \qquad \text{numeric } \angle \quad (31)$$

It will be evident that the geomean surge admittance of any three-terminal network, presented as a dissymmetrical T, will be the same in all three aspects. Again, as in (6)

$$q_A \cdot q_B \cdot q_C = 1 \angle 0^\circ \qquad \text{numeric} \quad (32)$$

Moreover, as in (9)

$$\sinh \theta_A + \sinh \theta_B + \sinh \theta_C = \sinh \theta_A \cdot \sinh \theta_B \cdot \sinh \theta_C \quad \text{numeric } \angle \quad (33)$$

If all the sines are real, we find a corresponding characteristic gudermannian triangle, in accordance with (13).

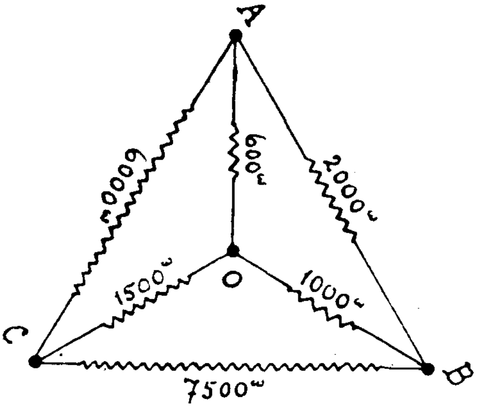


Figure 17. Six element network with four junction points.

Figure 17 shows a particular network of six elements AO, BO, CO, AB, BC, CA all taken as reactanceless, or pure resistances. There are just 12 ways in which, using the four junctions A B C and O for

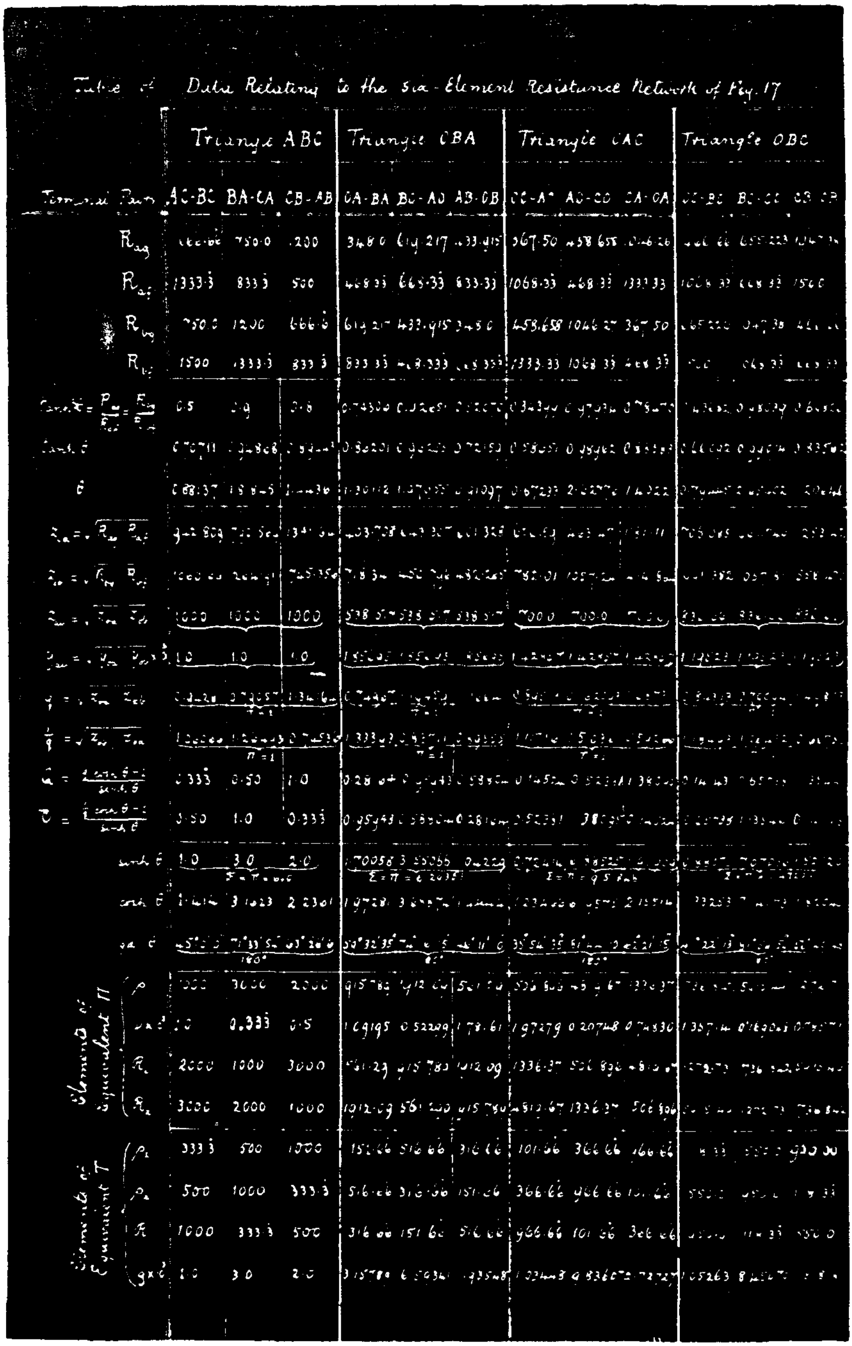


Table of Data Relating to the Six-Element Resistance Network of Fig. 17

	Triangle ABC			Triangle OBA			Triangle OAC			Triangle OBC		
Terminal Pairs	AC-BC	BA-CA	CB-AB	OA-BA	BO-AO	AB-OB	OC-AC	AO-CO	CA-OA	OC-BC	BO-CO	CB-OB
R_{ag}	666·66	750·0	1200	348·0	619·217	433·915	367·50	458·658	1046·26	[illegible]	655·223	[illegible]
R_{af}	1333·3	833·3	500	468·33	665·33	833·33	1068·33	468·33	1333·33	1066·33	668·33	1500
R_{bg}	750·0	1200	666·6	619·217	433·915	348·0	458·658	1046·27	367·50	[illegible]	[illegible]	[illegible]
R_{bf}	1500	1333·3	833·3	833·33	468·333	665·333	1333·33	1068·33	468·33	[illegible]	665·33	[illegible]
$\tan\psi = \frac{R_{ag}}{R_{af}} = \frac{R_{bg}}{R_{bf}}$	0·5	0·9	0·6	0·74308	[illegible]	[illegible]	0·34344	0·97934	[illegible]	[illegible]	0·98079	[illegible]
$\tanh\theta$	0·70711	0·94868	[illegible]	0·86201	[illegible]	0·72152	0·58[illegible]	0·98962	[illegible]	[illegible]	[illegible]	0·8350[illegible]
θ	0·88137	1·8185	1·4436	1·30112	[illegible]	0·91097	0·67233	2·6277[illegible]	1·4022	[illegible]	[illegible]	[illegible]
$z_a = \sqrt{R_{ag} R_{af}}$	942·809	[illegible]	[illegible]	403·728	[illegible]	601·326	[illegible]	[illegible]	[illegible]	705·065	[illegible]	[illegible]
$z_b = \sqrt{R_{bg} R_{bf}}$	1060·66	1264·91	745·356	718·34	[illegible]	[illegible]	782·01	1057·2[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$z_{ab} = \sqrt{z_a z_b}$	1000	1000	1000	538·517	538·517	538·517	700·0	700·0	[illegible]	[illegible]	[illegible]	[illegible]
$y_{ab} = \sqrt{y_a y_b} \times 10^3$	1·0	1·0	1·0	1·8[illegible]	[illegible]	[illegible]	1·42857	1·42857	1·42857	[illegible]	[illegible]	[illegible]
$q = \sqrt{z_a / z_b}$	[illegible]	0·79057	[illegible]	0·7486[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		$\Pi = 1$			$\Pi = 1$			$\Pi = 1$			$\Pi = 1$	
$\frac{1}{q} = \sqrt{z_b / z_a}$	1·06066	1·26491	0·74536	1·3358[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		$\Pi = 1$			$\Pi = 1$			$\Pi = 1$				
$a = \frac{q\cosh\theta - 1}{\sinh\theta}$	0·333̇	0·50	1·0	0·28504	[illegible]	0·5396[illegible]	0·1452[illegible]	0·523[illegible]	1·380[illegible]	0·14[illegible]	[illegible]	[illegible]
$b = \frac{\frac{1}{q}\cosh\theta - 1}{\sinh\theta}$	0·50	1·0	0·333̇	0·95948	[illegible]	0·2816[illegible]	2·52331	3·8[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\sinh\theta$	1·0	3·0	2·0	1·70058	3·55266	0·4229	0·7241[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		$\Sigma = \Pi = 6·0$			$\Sigma = \Pi = 2·20[illegible]$			$\Sigma = \Pi = 9·5[illegible]$			$\Sigma = \Pi =$ [illegible]	
$\cosh\theta$	1·414	3·1623	2·2361	1·9728[illegible]	3·69[illegible]	[illegible]	1·2344[illegible]	[illegible]	2·1557[illegible]	[illegible]	[illegible]	[illegible]
$\operatorname{gd}\theta$	45°0′0″	71°33′54″	63°26′6″	59°32′35″	[illegible]	[illegible]	35°54′35″	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		180°			90°			180°			90°	
Elements of Equivalent Π: ρ	1000	3000	2000	915·782	[illegible]	561·20	506·[illegible]	[illegible]	1336·37	[illegible]	[illegible]	[illegible]
[illegible]	1·0	0.333̇	0·5	1·69195	0·52209	1·7861	1·97279	0·20748	0·74330	1·357[illegible]	[illegible]	[illegible]
R_1	2000	1000	3000	[illegible]	915·782	1412·09	1336·37	506·896	[illegible]	[illegible]	[illegible]	[illegible]
R_2	3000	2000	1000	1412·09	561·220	915·780	[illegible]	1336·37	506·896	[illegible]	[illegible]	[illegible]
Elements of Equivalent T: ρ_1	333·3̇	500	1000	151·66	516·66	316·66	101·66	366·66	166·66	[illegible]	[illegible]	930·00
ρ_2	500	1000	333·3̇	516·66	316·66	151·66	366·66	966·66	101·66	[illegible]	[illegible]	[illegible]
R	1000	333·3̇	500	316·66	151·66	516·66	966·66	101·66	366·66	[illegible]	[illegible]	[illegible]
$g \times c^2$	1·0	3·0	2·0	3·1578[illegible]	6·5034[illegible]	0·93548	1·0244[illegible]	9·836[illegible]	[illegible]	1·05263	[illegible]	[illegible]

terminal points, this network can be connected to three terminals so as to form a three-terminal delta network; i. e. three aspects to each of the four triangles *ABC*, *AOB*, *AOC* and *BOC*. Each of these 12 deltas provides also an equivalent star network. They are all dissymmetrical. They are analyzed and presented in the accompanying Table. It will be seen that in each of the four triple-aspect groups, the following properties are found:—

(1) z_{ab} and its reciprocal y_{ab} are constant for each group.

(2) The triple q product of each group is equal to unity.

(3) " " $1/q$ " " " " " " " "

(4) There are three values for Q and ∂, distributed among each group.

(5) There is equality between the sums and products of $\sinh\theta$ in each group.

(6) The three gudermannian angles of each group sum to 180°, or there exists a corresponding plane triangle for each group.

Summary of Conclusions, relating to a Three-Terminal Network, at a single alternating-current frequency (including zero frequency) in the steady state.

(1) In the three aspects of the three terminals, the geomean surge impedance z_{ab} and its reciprocal, the geomean surge admittance y_{ab}, are constant.

(2) In the three aspects of a delta network, $\sinh\theta$, the sine of the angle of the network is proportional to the impedance connecting the a, b terminals.

(3) The product $q_A \cdot q_B \cdot q_C$ of the three inequality ratios is equal to unity.

(4) The planevector sum of the sines of the three angles of the network is equal to their planevector product.

(5) When the three hyperbolic angles θ_A, θ_B, θ_C are real, the three corresponding gudermannian circular angles make up a sum of π radians or 180°. Consequently, these gudermannians define a triangle, or family of similar plane triangles.

(6) When two of the delta impedances of a network are equal, and form the pillars of the equivalent π, the versed sine of the angle of the network is m where $\bar{m} = \rho/R$.

(7) When all three of the delta impedances of a network are equal in size and in slope, the angles in the three aspects are equal, real, and have a versed sine of unity. The angle is 1.317 hyps, nearly, and the gudermannian triangle is equilateral.

(8) Similar relations affect the equivalent T or star of a network.

List of Symbols Employed.

a, b, c	lengths of sides of a plane triangle.
β	a circular angle, the gudermannian of a network hyperbolic angle (circular radians or degrees)
$D = R_f - R_g$	difference in impedance due to shorting at the opposite end terminals (ohms $\angle$).
$\Delta = G_g - G_f$	difference in admittance due to shorting at the opposite end terminals (mhos $\angle$).
G_{ag}	admittance measured at terminals a, g, when the bh pair are shorted (mhos $\angle$).
G_{af}	admittance measured at terminals a, g, when the b, h, pair are freed (mhos $\angle$).
G_{bg}	admittance measured at terminals a, g, when the b, h, pair are shorted (mhos $\angle$).
G_{bf}	admittance measured at terminals b, h, when the a, g, pair are freed (mhos $\angle$).
g	admittance of the staff leak in an equivalent T (mhos $\angle$).
g_1, g_2	admittance of the two pillar leaks of an equivalent π (mhos $\angle$).
$gd\ \theta$	gudermannian of a real hyperbolic angle θ. (Circular radians).
θ	hyperbolic angle, real or complex (hyperbolic radians or hyps $\angle$).
$\theta_A, \theta_B, \theta_C$,	hyperbolic angles of a network in three successive aspects (hyps $\angle$).
m, n	real numbers, integral or fractional for equislope impedances, also m a complex number for case of two equal pillar impedances.
ν	admittance of the architave of an equivalent Π or delta (mhos $\angle$).

ν_1, ν_2	admittance of the arms of an equivalent *T* or star (mhos $\angle$).
Π	product of three impedances forming the sides of an equivalent delta (ohms^3 $\angle$).
Π'	product of three admittances forming the branches of an equivalent star (mhos^3 $\angle$).
Π	a delta connection of three impedances simulating the properties of a network at three terminals.
$\pi = 3.14159\ldots$	
$Q = \dfrac{q \cosh\theta - 1}{\sinh\theta}$	a factor in the determination of an equivalent *T* or Π (numeric $\angle$).
$\vartheta = \dfrac{\frac{1}{q}\cosh\theta - 1}{\sinh\theta}$	a factor in the determination of an equivalent *T* or Π (numeric $\angle$).
$q = \sqrt{z_{oa}\ z_{ob}}$	inequality ratio of a system.
q_A, q_B, q_C	inequality ratios of a three-terminal system in its three successive aspects. (Numeric $\angle$).
$\mathcal{R}$	impedance of a *T* staff leak (ohms $\angle$).
R_1, R_2	impedances of the pillar leaks of a Π (ohms $\angle$).
R_{af}	impedance of a network measured at terminals *a*, *g*, when freed at *b*, *h*, (ohms $\angle$).
R_{ag}	impedance of a network measured at terminals *a*, *g*, when shorted at *b*, *h*, (ohms $\angle$).
R_{bf}	impedance of a network at terminals *b*, *h*, when freed at *a*, *g*, (ohms $\angle$).
R_{bg}	impedance of a network measured at terminals *b*, *h*, shorted at *a*, *g*, (ohms $\angle$).
ρ	impedance of the architrave of a Π (ohms $\angle$).
Σ	sum of three impedances forming the sides of a Π or delta (ohms $\angle$).
Σ'	sum of three admittances forming the branches of a *T* (mhos $\angle$).
$y_{ab} = 1/z_{ab}$	surge admittance of a system (mhos $\angle$).
z_o	surge impedance of a symmetrical system (ohms $\angle$).

z_{oa}, z_{ob}	surge impedances of a dissymmetrical system from the a, g, and b, h, terminals, respectively (ohms ∠).
$z_{ab} = \sqrt{z_{oa} \cdot z_{ob}}$	geomean surge impedance of a dissymmetrical system (ohms ∠).
$\lvert z \rvert$	size or modulus of a complex quantity z (numeric).
$\underline{z}$	slope or argument of a complex quantity z (radians or degrees.)

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CONTRIBUTION FROM THE GEOLOGICAL DEPARTMENT OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASS.

THE GRANITES AND PEGMATITES OF CAPE ANN, MASSACHUSETTS.

By Charles H. Warren and Hugh E. McKinstry.

With Five Plates.

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CONTENTS.

	Page.
Introduction	315
Part I. *Granitoid Rocks of Cape Ann, Mass.*	
Previous Geologic Work	317
Field Relations	318
Other Rocks in the Area, chiefly Later Dike Rocks	320
Petrographic Descriptions	321
Cape Ann Granite	321
Bay View Granite	325
Nordmarkitic Granite	326
Fine-granite	331
Nordmarkite of West Gloucester	334
Inclusions	337
History of the Intrusion of the Igneous Rocks	339
Part II. *Pegmatites of Cape Ann, Mass.*	
Types and Occurrence	341
Mineralogy of the Pegmatites	344
Paragenesis of the Pegmatite Minerals	354
Bibliography	357

Introduction.

The igneous rocks of New England have been the subject of study from a number of localities and the results obtained have furnished petrologists with material of considerable interest and importance including a knowledge of several new rock types. The glaciated island of Cape Ann (Plate I shows the location of Cape Ann on the New England coast and also the distribution of the principal formations in eastern Massachusetts), with its margin bared by wave erosion and its extensively exploited quarries, offers exceptional opportunities for the study of the rock types found there. Attention was long ago directed to the locality both because of the commercial importance of the granites and the mineralogic interest of the minerals found in the granites and associated pegmatites.

It is the purpose of the present paper to describe the more important types of granitoid igneous rocks and the pegmatites exposed on the island of Cape Ann, to note their field relations and to discuss in so far as the facts available warrant, the history of the intrusion of these rocks and their genetic relations.

In stating the facts herein set forth, the authors have drawn freely on the publications of previous workers in this field. They have, however, gone over the entire area personally and have restudied the various rock types in the laboratory and while many of the statements regarding petrographic details may be found elsewhere, the attempt has been made to present a comprehensive statement of the petrology of the entire area and to add to preëxisting data considerable new material, particularly with regard to the relation of the various types. Earlier work on the area appears to have left much to be desired in this latter respect.

The rocks here described form a part of a great intrusion of igneous rocks which appear to have originated simultaneously, or approximately so, at a considerable number of points throughout New England and adjoining parts of Canada, and which taken together have been appropriately designated as the "Novanglian Petrographic Province" by Washington and Pirsson.[1] The rocks are in the main granitoid types, but in part also of porphyritic or other texture. The latter are in the main intrusive as dikes, or occur as marginal phases; some of them are extrusive in character. They are predominately high in silica, alkalies and ferrous iron. Lime is in general subordinate while the magnesia is conspicuously low.

These rocks are intrusive through an older series of granite, diorites and gabbros and their associated sedimentary rocks, quartzites, schists, etc. They are overlaid in the immediate vicinity of Boston by carboniferous rocks, conglomerates, slates and the contemporaneous volcanics. They may be of late Devonic or early Carboniferous age.

Many trap dikes, mostly if not entirely diabasic in character, and possibly of Triassic age, cut all of these rock formations.

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[1] See Amer. Jour. Sci., vol. 22, p. 514, 1906.

PART I.

Granitoid Rocks of Cape Ann, Mass.

PREVIOUS GEOLOGIC WORK.

The rocks of Cape Ann first attracted scientific attention on the mineralogic side. In 1866–67 and again in 1886 Cooke [2] two new mineral species from the Rockport pegmatites. Throughout the mineralogical literature of the period are scattered references to the granitic rocks, notably Wadsworth's [3] description of the biotite and hornblende granites — the latter he described as syenite, in accordance with the older nomenclature. Sears' geologic map in 1895 was accompanied by a description of certain rock types. Shaler,[4] in 1899, published a description of the glacial features of Cape Ann, together with a detailed study of the location of the dikes and joint planes in the granite and a brief description of the igneous rocks.

However, the first modern study of the igneous rocks of the district was published by H. S. Washington [5] in 1898. This admirable piece of work is primarily concerned with the petrographic description of the various rock types found in the general region about Salem, Massachusetts, Marblehead, and the North Shore as far as, and including, Cape Ann. While adding some field observations of his own, he in the main accepts the field relations as given by Sears [6] in his map of the region published somewhat later. Unfortunately, Sears' map is unreliable. Washington established beyond doubt the consanguinity of the rocks studied and the general similarity of the rocks to other alkaline types, not only in New England but to types occurring in Norway and described by W. C. Brogger in his classic papers.

Clapp,[7] in 1910, described the general relations of the igneous rocks of Essex County, devoting detailed study to a special area which did not include, except by incidental reference, Cape Ann. He regards the nordmarkitic granite (quartz-syenite) as a marginal differentiate of the granite magma and considers the fine-granite and pegmatite as apophysal phases of the final granite intrusion. His data on Cape Ann are based partly on a reconnaissance of the area made by Maxwell and Chipman.[8]

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[7] Clapp, C. H. Igneous Rocks of Essex County, Mass. Manuscript Thesis, Mass. Inst. Technology, 1910. Also U. S. G. S. Bull. 704, 1921.

FIELD RELATIONS.

By far the greater portion of the district is underlain by the Rockport, or normal, granite. The only other rock which occupies any considerable area is the nordmarkitic granite, which so far as we have been able to discover, occurs chiefly along the west coast of the Cape, extending from Squam Light to a short distance north of Lanesville. Besides this narrow strip, a small area, not more than fifteen meters wide, of dark coarse-grained rock occurs surrounded by the granite on Andrew's Point on the extreme end of the Cape. North of Rockport to Andrew's Point small areas of nordmarkitic granite are exposed along the coast and there is a somewhat poorly defined area on Briar Neck near Gloucester.

The nordmarkitic granite alone the west coast could not be traced far inland. Along a valley between Bay View and Lanesville it extends eastward about half a mile from the coast, but farther inland large granite quarries show none. The general absence of exposures of the rock inland may be due not so much to its actual absence as to its relative ease of disintegration which would cause it to be covered with a mantle of soil, while the granite, which everywhere intrudes it, would stand out in bare knobs. The comparative instability of the nordmarkitic phase under weathering is very apparent in the glacial moranes, where boulders of the granite appear fresh, while the other shows an advanced state of disintegration. Yet the continuous exposures of rock along the coast from Folly Cove around Halibut Point to Pigeon Cove show practically no nordmarkitic granite nor is any exposed in any of the numerous quarries, except in the form of altered inclusions. Thus, there are large areas over which the rock does not occur.

Along the west shore, from Squam Light to Folly Cove, the relations of the nordmarkitic phase to the granite are very clearly shown. The nordmarkitic phase is cut by an irregular system of dikes ranging from a few centimeters to three meters or more in width. These dikes consist of granite, fine-granite, and aplite, branching in intricate patterns, and in places the granite is the more abundant of the two rocks, the nordmarkitic rock occurring in the form of angular included blocks. The penetration is so complicated that it is impossible to draw a line between areas of the nordmarkitic granite intruded by granite and areas of granite carrying inclusions, hence boundary lines must be somewhat arbitrarily located. Large inclusions in granite may be seen in vertical sections in a low cliff about half a mile north of Lanesville (Plate III, Figs. A and C) where three large blocks are surrounded on both sides and below by granite. Along the west coast, both rocks are cut by veins of fine-granite, passing into aplite and pegmatite.

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[8] Chipman, K. G., and Maxwell, J. W. Geology of the Cape Ann Granite. Manuscript Thesis, Mass. Inst. Technology, 1908.

Between Rockport and Andrew's Point nordmarkitic granite is exposed along the coast, in the cut along the road, and on the hill above. It occurs here associated with normal granite, a fine-grained porphyritic granite and fine-granite. Contacts here are not sharply defined, and no convincing evidence was here found to show whether or not the nordmarkitic portions were inclusions.

Three-fourths of a mile north of Bay View, a dike of light colored granite cuts the nordmarkitic granite with straight, sharp walls, and in one place sends out a well-marked tongue. This dike graded into fine-granite and at the center bears a large mass of bluish quartz.

At a number of places along the west coast, the nordmarkitic granite contains large angular blocks of a dike rock which usually carries abundant and very large phenocrysts of a dark purple labradorite. These are commonly 6 to 10 cm. long by 2 to 5 cm. wide; one crystal 25 cm. long and 10 to 15 cm. was observed. These porphyry inclusions are elongate in form and usually have two parallel straight walls. There appears to be little doubt but that they are sections of an old dike torn apart by subsequent movement of the nordmarkitic granite into which the dike was intruded. They are cut by dikes of granite and fine-granite which also cut the nordmarkitic phase. (See Plate IV, Figs. A and B).

The relations of the rocks so far described are quite clear. Nordmarkitic granite is cut by basic dike rocks. These are both cut by the granite, fine-granite, and the latter phases, aplite and pegmatite. On Davis Neck, however, a new factor enters. The nordmarkitic granite, as elsewhere, is cut by fine granite and pegmatite, but contains large masses of fine-granite containing areas of pegmatite. A few small masses of fine-granite are entirely surrounded.

Passing to the east shore, we find similar relations on Andrew's Point.

(See Plate II.) The fine-granite here contains much pegmatite, not in dikes, but forming irregular masses of coarse feldspar and bluish quartz. The contact between normal granite and fine-granite (and its pegmatitic phase) is irregular, but essentially a plane surface, dipping eastward, the fine-granite passing below the normal granite and being exposed where the overlying rock has been removed by wave erosion. Farther north, the fine-granite takes on a coarser and less siliceous character, perfectly continuous with the fine-granite to the south, and, like it, containing small areas of pegmatite. In the coarser, more syenitic phase are dikes of a black, highly biotitic dike rock, faulted and torn apart like those on the western coast. (See Plate IV, Fig. C.) The fault planes show no trace of fracture nor shear zones, but are perfectly sealed, indicating that either the faulting took place while the granite was somewhat plastic or that there was reheating subsequent to the intrusion. The basic rock dike is cut by pegmatite veins and in one place a tongue of fine-granite extends into the basic rock.

Both the fine-grained syenitic granite and the basic dikes are cut by a well-defined dike of coarse normal granite five feet wide, having gneissic texture along the margins. One of the basic dike-masses is cut in two by the granite and portions of it abut against the granite dike on both sides. Following the granite dike northwestward, it ends at the contact between fine and normal granite, the latter being lithologically similar to the dike. The contact is nearly vertical and is marked by joint planes, so that it is impossible to tell whether the dike extends uninterruptedly into the main body of the granite or is cut off by a fault.

Other Rocks in the Area, chiefly Later Dike Rocks.

Besides the above-described rocks, all of the rest are dike rocks of clearly later age than the granites, etc. The greater proportion are diabases and these are very numerous indeed as may be seen by a glance at the map made by Tarr and published in Shaler's monograph. They are perhaps of Triassic age and have so far as known no immediate genetic connection with the alkaline rocks.

The huge labradorite porphry dike, occurring at Pigeon Cove, together with one or two other smaller dikes of the same character appear from field relations elsewhere in the adjoining regions where similar dikes occur to be earlier than the diabase dikes. They are the same rock petrographically as the inclusions found in the nordmarkitic granite on the west coast, from which it appears that the magma which gave rise to them was present and was intruded both before and after the intrusion of the alkaline granites. These labradorite porphyry dikes will be described more in detail in a later paper.

Washington has described several dike rocks belonging to the alkaline suite. His solvsbergite from near Andrew's Point, Pigeon Cove and elsewhere, and tinguiaite from near Squam Light, represent the acid pole of a complementary series to which there are doubtless complementary basic dikes among the very many dark dikes which occur in the area. These occur frequently outside the area to the south in Manchester and Salem. Some of these have been described by Washington, Clapp and MacKenzie, in papers already referred to.

There are also in the area several quartz-porphyry dikes which are not, so far as they have been studied, closely related to the alkaline rocks. The most interesting of these occurs on Briar Neck and is remarkable for the inclusion of a basic rock which it contains.

Petrographic Descriptions.

Cape Ann Granite.

This rock, which is the normal granite of the area, has been described in detail by Washington[9] so that its characteristics are well known to petrographers. The present description is given mainly for the sake of completeness, but also to bring out certain additional facts regarding this rock which are of interest particularly with reference to the associated rocks.

While the granite is quite uniform throughout the extensive exposures over the island, it is seen in its most typical form in the large quarries near Pigeon Cove and on the Babson Farm near the extreme end of the Cape. Megascopically it is a medium-grained (5 mm.) hornblende granite of a mottled gray color when fresh. It is on the whole even-grained but with a suggestion of a porphyritic texture.

The minerals are: pearly white or gray alkalic feldspar (66.0%); gray or bluish to smoky quartz (27.0%); black shiny hornblende (7.0%), associated with which is often a variable amount of black mica. Locally there are variations: the dark minerals may be almost absent, the quartz more abundant and the grain finer; the color may be distinctly greenish (Bay View), the dark minerals more abundant, the quartz less, thus grading toward nordmarkite or syenite.

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[9] Washington, H. S., op. cit., pp. 769–794.

The weathering varies with the location. Inland, the feldspar kaolinizes, the hornblende develops limonite, and we get a whitish rock with a brown staining and a rough surface; near the coast, where the salt water acts on it, the dark minerals weather out completely, leaving a slightly pitted but otherwise smooth surface.

The rock makes a strong, very durable and handsome building stone. Its smooth or polished surfaces do not, however, possess the pleasing character of the Quincy granite.

Microscopically, the granite consists essentially of a microcline-albite microperthite, quartz, an alkali-hornblende and lepidomelane. The accessory minerals are: hedenbergite, fayalite, magnetite, ilmenite, zircon, apatite (rare), allanite (very rare), and fluorite; secondary minerals are: kaolinite and limonite.

The main part of the feldspar consists of an intergrowth in about equal proportions of microcline and a plagioclase very near albite. (See Plate V, Fig. C.) The crystals are commonly twinned after the carlsbad, less commonly after the Mannebach law, and are rectangular in habit although the outlines are always minutely irregular. The potassic member commonly shows the characteristic microcline structure adjacent to the albite lammellae, though it is often untwinned. It shows some incipient kaolinization and many minute cavities.

The quartz is wholly xenomorphic, generally shows an undulatory extinction or else is broken down into a mosaic. It usually contains fluid inclusions.

The hornblende is clearly a soda-iron variety, as shown by its optical properties and by the analysis of the rock as a whole. Its optical properties as given by Washington and confirmed by us are: α bright yellowish-green to light brownish-yellow; β dark greenish-brown; γ dark olive green. Absorption, $\gamma = \beta > \alpha$; the extinction $\gamma \wedge C'$ about 30°; optic angle small; optically negative. Its grains are commensurate with the feldspar, but usually have extremely ragged edges with numerous attached shreds of biotite. The biotite extends out into the feldspars, strings of flakes following along fractures away from the hornblende. Some of the hornblende appears to have been very strongly attacked by mineralizing solutions and recrystallized into columnar or radiating aggregates which are associated with more or less biotite. There has doubtless been a removal of the hornblende substance during the process since quartz and biotite often appear in the centers of the hornblende areas and they are at times almost or entirely replaced. The recrystallization has to some extent been accompanied by a change in composition for the γ-ray often assumes a strong blue color with a smaller extinction angle. Occasionally an irregular core of pyroxene, hedenbergite, may be seen in the hornblende; also magnetite and less often fayalite.

Most of the mica of the rock seems to be a secondary biotite, but there are occasionally large crystals which may be primary. It is closely similar to the lepidomelane of the pegmatites.

Hedenbergite and fayalite have been referred to above as present in the hornblende. Both occur rarely as separate crystals of early formation. They appear to be the same minerals that occur in the nordmarkitic phase where they are much more important constituents (see beyond). Clapp mentions diallage and glaucophane, and Washington also refers to a blue amphibole in the granite of Eastern Point, Gloucester, and at Magnolia. We have not been able to confirm the presence of diallage.

A striking feature of the granite is the fine material interstitial to the feldspar crystals. (See Plate V, Fig. C.) It is of granular texture and the grains rarely exceed 0.1 mm. in diameter. The material consists partly of quartz and partly of albite. Where contacts between grains of microperthite are invaded, the microcline member seems to be more readily replaced while the albite plates extend out into the interstitial matter. More rarely the finely granular material is not confined to the boundaries of the feldspars, but occupies narrow areas extending into the microperthite crystals. The texture of these fine-grained areas and seams is strongly suggestive of replacement.

The fine-grained material just described, taken in connection with the straining and granulation of the quartz and the alteration of the hornblende, fayalite, etc., seem to the authors to indicate quite clearly that there was movement in the granite during or immediately following the later stages of consolidation. While the temperature was still high, the residual liquors still present, a period obtained in which reactions, replacements, and recrystallization took place. This action occurred chiefly along the crystal boundaries and along the minute fractures or openings caused by slight movements in the mass as a whole.

A chemical analysis [10] of a specimen taken from the large quarry on the Babson Farm near the end of the Cape has been made for the authors by Professor R. S. Hamilton and is given below together with the well-known analysis by Washington. For comparison there is also given the analysis of the Quincy, Mass., granite by Warren.[11] The specimen analyzed is, in our opinion, very fairly representative of the greater bulk of the granite of Cape Ann. It is notably lower in silica than that analyzed by Washington which came from the large pit of the Rockport Granite Company's quarry near Pigeon Cove. A good deal of the granite in this quarry is, we believe, as a result of our observations, more quartzose, as its analysis indicates, than the general average of the granite of the island as a whole.

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[10] The expense of this analysis as well as that of the other new analyses recorded in this paper was defrayed from a grant made to the senior author from the Bache Fund of the National Academy of Science. The authors desire here to express their thanks to the Academy for this grant.

Chemical Analyses.

	I	II	III
SiO_2	72.56	77.61	74.86
TiO_2	.25	.25	.20
ZrO_2	n.d.t.	n.d.t.	.20
Al_2O_3	13.21	11.94	11.61
Fe_2O_3	.78	.55	2.29
FeO	2.14	.87	1.25
MnO	.05	tr.	.02
MgO	.12	tr.	.05
CaO	.89	.31	.41
Na_2O	3.82	3.80	4.30
K_2O	5.53	4.98	4.64
H_2O+	.25	.23	.31
H_2O-	.04	tr.	.04
P_2O_5	tr.	tr.	tr.
	99.64	100.54	100.18

I. Hornblende-granite, Quarry on the Babson Farm near the Beacon. Cape Ann, Mass. L. H. Hamilton, Analyst.
II. Hornblende-granite, Rockport Granite Company's quarry, Cape Ann, Mass. H. S. Washington, Analyst.
III. Hornblende-granite, Quincy, Mass. C. H. Warren, Analyst.

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[11] Warren, C. H. Proc. Amer. Acad. of Arts and Sciences, vol. 49, p. 227, 1913.

In the quantitative classification the Cape Ann granite falls into the same groups as does the Quincy, viz., Class I, Order 4, Rang 1, Subrang 3. Comparing I and III, while they are obviously alike in type, it may be noted that the total iron is much lower in the Cape Ann granite, and that ferrous iron predominates in the Cape Ann rock, the ferric in the Quincy. The total alkalies are slightly greater (9.35% to 8.94) and the ratio K_2O/Na_2O is 0.96 in I and 0.75 in III, that is to say, the Rockport granite is distinctly more potassic. The lime is higher in I, due doubtless to the presence of some hedenbergite.

The approximate mineral composition of I, II, and III is as follows:

Mineral	I	II	III
Quartz	27.0	35.8	32.1
Microcline	32.8	29.5	27.2
Albite, ab + an	33.2	32.7	34.1
Hornblende, etc.	7.0	2.0	6.6
	100.0	100.0	100.0
Total feldspar	66.0	62.2	61.3

The Cape Ann granite is probably on the average a somewhat less quartzose rock than the Quincy, and is somewhat higher in feldspar. It contains about the same proportion of dark minerals, although these are of different kind, the Quincy granite containing only a riebeckitic hornblende and acmite.

Bay View Granite.

This variation, known commonly as the "Rockport green-granite" is developed in several places on the island and appears to grade gradually into the more normal type. Its largest and best exposed development is on the west side of the island near Bay View. It differs from the Cape Ann granite in appearance chiefly in its color which is distinctly greenish after a brief exposure to the air. It is rather gray than green when freshly broken from large blocks well within the quarry. Its further weathering is so like the regular granite that it cannot be distinguished megascopically from the latter.

Microscopically it is seen to contain less quartz, more hedenbergite and hornblende, but is otherwise quite similar. It is intermediate in many respects between the normal granite and the nordmarkitic phase.

Nordmarkitic Granite.

We give next a description of a type which grades on the one hand toward the normal granite and on the other toward an alkali-syenite. Although in general somewhat richer in quartz, and a little coarser in grain, it resembles in color and in its mineralogy the nordmarkite from Ascuteny Mt., Vt., described by Daly.[12] Clapp has also termed it a nordmarkite. Locally quartz is lacking and the rock is then an alkali-syenite. This rock is of earlier origin than the granite, since the latter cuts it. Washington [13] has described a variation of it lower in silica from near Gloucester as an akerite, and states that, in his opinion, it is about as abundant as the granites in Essex County. We are inclined to differ with him. Certainly so far as Cape Ann is concerned it is distinctly subordinate to the more quartz-rich type. He apparently includes all of what we have here called nordmarkitic granite under akerite. It should be noted here that this rock is quite distinct in occurrence, constitution and appearance from the nordmarkite described by Washington [14] which occurs typically in West Gloucester at Wolf's Hill and to the west of Annisquam.

The nordmarkitic granite is characterized by a greenish-gray color on fresh surfaces, but on exposure it assumes a dull green color which is characteristic. It is of medium to rather coarse grain, and commonly has a very distinct tendency toward a porphyritic texture. The quartz is dark, even oily in appearance, and is distinctly less in amount than in the normal granite. The dark minerals are more abundant. What has been written about the feldspar of the granite applies without essential change to this rock. There is the same extreme raggedness of crystal boundaries and the same interstitial granular material, except that in this rock this material is larger in amount and is in part a true groundmass whose original texture is complicated, and in part destroyed, by movement and granulation and probably by replacement.

The dark minerals are hedenbergite, an alkali-hornblende, fayalite, and a little biotite. The relative amounts of the hedenbergite and hornblende vary considerably in different specimens but the pyroxene often predominates. The hedenbergite occurs in irregular formless grains, often of good size, but again as small crystals, being most abundant as such in those varieties of the rock which possess a more distinct groundmass. The pyroxene shows an almost universal tendency to replacement by hornblende. All stages of this may be seen, from scattered shreds in the body of the pyroxene, or a little about the margins, to almost complete replacement. The hornblende is thus a later mineral formed, we believe, at the expense of the hedenbergite, by a reaction between the material of the pyroxene and the magmatic solutions during a late stage when soda, together with water and other mineralizers, were strongly concentrated. Indeed, we suspect that most, if not all, of the hornblende in all of these rocks may have been formed through a resorption process involving the preëxisting pyroxene.

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[12] U. S. G. S. Bull. 203, p. 59.
[13] Washington, op. cit., p. 796.
[14] Op. cit., p. 799.

The color of the pyroxene is a very light green when unaltered to hornblende. Its extinction is $\gamma = 43°$ measured on 010; optically positive, with a fairly large optic angle (estimated at from 50 to 60 degrees); $\gamma = 1.750$, $a = 1.730$. These properties point to a pyroxene near hedenbergite. This is in keeping with the fact that magnesium is too low in the rock as a whole to permit the presence of more than a very little of the diopside molecule. It is supported by the fact that hedenbergite has also been identified in the associated pegmatites. Washington calls the pyroxene of the akerite a diopside. In all slides examined by us it appears to be also hedenbergite.

The hornblende is of substantially the same character as that of the granite, deeply colored in olive-green tones, though in the akerite and syenitic phases, strong brown tones appear.

The fayalite is perhaps the most interesting mineral in the rock. While subordinate to the pyroxene and hornblende in amount, it is fairly common, although it does not appear to have been previously noted except in the pegmatites. It is present sparingly in the granites as noted, and has the same characteristics wherever found. It occurs in rounded grains, seldom with any definite crystal outline, and is generally closely associated with the hedenbergite, being apparently almost contemporaneous with it in time of formation — that is, an early mineral. It is enclosed in hornblende when this mineral is present. The grains when fresh are colorless. The indices of refraction are very high (over 1.8), the double refraction is strong (ca. 0.050); optically negative. The grains of the mineral are however seldom unaltered and frequently only reaction or decomposition products remain, but these are so characteristic that they afford unmistakable evidences of the original presence of the fayalite. Strong orange colored stains are present in and about even the freshest grains, and usually the mineral is replaced to a greater or less extent by a dark red, and nearly or quite isotrophic material which appears to be its ordinary decomposition product. Besides this decomposition the mineral has often undergone another and more interesting transformation (the same has been noted in the case of the fayalite of the pegmatites). In this type of alteration the fayalite is replaced wholly or in part, first by a compact mass, often with a radiating structure of brightly polarizing fibers of ferroanthophyllite. Outwardly this gives place to a green, pleochroic and monoclinic amphibole probably grünerite (Part II) with an extinction of about 15°. This amphibole occurs often without the ferroanthophyllite as a border about the fayalite grains. It has also been observed that where feldspar borders the amphibole, a distinctly stronger blue color is characteristic of it. Magnetite in minute grains is thickly distributed through the ferroanthophyllite and the monoclinic amphibole. Outside of the amphibole zone comes generally one composed of a dark brown biotite.

The fayalite appears to have crystallized as a stable mineral early in the consolidation of the magma, but became unstable during a later stage, perhaps the same in which the pyroxene also became an unstable phase changing in part at least to hornblende. Under the influence of an increasing silica concentration in the magma solutions, the fayalite alters to ferroanthophyllite or grünerite. The reaction may be a simple addition of silica, but the constant presence of magnetite is suggestive of a more complicated reaction in which the magnetite is a by-product. The alkali, sodium, appears to be involved in the reaction, for we note the presence of a blue-green monoclinic amphibole and this appears to favor the fayalite-feldspar contacts, which suggests that the feldspar was also involved in the reaction.

These reaction products are not always confined to the area originally occupied by the fayalite, but tend to wander out along boundaries and cracks. The red and yellowish decomposition products noted above, also sometimes extend away from the immediate neighborhood of the affected grain, and yellowish or greenish stains extend very generally out along crystal contacts and cracks, giving the entire rock a yellowish-green color. This is apparently the main cause of the peculiar color of the rock which is the same color characterizing the nordmarkite of Ascutney Mt., Vt. Daly [15] discussed the cause of the color in this rock and ascribed it to an oxidation of the ferrous oxide contained in the feldspar to a yellowish-green ferric compound, which, with the natural gray color of the rock, gave it the peculiar and characteristic greenish color. Daly does not mention fayalite, but we have identified fayalite in several sections of the Ascutney rock, and they show much the same type of decomposition as is noted here. We suspect that ferrous iron-bearing solutions emanating from the decomposed fayalite crystals spread through the rock were responsible for the color which is common in rocks of this class.

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[15] Daly, R. A., op. cit., p. 52.

Regarding the accessory minerals we may note that magnetite is common in and with the pyroxene, the hornblende, and fayalite, apparently as an original mineral. Zircon (or cyrtolite) is quite abundant and is particularly apt to occur in the form of numerous crystals in the hornblende, there being sometimes as many as fifteen or twenty crystals in a single hornblende; they have not been noted in the pyroxene. This suggests that the period of zircon formation did not begin until a relatively late stage. It may be stated here that the same thing has been noted in regard to the zircon in the Quincy granite which is, in many respects, closely related to the Rockport granite. In this connection we should also note the occurrence on Andrew's Point of a small dark and rather coarse-grained inclusion in the granite, very rich in black hornblende. The hornblende is literally peppered with small zircon crystals, distinctly recognizable under the hand-lens. We suspect that this zircon was introduced into this inclusion from solutions emanating from the granite.

Apatite is present but never abundant. Fluorite has been noted, as in the granite, usually in the hornblende or about it. Washington does not mention fayalite as being present in the variation of the quartz-syenite which he described, nor have we noted it in the syenitic phases. It seems to be confined particularly to the types richer in quartz. Washington mentions titanite as common in the akerite. This mineral has been identified only rarely in our study and appears to be present only in quantity in those phases of the rocks which are lower in their silica content (outside the present area).

Chemical Analyses.

	IV	V	VI	VII
SiO_2	69.42	68.36	66.60	65.43
TiO_2	.20	tr.	.76	.50
Al_2O_3	15.15	16.58	15.05	16.11
Fe_2O_3	1.49	.90	1.07	1.15
FeO	3.50	3.24	4.42	2.85
MnO	.10	tr.	tr.	.23
MgO	.09	.45	.36	.40
CaO	.96	1.85	2.21	1.40
Na_2O	3.35	3.97	4.03	5.00
K_2O	5.12	5.27	5.42	5.97
H_2O	.47	.35	.41	.49
P_2O_5	tr.	tr.		.33
Miscell.				.28
	99.85	100.97	100.33	100.14

IV. Nordmarkitic hornblende-granite, Cape Ann, Mass. F. L. Hamilton, Analyst.

V. Nordmarkite, Wolf's Head, near Gloucester, Mass. H. S. Washington, Analyst (op. cit., p. 800).

VI. Akerite, near Prospect Street, Gloucester, Mass. H. S. Washington, Analyst (op. cit., p. 798).

VII. Nordmarkite, Ascutney Mt., Vt. Hildebrand, Analyst (op. cit., p. 59).

The calculated mineral composition of the nordmarkitic granite (II), also that of the normal granite (I) is —

	I	II
Quartz	27.0	24.4
Microcline	32.8	30.0
Albite	33.2	32.5
Hornblende		
Pyroxene	7.0	13.1
Fayalite		
	100.0	100.0

Compared with the normal granite we see that the quartz is lower by about 3%, the feldspar by about 4%, while the dark minerals are decidedly higher, being nearly twice as abundant. The silica is somewhat higher than the limit set by Brogger for the nordmarkite, which was 67%, and although the rock is undoubtedly related in type to nordmarkites like that of Ascutney Mt., Vt., it contains so much quartz that it must be classed with the granites. On the other hand it is very closely related to the rocks from Wolf's Hill (V) and from Prospect St., Gloucester (VI), the former called a nordmarkite by Washington and the latter akerite. Its relations to the Wolf's Hill rock will be discussed later.

Fine-granite.

According to its mode of occurrence the fine-granite may be divided into four groups as follows:

First, large areas which seem to be islands in the granite such as the one on Andrew's Point which measures some hundred meters in diameter. Smaller ones occur on Davis Neck and along the coast near Lanesville and on Briar Neck near Gloucester.

Second, as inclusions varying from small patches a few centimeters in diameter to ones two or three meters across. These are found in the normal granite in many places and probably differ from the larger masses grouped under the first group only in size.

Third, forming dikes in the granite and nordmarkitic granite in several localities.

Fourth, forming a marginal zone about the pegmatite. This occurs with the pegmatite when the latter is found as rounded knots or patches in the granite as well as in dikes.

Group 1. The fine-granite on Andrew's Point varies in character within the exposure, being fine-grained and highly quartzose at the southern end and coarser and quartz-poor toward the north, where it eventually appears to pass into a quartz-syenite carrying a considerable amount of dark silicates. In the syenitic phase are found disjointed dikes of a mafic rock. This fact is of interest and importance when considered in connection with the disjointed dikes of labradorite porphyry occurring in the nordmarkitic granite along the Lanesville shore previously referred to. This will be again considered when discussing the general relations of the granitoid rocks of the area. Only the extreme types here noted will be described, but all gradations between them may be found.

The fine-grained, silicious phase is white to light gray in color, weathering brownish or reddish-yellow. The hand specimen shows a somewhat irregular size of grain but has what may be termed a sugary texture. The feldspar is nearly white and the quartz rather bluish, both varying in size from very minute grains to those 5 mm. in diameter. The hand specimens frequently show patches of relatively coarse quartz and feldspar and not uncommonly composite patches of biotite occur. In the field a similar feature appears, where areas of coarse pegmatite a meter or more across are developed in the fine granite. In general there appears to be no definite boundary between the coarser and finer portions although in places the pegmatite occurs in angular areas with moderately sharp contacts against the fine-granite. (See Plate III, Fig. D.)

Scattered through the fine-grained portions are small flakes of biotite and hornblende. The amount is always small but these minerals appear to be never wholly absent. In the coarse, or pegmatitic portions, hornblende and biotite crystals 3 cm. or more in their largest dimension are found.

Thin sections of the finer portions show under the microscope that the size of grain and the texture are extremely irregular, the grain size ranging from 0.2 mm. to 2 mm. The essential minerals are quartz, microperthite, hornblende, and biotite. The hornblende and biotite occur in tiny flakes or shreds scattered through the rock. The feldspar is a microcline-albite microperthite, in part much crushed and disturbed and in part intergrown graphically with the quartz. Over a part of a thin section, this graphic intergrowth may be particularly striking in appearance. The quartz is rounded, subangular in outline, or in irregular rounded grains, similar to the coarser graphic-granite of the pegmatite. The intergrowth passes from one microperthite crystal to the next without apparent relation to the orientation of the feldspars. Quartz is also developed along the interstices between the feldspars and along the borders.

Biotite and hornblende occur in tiny shreds or flakes but are not associated with each other. Their appearance suggests that they have in many cases been derived from larger grains which have been torn apart and perhaps partly destroyed.

In the finer-grained, syenite phase, the grain is quite even and the mafic minerals are decidedly more abundant than in the fine granite. The dark minerals occur generally in patches one or two millimeters in diameter, each patch consisting of fine biotite flakes or hornblende crystals with more or less biotite. In the field there are no distinct boundaries between the coarse and fine phases. The coarse phase is cut by irregular veins of aplite and pegmatite and by dikes of the normal granite.

In thin sections, the fine-grained quartz-syenite is seen to consist, like the granite, of quartz, microperthite, hornblende and biotite. The dark silicates are much more abundant than in the fine granite and the intergrowth of quartz and feldspar is much less pronounced. While there are occasional feldspar crystals from 2 to 3 mm. across, most of the feldspar is much finer (0.2) and is granulated as if from crushing. The quartz grains run from 1 to 2 mm. in diameter, are angular and interlock with the feldspar. Hornblende is quite abundant and shows what suggests in appearance an advanced alteration to biotite; the latter penetrates along feldspar crystals and forms tiny replacement veins in it. Zircon is associated with the mafic minerals as an accessory.

Group 2. The inclusions of fine-granite seem to be identical in mineralogical character with the larger masses like that just described, and we believe they have the same origin. Their outlines are sharply angular to subangular or irregular and they exhibit an irregularity of grain, frequently showing patches of truly pegmatitic character.

Group 3. The fine-granite occurring as dikes cutting both the granite and the nordmarkitic granite is mineralogically similar to the fine-granite under Group 1, although it is even in grain. It appears to be highly quartzose, an observation borne out by Washington's analysis (No. X beyond) of an alpite dike from Bass Rocks, Gloucester.[16]

Group 4. The fine-granite occurring about the pegmatite knots and as margins to certain pegmatite dikes is very similar to the fine-granite of Group 1. It will be more properly discussed under Part II which deals with the pegmatites.

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[16] Op. cit., pp. 106–108.

Chemical Analyses.

	VIII	IX	X
SiO_2	75.65	67.50	77.14
TiO_2	.10	.20	.29
Al_2O_3	13.72	16.50	12.24
Fe_2O_3	.81	.79	.29
FeO	1.00	2.76	1.04
MnO	.02	.06	tr.
MgO	.04	.09	.06
CaO	.34	1.11	.35
Na_2O	3.22	3.89	4.64
K_2O	4.30	6.09*	4.47
$H_2O +$	.21	.32	
$H_2O -$	.12	.42	.14
(F)	(.10)	(.14)	
	99.63	99.73	100.66

* This seems too high.

VIII. Fine-granite, Andrew's Point, Cape Ann, Mass. F. L. Hamilton, Analyst.
IX. Quartz-syenite phase of fine-granite, Andrew's Point, Cape Ann, Mass. F. L. Hamilton, Analyst.
X. Aplite compound dike, Bass Rocks, Gloucester, Mass. H. S. Washington, Analyst.

The fine-granite resembles the more siliceous phases of the Cape Ann granite chemically, as does also the aplite dike. The syenitic phase is lower in silica and resembles the nordmarkitic phase, particularly that described by Washington from Wolf's Hill, Gloucester.

Nordmarkite of West Gloucester.

On Shaler's map of Cape Ann he outlines a considerable area of what he terms, very properly as a field term, diorite. As seen on the map this rock extends across the river for some distance to the west into Annisquam. It is well and typically exposed on the island of Cape Ann at Wolf's Hill and all along the east side of the river. Washington who described this rock from specimens collected at Wolf's Hill called it a nordmarkite. It differs in mode of occurrence and appearance from the nordmarkitic granite which is associated with the Rockport granite elsewhere on the island. It differs in appearance both in the hand specimen and under the microscope from the syenitic phase of the fine-granite of Andrew's Point although it resembles this rock closely in chemical composition. From our examination of the exposures and study of this rock and its variations we are unable to arrive at any satisfactory conclusion as to just what its relations to the other igneous rocks of Cape Ann are. That it is genetically connected with them seems likely, but whether earlier, later, or contemporaneous with them we cannot determine.

The eastern part of Wolf's Hill, which is a bold, rounded hill of rock, consists of Rockport granite. Toward its western side and running across it in a generally northerly direction the granite passes suddenly but without any discontinuous contact into a strongly and rather coarsely porphyritic rock, and as we pass away from this contact to the west within a few meters the porphyritic rock gives way, also without sharp contact, to a fine-grained dark rock of dioritic appearance, which is the rock called diorite by Shaler. This rock is, however, not homogeneous, but consists of two quite distinct types. One of these is quite dark gray in color and shows hornblende alone or biotite and hornblende as the dark mineral. The other clearly cuts the former, occurring as irregular stringers and masses in it, is much lighter in color, rather finer in grain, and contains only biotite as a dark mineral.

Microscopic study shows that the porphyry has the following characteristics: coarsely and rather strongly porphyritic; phenocrysts of white unstriated feldspar, rectangular and measuring from 3 mm. to 1 cm. on a side; a few black hornblende prisms; groundmass, light gray and rather fine with biotite as the dark mineral. The phenocrysts are a cryptoperthite or microperthite, the perthitic structure being more distinct about the margins. Small rounded quartz crystals are included in the feldspars in a zone about the margin. The groundmass is fine and quite irregular in grain and consists of microcline, albite or oligoclase, quartz and biotite. The quartz forms rounded grains and is included in the microcline. Hornblende has been noted in some sections.

Nordmarkite. A typical specimen collected from a small island opposite Wolf's Hill, on the Little River, shows an even and rather fine-grained, light gray rock. An estimate of the composition is as follows: microcline 35%, sodic plagioclase 40%, quartz 12%, hornblende 12%. The microcline is in large rectangular crystals finely twinned and contains very fine lammellae of soda feldspar perthitically intergrown. The plagioclase forms rectangular to lath-shaped crystals which are automorphic against the microcline and quartz. Its indices lie between that of the balsam and quartz, the extinction is nearly zero, indicating an oligoclase. Hornblende occurs in well-formed prisms, is pleochroic in dark olive green to yellowish-brown tones and shows an extinction of about 21 degrees. Very little biotite is present.

Another specimen collected from a ledge on the eastern shore of the Little River west of Wolf's Hill and representing the darker-colored variation of the rock, shows the same feldspars but with much less microcline and with biotite intergrown with the hornblende as in many typical diorites. An estimate of the mineral percentages made on one specimen gave: microcline 6%, oligoclase 58%, hornblende and biotite 26%, quartz 10%. The plagioclase is somewhat zoned in this type and the microcline crystals are rimmed with plagioclase.

Lighter colored type. A specimen of the lighter colored rock which cuts the darker was collected from the island west of Wolf's Hill on the Squam River. Besides being lighter in color, this rock is finer in grain. An estimate of its mineral composition gave: quartz 20–25%, microcline 70–75%, oligoclase 5%, biotite 5%. The microcline and quartz appear to be more abundant; the plagioclase is included in the microcline and is considerably seriticized. Epidote and allanite were noted, and biotite, but no hornblende is present.

These rocks differ so strongly in their characteristics from the other rocks of the area that we have a strong suspicion that they represent a separate intrusion closely connected in time with the intrusion of the Rockport granite, and probably preceding it. The absence of sharp contacts perhaps favors the view that the great mass of the Rockport intrusion engulfed an earlier rock, just as it did the nordmarkitic granite and fine-granite of Cape Ann as described by us, obliterating sharp contacts and possibly causing the development of the phenocrysts of the contact zone. This is of course purely speculation and we offer it only as a suggestion of a relationship which is in any case extremely puzzling. The rock seems to connect itself chemically (see Analyses V and VIII) most closely to the syenitic phase of the fine granite on Andrew's Point. Washington mentions a porphyritic type from Hospital Point near Beverly and on Salem Neck. We have seen in several localities a very similar rock forming dikes which are in turn cut by the alkaline syenite. This occurrence would also lead to the conclusion that the West Gloucester rock is of earlier origin than the main mass of the alkaline granites.

Inclusions.

The normal granite contains inclusions varying in size from a few centimeters to many meters. The most common varieties will be described under the following types:

I. Fine-granite.
II. Pegmatite, with and without fine granite.
III. Nordmarkitic granite.
IV. Fine to medium-grained dark inclusions.
V. Fine-grained, black, highly mafic inclusions.

Fine-granite inclusions. The material composing the inclusions of this type has already been described. The contacts are often very sharp and angular; again the rocks appear to have welded so that the contact is rapidly transitional and more or less rounded. As indicated in our discussion of the relation of the fine granite to the coarse granite, we are inclined to the opinion that all of the fine granite, except that which occurs definitely as dikes has the same origin, viz., is included in the granite magma.

Pegmatite inclusions. One remarkable inclusion of very coarse pegmatite has been found in the granite in the quarry on the Babson Farm near the end of the Cape. This was first noted by Professor Charles Palache of Cambridge and from it came the remarkable crystals of fayalite now in the Harvard University collection. This inclusion was sharply rectangular in outline and must have originally been of such size as to weigh about two tons. It consisted essentially of huge crystals of quartz and microperthite, and contained also a few large crystals of fayalite and hedenbergite, together with black mica and rare accessory minerals. Other inclusions containing pegmatite have been noted, but these have all been largely composed of fine granite. They lack the zoning characteristic of the pegmatitic schlieren found in the granite and are essentially identical with the fine granite with its pegmatite described as occurring on Andrew's Point. They are thought to have the same origin.

Nordmarkitic granite inclusions. The invasion of the nordmarkitic granite by the granite, particularly well exposed on the Laneville shore, has been described earlier. The invasion resulted in a huge breccia in which all stages of immersion in the granite may be seen. The smaller knots under Group IV are undoubtedly most of them of the same origin.

Fine to medium-grained dark inclusions. In nearly all of the quarries in the granite are found angular to rounded patches of material, darker and finer grained than the granite. Some are evenly fine-grained, others show a porphyritic texture, due to the presence of rather large tabular feldspars in a fine groundmass. A few are comparatively coarse-grained and resemble the nordmarkitic granite. A series was collected showing all gradations from this latter type to the fine-grained type. The coarser type is found under the microscope to be identical in all essential particulars with the nordmarkitic granite previously described. Without going into detail, it may be said that a microscopic study of the various types exhibited by these "knots" indicates that they are inclusions of the nordmarkitic granite in the granite and that the differences in texture and present mineral composition can all be ascribed to the metamorphosing action of the granite magma on the original inclusion. The process of metamorphism as illustrated by this series is as follows: first, development of fine-grained quartz, albite, and microcline along boundaries between the larger microperthite crystals, together with the alteration of the hedenbergite to hornblende and biotite and of fayalite to amphibole, etc., and the mechanical crushing of the feldspar and quartz grains; second, replacement of microperthite crystals at the edges by quartz-albite material and fine-grained biotite; third, development of quartz-albite-biotite veinlets within the microperthite crystals, accompanied by further alteration of the hornblende; fourth, complete disappearance of the large feldspar grains and a further development of biotite.

Washington described inclusions of this type. He analyzed a "rather dark fine-grained specimen from Pigeon Hill quarry" and observes that "from its analogy with the analysis of rocks described below, it might be called a quartz-syenite." We give Washington's analysis together with his analysis of the nordmarkite from Wolf's Hill and ours of the nordmarkitic granite. Their similarity in chemical composition is obvious.

Chemical Analyses

	XI	IV	V
SiO_2	67.35	69.42	68.36
TiO_2	0.60	0.20	tr.
Al_2O_3	15.05	15.15	16.58
Fe_2O_3	1.23	1.49	0.90
FeO	4.76	3.50	3.24
MnO	0.05	0.10	tr.
MgO	0.03	0.09	0.45
CaO	0.55	0.96	1.85
Na_2O	4.42	3.35	3.97
K_2O	6.08	5.12	5.27
H_2O −	0.18	0.05	0.35
H_2O −	0.17	0.42	
	100.45	100.35	100.97

XI. Inclusion from Pigeon Hill quarry. H. S. Washington, Analyst.
IV. Nordmarkitic granite, Cape Ann, Mass. L. H. Hamilton, Analyst.
V. Nordmarkite, Wolf's Hill, West Gloucester, Mass. H. S. Washington, Analyst.

Dark fine-grained inclusions rich in mafic constituents. These inclusions, several types of which occur in the granite, have not been studied microscopically. They are fine-grained, sometimes feebly porphyritic, and always are very rich in biotite, and sometimes zircon. Their appearance leads us to believe that they are all of them inclusions more or less metamorphosed of basic dike rocks similar in origin to the fragments of faulted and broken dikes found in several localities and earlier referred to.

History of the Intrusion of the Igneous Rocks.

The intrusion of the igneous rocks of Cape Ann appears to have begun with an upwelling of a magma which solidified as the nordmarkitic granite although in places (Andrew's Point) it may be akeritic or even syenitic. It appears to have solidified sufficiently to have allowed of the intrusion of the labradorite porphyry dikes and others which, as noted, are now found in it. Following this, probably at no great interval, came the intrusion of the main mass of the normal granite. This invaded the nordmarkitic granite, breaking it up and including it together with its contained dikes. Portions of it were doubtless entirely engulfed. It seems also necessary to believe that the nordmarkitic granite was rendered sufficiently plastic as a result of this intrusion to permit of its flowing and filling in about the disjointed blocks of the basic dike rocks now found in it.

Associated with the nordmarkitic granite intrusion there seems to have been a fine-granite phase, for fragments of fine-granite passing into nordmarkite or syenite are found included in the normal granite. There was also a pegmatitic phase, apparently associated with the fine-granite, for inclusions of pegmatite with and without fine-granite are also found included in the granite.

There was also a pegmatite fine-granite phase of the normal granite, of which we find evidence in the numerous blebs of coarse pegmatite with their margins of graphic-granite and fine granite scattered through the normal granite in many localities, also in the dikes and veins of pegmatitic character found in the granite, particularly in some of the quarries. There also seems to have been a later intrusion of aplite following the main granite intrusion.

Subsequently came the intrusion of the solvebergites and complementary dikes genetically related to the alkaline rocks, and still later came certain quartz porphyries of unknown relationships, and the distinctly later diabase dikes which appear generally throughout Essex County. As Clapp has shown, the alkaline intrusion in Essex County was opened by the extrusion of alkaline volcanics which are so strongly developed about Lynn, Mass., and Marblehead Neck, but all trace of any such earlier stage has disappeared from Cape Ann if it were ever present.

We may note here that there is good evidence of two distinct periods in the intrusion in the granitoid rocks along the West Manchester and Beverly shore, such as we have noted for the Cape Ann area, and the same doubtless applies to Salem and Marblehead. On the Beverly shore there are included in the coarse alkaline syenite which occurs there, basic dikes which are faulted and moved apart in exactly the same manner as are the basic dikes of labradorite porphyry near Lanesville in the Cape Ann area. The syenite here is also cut by granite. While at Beverly and Salem the history of the intrusion of the main alkaline rock types is doubtless more complicated than on Cape Ann, there is evidence, we believe, of two major periods of intrusion, an earlier one which consisted of nordmarkitic or an alkali syenite type, and a second one of a siliceous alkaline granite of the

Cape Ann, or what Clapp has called the "Peabody," type. These rocks are essentially the same, and in view of the greater importance of the Cape Ann granite commercially and the fact that it is well known as a petrologic type, we venture to suggest that it would be better to use the name Cape Ann, instead of Peabody, as a descriptive term when discussing the igneous rocks of Essex County.

PART II.

Pegmatites of Cape Ann, Mass.

The mineralogy of the pegmatites which occur with the alkali-granites of Cape Ann, Massachusetts, has been the subject of a number of papers by various authors. Recently, one of the authors (McKinstry) published a summary account of the minerals of these pegmatites.[17] It is the purpose of the present paper to deal somewhat more fully with these interesting mineral occurrences.

The authors wish to express here their indebtedness to Professor Charles Palache of Cambridge, Mass., for permission to make use of a number of observations which he has from time to time made on these pegmatites, and for the loan of a number of specimens from the Harvard collection. His kindly interest in the study of the minerals has also been of material assistance to us.

TYPES AND OCCURRENCE.

We may classify the pegmatite occurrences under the following heads:

I. True dikes with well-defined walls.
II. Dikes or veins without well-defined walls.
III. Rounded masses with margins of fine granite enclosed in the granite.
IV. Angular or sub-angular blocks included in the granite with or without fine-granite.

I. True dikes with well-defined walls are found in several localities. The dikes are all small, being with one exception, to be referred to below, measured in centimeters, and, with the same exception, consist of a central zone (a) of bluish, opalescent or smoky quartz, (b) a zone of pure microperthite feldspar, (c) a zone of graphic-granite often containing large lepidomelane crystals, and (d) an outer zone of fine-granite. (See Plate III, Fig. B.) Zones (c) or (d) may be lacking. For example, a pegmatite dike on Andrew's Point has a border-zone of beautiful graphic-granite one-half meter wide containing a little biotite; in another type the marginal phase consists of a fine-grained feldspar-biotite rock with little quartz, the feldspar being lath-shaped and the biotite abundant in tiny well-distributed flakes. Again, the quartz and feldspar may form a graphic intergrowth and the biotite occur in plates several centimeters in diameter, but without any particular orientation. The microscope shows that the fine-granite consists essentially of quartz, microcline or microperthite, a little albite and biotite. Pneumatolytic minerals such as fluorite and molybdenite may be present. The quartz is very often enclosed in the feldspar and there is always a strong tendency toward the graphic structure.

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[17] American Mineralogist, vol. 6, p. 56, 1921.

The exception referred to above is that of coarse-grained pegmatite some 6 meters long by ¾ of a meter thick which occurs in the granite on Briar Neck near Gloucester. This pegmatite shows no zonal structure and is noteworthy because it contains numerous large crystals of fayalite or its alteration products. Much of the fayalite is now strongly altered, but one large crystal perhaps three kilograms in weight was broken out with the hammer and a mass of unaltered mineral weighing about two kilograms was secured from the central part of the fragment. Whether this pegmatite is a true dike as it appears to be, or whether it is a great inclusion like that found in the Babson Farm quarry cannot be told with certainty from the outcrop. We suspect that it is an inclusion although it must be admitted that had we never seen the inclusion in the quarries we should not have thought of this pegmatite as being anything but an ordinary dike. If it is a true dike, it is unique in the area, both as to size and structure.

II. Narrow pegmatite veins or streaks are occasionally encountered in the normal granite. Several have been well exposed for observation in the quarries. They are generally irregularly continuous for long distances, being exposed from the bottom to the top surface of the quarry walls which in places are as much as 30 meters high. They have the appearance of being seams or openings in the granite through which mineralizing solutions or vapors have passed depositing mineral material and effecting a recrystallization and modification of the granite along their course. This mode of formation seems more plausible to the writers than to suppose that a pegmatite solution has been forced as such into the rock and then crystallized into its present form. Their margins appear generally to lack sharp definition and to merge with the granite. The granite on one or both sides often shows strong evidences of the action of modifying solutions. It is from these veins that many of the green microcline crystals commonly found in collections have come.

III. In many places in the granite small spots may be noticed which consist of a center of coarse quartz into which penetrate crystals of microcline. When these spots attain a larger size, a zonal structure may in general be seen. The zoning is of the same type as that described above for the zoned dikes. One finds about the same variations in different spots, and the mineralogy appears to be substantially the same as in the dikes. In size they will vary from those a few centimeters in diameter to those of relatively large masses nearly a meter across. The margin of granitic material is in general relatively narrow, while the quartz center makes up the greater part of the mass. They appear, therefore, as blebs of coarse quartz margined with granitic material which outwardly merges suddenly but almost insensibly into the surrounding granite.

In the granite of Andrew's Point there is a large development of fine-granite containing much pegmatite. In part this appears to have dike-like relations to the granite and might come under Group I; again its outlines and contacts suggest that it is a large included block or blocks, or possibly huge schlieren of irregular shape. We are inclined to favor the idea that it is an included block, particularly in view of its close association with patches of the nordmarkitic granite and its strong resemblance to some undoubted inclusions in the near-by quarries.

In the narrower portions of this fine-granite the quartz and coarse feldspar seem to occupy a central position with fine-granite outside, but where the fine-granite is greater in amount the coarse-grained portions seem to occur without any particular method of arrangement. They form isolated areas of varying size up to those over a meter across (see Plate III, Fig. D), with sharp and somewhat irregular contacts, and consist of large microperthite and quartz crystals related to each other texturally just as in the ordinary granite the quartz in general predominates in amount and is of the opalescent variety. All together they present a rather striking appearance. While one is conscious of the very great contrast in grain as compared with the fine-granite host one cannot altogether escape the impression that the difference must after all be brought about by some rather slight differences in physical conditions at the time of crystallization; the fine-granite always seems about to pass into coarser pegmatite for one notes here and there a sudden variation in the fine-granite in the direction of a coarser texture.

IV. Angular or subangular inclusions of fine-granite and of fine-granite with a variable amount of pegmatite are found in the normal granite, and one large and remarkable inclusion of coarse pegmatite has been found, as noted earlier. These are undoubtedly inclusions of a pegmatitic phase which accompanied or followed the first invasion of the alkaline magma, which as noted, was in the main nordmarkitic in character.

Attention was first directed to the large inclusion of coarse pegmatite in the granite of the quarry on the Babson Farm by Professor Charles Palache. The best specimens from it are now in the mineralogical collection of Harvard University. This inclusion was rectangular in outline and its original dimensions must have been 2 by 1 by 1 meters. It consisted of very large crystals of quartz and microcline-microperthite with occasional large crystals of fayalite and hedenbergite, or their decomposition products, lepidomenale together with accessories such as ilmenite and several rare minerals. The material of this included mass is the same as that of the fragments of fayalite with attached feldspar and quartz which were found in the Rockport quarry many years ago and described by Penfield and Forbes and somewhat later by Warren. This material, though never seen in place, is, from its appearance and the attached granite, believed to be an inclusion similar to the one described above.

MINERALOGY OF THE PEGMATITES.

Beyond is given a complete list of the minerals occurring in these pegmatites. Many pegmatite minerals typical of the less alkaline rocks are entirely absent here. Among these many be mentioned tourmalin, beryl, lepidolite, garnet, and muscovite. The mineral association is that of an alkaline granite pegmatite in which the iron is high, magnesium low, and boron absent. Hence we have lepidomelane in place of muscovite or biotite, cryophyllite in place of lepidolite, and danalite instead of beryl.

Analogous localities. The pegmatites of Quincy, Mass., occurring in a granite closely related to the present one in type, present certain features in common but also some striking differences. The Quincy pegmatites carry acmite and riebeckite without lepidomelane; the

Cape Ann, alkali-hornblende, hedenbergite, lepidomelane, and fayalite. Structurally the main pegmatites of the Quincy granite are pipe-like and have open cavities, features not found on Cape Ann. The zonal arrangement as described by Warren[18] for the Quincy pegmatites is similar however to that found at Rockport, and like the latter, the Quincy also carries rare-earth minerals, although different in type.

Some resemblance is indeed to be expected in similar stocks or batholiths of the same petrographic province. It is interesting to note, however, the analogy in occurrence at a remote locality, the famous Barringer Hill in Texas, which seems to be a pegmatite body somewhat analogous to the Rockport type but magnified to enormous proportions, as the following extract from a description of the Texas locality will show:[19]

"At the edge of the intrusion is a graphic-granite of peculiar beauty . . . the feldspar is an intergrowth of microcline and albite, (1) in large masses . . . (2) as huge crystals, many of which have one or more sharply defined edges, especially where surrounded by quartz.

"Ilmenite occurs in radiating bunches of sheets or blades. The mica . . . is reported to be close to lepidomelane in composition. Small flakes of lithia mica are found" (whether this is lepidomelane or cryophyllite is not stated) "no muscovite was seen, though it is said to be found occasionally.

"Cyrtolite is rather common . . . in polysynthetic groupings with curved edges."

Other minerals common to the two localities are: allanite, fergusonite, magnetite, smoky quartz, pyrite, sphalerite, molybdenite, and gadolinite.

The following minerals are definitely reported as absent at Barringer Hill and all of them are also absent at Rockport: tourmaline, zircon (though cyrtolite is common at both localities) beryl, monazite, cassiterite, and tungsten minerals.

Hidden[20] describes as very conspicuous the radial lines or "stars" about the rare-earth ore masses at Barringer Hill, a phenomenon very characteristic about the "strained zones" surrounding the cyrtolite and gadolinite crystals at Rockport.

The pegmatite minerals of the Cape Ann granite may be grouped as follows:

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[18] Warren, C. H. Am. Jour. Sci., vol. 28, p. 449, 1919.
[19] Hess, U. S. G. S. Bull. 340.
[20] Hidden, W. E. Am. Jour. Sci., vol. 19, p. 425, 1905.

a. Essential, major constituents.
- Microcline
- Albite
- Quartz
- Lepidomelane

Subordinate constituents.
- Hedenbergite
- Hornblende
- Fayalite

b. Minerals characteristic of pneumatolitic processes.

Element	*Represented by*
F	Fluorite
Li	Cryophyllite
Rare-earths, Ti, Nb, Ta, etc.	Cyrtolite
	Allanite
	Fergusonite
	Yttrocerite
	Tantalite
	Thorite
	Gadolinite
Ti	Ilmenite
Mo	Molybdenite
Pb	Galena
Zn	Sphalerite
Be	Danalite
Fe	Pyrite
	Pyrrhotite

c. Secondary minerals.

Formed at high temperatures.
- Grünerite
- Ferroanthophyllite
- Chlorite

Formed at low temperatures.
- Sphalerite
- Siderite
- Calcite
- Hematite

Final products of oxidation
- Limonite
- Kaolin.

Feldspars. As already mentioned the feldspar of the pegmatite is chiefly a microcline-microperthite. In places the feldspar approaches amazon stone in color and shows crystal planes, particularly where surrounded by quartz. More commonly the feldspar is gray or greenish-gray. In some of the pegmatites of Group IV the feldspar is somewhat platty in habit with a strong tendency toward curved cleavage surfaces. Frequently it includes plates of lepidomelane and ilmenite or is fractured and carries stringers of black chloritic material.

The curved feldspar from the included pegmatite in the Babson Farm quarry occurs in masses of considerable size and presents a curious appearance; perhaps it may be said to resemble a curved shell as nearly as anything. In basal section, under the polarizing microscope (see Plate V, Fig. B) the traces of the 010 composition planes, instead of being parallel, are seen to radiate slightly, forming long narrow segments that extinguish successively as the stage of the microscope is rotated through a small angle. The traces of the basal cleavage change their direction slightly in each segment, the aggregate forming the curved surfaces seen in the broken fragments. The albite member of the microperthite follows the usual mode of intergrowth in each segment. Included in the feldspar are tiny greenish needles, orientated in the direction of the edge 010–001. They resemble amphibole, but are so small that their properties could not be determined.

Quartz. Quartz occurs (1) in graphic intergrowth with feldspar, (2) in large masses, usually at the center of the pegmatite. The quartz is usually fractured or shattered and varies in color from that which is clear and glassy to that which is blue, purplish, or smoky. The large masses found in the Babson Farm quarry showed a very beautiful bluish to purplish opalescent tint. This variety when heated becomes filled with minute cracks, and splinters are spalled off with violent decrepitation. Quartz in distinct crystals occurs only as a secondary product.

Micas. The most common mica is lepidomelane. The lithia mica, cryophyllite, appears to be quite rare. It is distinguished by greater fusibility, stronger lithium flame, greener color and the greater elasticity of its plates. Since much of the lepidomelane also gives a lithium flame, it is probable that it contains admixed cryophyllite in small quantities.

Phlogopite has been reported by Sears, but the writers are inclined to question the accuracy of this report, since a highly magnesian mica would not be expected here.

For comparison the optical properties of the micaceous minerals of the pegmatites are tabulated:

	Lepidomelane	Cryophyllite	Chlorite	"Chlorotoid"
Color in basal section	grayish green	apple green	grayish-green	brown
Pleochroism	olive green to brown (none in basal section)	apple green to brown	same	dark brown to colorless
β	1.656	1.56	1.677	1.67
α				1.57
Biref. in basal section	very low	very low	very low	
2 E	very small	57°	55°	very small
Optic Sign	(−)	(−)	(−)	(−)?

Lepidomelane occurs in the pegmatite in large tabular masses up to a decimeter in diameter and 35 mm. in thickness. It was early noted by Cooke[21] in describing cryophyllite from this locality, and was named "annite" by Dana. In 1886 Clarke,[22] who published analyses by Riggs and Cooke, discussed its probable formula as deduced from the analyses. It is brilliant black in hand specimens, rather brittle and gives a greenish-black streak. In thin section, basal cleavage flakes are grayish-green and show double refraction. The optic angle is very small and scarcely any separation of the hyperbolae is discernible. β is close to 1.656, hence considerably greater than in ordinary biotite.

One mass of lepidomelane on the border of a large pegmatite body shows considerable granular magnetite between its foliae associated with chlorite. Some plates of lepidomelane end abruptly against magnetite, while others continue through it. Isolated shreds of lepidomelane are enclosed in the chlorite-magnetite mixture. The relations suggest that the lepidomelane has been replaced along the cleavage directions by the magnetite and chlorite.

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[21] Cooke. Am. Jour. Sci., vol. 43, pp. 217, 230, 1867.
[22] Clarke, F. W. Am. Jour. Sci., vol. 37, p. 358, 1889.

Some of the lepidomelane shows replacement and injection by quartz, sphalerite, and fluorite. One thin section showed fine needle-like crystals probably of ferroanthophyllite, extending from lepidomelane into quartz and another shows a granular alteration rim of chlorite between feldspar and mica, stringers of chlorite running off into cracks in the feldspar.

The cryophyllite occurs in plates in the feldspar, with which it apparently crystallized simultaneously. Its physical and optical properties have already been mentioned. The optic angle (2E) was found to be 57° which agrees substantially with Dana's value of 56.

Fayalite. The occurrence of large crystals of fayalite several kilograms in weight in these pegmatites has already been mentioned. They invariably show more or less alteration, part of which is due to weathering but the greater part to reaction with magmatic or late mineralizing solutions. The mineral changes which have occurred in the fayalite of the pegmatites are substantially the same as those seen in the thin sections of the granite and the nordmarkitic granite. The fayalite forms only rough crystals, is of a very dark, oily green to almost black color, and appears to have contained some finely divided magnetite as an original impurity. The composition as found by Penfield and Forbes is as follows:

	I	II	Average	Ratio		
SiO_2	30.11	30.05	30.08	.501		1.00
FeO	68.04	68.19	68.12	.946	1.004	2.00
MnO	.77	.65	.72	.010		
H_2O	.88	.87	.80	.048		
	99.80	99.76	99.72			

Specific gravity 4.323, 4.316. Average 4.318.[23]

About the margins and to a greater or less extent throughout the body of the crystals along cracks and cleavages, there is very apt to be alteration which appears to be in part, at least, limonitic in character, and to be the result of surface alteration. In places (the Briar Neck pegmatite in particular) the fayalite crystals have changed almost entirely to a limonitic mass.

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[23] Am. Jour. Sci., vol. 1, p. 129, 1896.

Two somewhat different types of reaction products have been noted surrounding the fayalite. These, however, probably grade into each other. The first is that described earlier by Warren, and consists of a narrow border of radiating fibers of what he described as an iron-rich anthophyllite but which should now be called ferroanthophyllite.[24] With this is associated a considerable amount of fine magnetite which appeared to be segregated thickly about the outer edge of the fayalite and where the ferroanthophyllite makes its beginning. (See Plate V, Fig. A.)

The other type of reaction which our recent study shows to be probably more common, is as follows: (1) a zone, in one instance 3 mm. wide, of matted greenish to colorless amphibole crystals with much disseminated magnetite; (2) adjoining this on the outside is a zone of about equal width of matted biotite shreds with some greenish hornblende; (3) microcline-microperthite of the curved habit and carrying small stringers of quartz. The amphibole of (1) is colorless to very faintly pleochroic and forms rather stout bladed crystals instead of the needle-like habit of the anthophyllite. The extinction angle is close to 15°; its index of refraction was found by the immersion method to be above 1.67, and therefore too high for tremolite or any other non-pleochroic amphibole except grünerite. Here, as seems to be also the case in the granite and nordmarkitic granite, the reaction products appear to differ according as the fayalite borders on quartz or on the feldspar. If quartz was originally in contact with the fayalite, the reaction product is the ferroanthophyllite and magnetite; if the feldspar adjoined the fayalite, then we find the amphibole and biotite with magnetite. The presence of the alumina and alkali in the feldspar are doubtless responsible for the differences, so that it seems probable that the adjacent mineral took part in the reaction under the influence of the active solutions emanating from the surrounding rock. It seems to have been a reaction which took place in situ. The magnetite associated with the grünerite is in part, at least, secondary, since it clearly replaces the grünerite along fracture and cleavage directions. Along narrow fractures well within the fayalite, magnetite and grünerite are developed in long stringers. The reactions here noted would seem at first thought capable of representation by rather simple equations expressing the conversion of an orthosilicate to a metasilicate. The presence of magnetite, however, complicates the matter and it seems idle to attempt to express the reaction in detail.

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[24] See Shannon, E. V. Proc. U. S. Nat. Museum, vol. 59, p. 397, 1921.

Fluorite. Crystals of sea-green fluorite as large as 3 cm. in diameter are sometimes found embedded in the feldspar and apparently of early formation. Purplish fluorite occurs and is usually associated with granular chlorite and metallic sulphides.

Pyroxene. Crystals of pyroxene are not uncommon. The usual habit is that of a twin flattened parallel to the orthopinacoid, the basal partings meeting at an angle along this plane. These crystals are embedded in the feldspar. Qualitative as well as optical examination shows that this pyroxene is hedenbergite. The crystals are frequently heavily altered to limonite.

Hornblende. This mineral occurs as a marginal development about the pyroxene and is an alkalic variety.

Ilmenite. This mineral occurs in plates or in narrow veins in the feldspar, associated with granular chlorite which frequently coats the surface of the ilmenite adjoining the feldspar. Pyrite has been noted with the ilmenite along the feldspar contact.

Cyrtolite. The cyrtolite forms small groups of brown tetragonal crystals having curved faces and is usually associated with scattered white quartz and granular black chlorite.

Tantalite. Tantalite has been found by Professor Palache in the form of a prismatic crystal about 25 mm. long embedded in feldspar of the graphic-granite zone. The front and side pinacoids were well developed and were terminated with domes and by the basal pinacoid.

Thorite. This mineral occurs as the variety orangite and is associated with fluorite.

Fergusonite. Fergusonite occurs in the form of small irregular grains and narrow lens-shaped plates in the feldspar with the curved-plate habit, and also in the graphic-granite. One slender crystal 15 mm. in length was found in the feldspar. In another specimen from a zone of large mica plates, fergusonite is marginal on cyrtolite.

Gadolinite. A crystal 1 by 2 cm. embedded in a "strained zone" in the quartz was collected by Professor Palache and corresponds closely to the description of this mineral from Llano County, Texas. It is dark, lustrous and glassy, with a conchoidal fracture, black in ordinary light, but green in thin splinters by transmitted light. The crystal is rounded in outline with no well-defined faces, and is coated with a brick-red alteration product.

Yttrocerite. This mineral is reported by Sears [25] from Rockport on massive quartz.

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[25] Op. cit.

Molybdenite. Molybdenite occurs in flakes up to 12 cm. in diameter, although it is usually of much smaller size. It appears to favor the outer zone of the pegmatite bodies, where its characteristic mode of occurrence is in the feldspar containing large biotite crystals. It is often found in the fine-granite masses which contain no pegmatitic material at the center. Purple fluorite is a constant associate, though the two minerals are rarely found in contact. A granular black chlorite is also a companion mineral. In one specimen, flakes of molybdenite were noted in the chlorite which forms a small vein in fayalite alternation products.

Sulphides. Galena, sphalerite, and danalite are all found in similar associations, i. e. surrounded by quartz and particularly feldspar. The grains or masses of the sulphide minerals are separated from the feldspar by sharp walls, usually parallel to the crystal or cleavage planes. Well-formed crystals of feldspar were noted completely surrounded by danalite and in other cases by sphalerite. Usually the surface of the feldspar is corroded and discolored by minute chlorite flakes. Sphalerite also occurs in small stringers with chlorite, penetrating fractures in the feldspar.

Where galena and sphalerite occur together, galena is usually enclosed in the sphalerite and does not form stringers or veinlets in the gangue minerals. Polished sections show the galena replacing sphalerite in irregular veinlets, one vein enclosing a minute "horse" of sphalerite. Pyrite and pyrrhotite occur very sparingly, sometimes in poorly defined grains along the periphery of the crystals of the dark silicates such as fayalite and pyroxene. Pyrite was observed filling cleavage cracks in sphalerite and forming thin films in factures in bluish quartz.

Danalite. Rockport is the type locality for this mineral which was first described by Cooke.[26] It occurs in small grains to large masses, pink in color and usually irregular in shape, although rarely it shows a suggestion of octahedral form. The mode of occurrence resembles that of the sphalerite in (1) angular walls against feldspar, (2) surrounding of feldspar crystals, (3) association with galena and chlorite. Often danalite and sphalerite are found in contact, in which case the sphalerite is marginal on the danalite. Evidently the minerals were deposited by solutions containing silica and metallic sulphides. Where beryllium was present, and as long as it lasted, danalite was formed. In the absence of beryllium, sphalerite seems to have been the mineral deposited.

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[26] Cooke, J. P. Am. Jour. Sci., vol. 43, p. 75, 1866.

Magnetite. Magnetite in small grains is found as an original mineral in the fayalite, possibly elsewhere. It is common as the result of the alteration of the fayalite and lepidomelane as elsewhere noted.

Chlorite. The occurrence of black chlorite as a decomposition product of the lepidomelane and an associate of the sulphides has already been alluded to. It is easily fusible, often exfoliating like a vermiculite, yields much water on heating and frequently gives a small lithia flame coloration. There appear to be all gradations between lepidomelane and fine-grained chlorite.

A similar form of chlorite is found in compact masses of long, narrow pseudo-hexagonal crystals a few millimeters in diameter. Cyrtolite is frequently embedded in this variety and the chlorite often forms embayments into the cyrtolite as well as into the adjoining feldspar. Flakes of the chlorite are grayish-green by transmitted light, optically negative with an optic angle, $2E = 55°$, while the index of refraction for the rays vibrating parallel to the plane of the cleavage is 1.67.

Chlorotoid. A black, cleavable mineral somewhat resembling hornblende in appearance, but showing only one cleavage, is frequently noted associated with chlorite and fluorite. Often it is seen in irregular masses, containing fragments of feldspar, thus forming the cementing material of a veritable breccia. This mineral was labeled "chlorotoid" in the collection of Harvard University, and is quite similar in appearance to the masonite from Rhode Island. The mineral was studied with a view to obtaining a more accurate identification but was found not to correspond to any described species. It is strongly plechroic, dark brown to colorless. The mineral is very opaque and attempts to make a thin section were not successful. Tiny grains, however, show high interference colors at their edges. The indices of refraction as determined by the immersion method were found to be $\alpha = 1.67$, and $\gamma = 1.57$, which would give a birefringence in the neighborhood of 0.10. Fragments are positively elongated. In thin sections of one of the fine-granites, a mineral having a similar color and pleochroism was noted and this afforded an interference figure which was apparently uniaxial and negative in optical character.

The mineral fuses with difficulty to a black magnetic globule, giving no lithium flame. It is not decomposed by sulphuric acid. A qualitative analysis gave much iron and aluminium, but no calcium nor magnesium.

Except for the high indices of refraction, the optical properties correspond to a biotite high in iron, but the mineral in question does not have a cleavage like a micaceous mineral.

Astrophyllite. Reported by Sears but not identified by us.

Ferroanthophyllite and *Grünerite.* These minerals have already been described as to occurrence and characters under the description of the alteration products of the fayalite.

Calcite. Calcite occurs in film-like plates along cleavage planes in decomposed pyroxene.

Quartz (secondary). Flattened crystals are found showing prismatic and terminal planes in decomposed pyroxene.

Phenacite and the decomposition products of Danalite. Palache[27] has described an interesting series of decomposition products of danalite from a quarry along the railroad between Rockport and Gloucester. Phenacite occurs in small crystals up to 5 mm. in diameter lining cavities, together with pale brown anhedral crystals of siderite, imperfect sphalerite crystals, minute pyrite crystals showing the faces of the cube and octahedron, and hematite in lustrous scales or as red stains on quartz crystals.

Kaolin and *Limonite.* These minerals are mentioned for the sake of completeness and are the final products of weathering of the feldspars and the dark silicates respectively.

Associated with the limonite in the somewhat tabular masses which form the decomposition products of the fayalite and pyroxene where these minerals have suffered decomposition on a large scale, is a yellowish to reddish-brown mineral with a conchoidal fracture which has not been identified. It is nearly or quite isotropic, index about 1.60, streak brown but lighter than that of limonite. In hydrochloric acid it loses its color and the acid reacts for iron and a trace of calcium. It is doubtless a mixture of iron oxides.

Xanthosiderite. This mineral has been described by Sears as occurring in segregated masses, and with a botryoidal habit in crevices at the Rockport Granite Company's quarry. We have not verified this occurrence.

PARAGENESIS OF THE PEGMATITE MINERALS.

From what may be observed regarding the mode of occurrence, structure, and composition of the pegmatite bodies it seems clear that they represent a fraction of the granitic magma produced by the normal progress of crystallization of the magma, and that they differ from the original magma chiefly in the greater concentration of silica, certain rarer elements and probably in water or water vapor.

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[27] Palache, C. Am. Jour. Sci., vol. 24, p. 249, 1907.

Some portions remained near their place of origin and eventually formed the pegmatitic segregations of a generally rounded form. Other portions, of larger size, were forced away from their places of origin and formed larger and more irregularly outlined bodies in the granite or were intruded into portions of the granite already solidified forming dikes or dike-like masses with true cross-cutting relations. Those pegmatites which were associated with the first and nordmarkitic intrusion were involved in the general breaking and inclusion of that rock in the later normal granite and now appear as inclusions in it. We may also suppose that from deeper portions of the granite mass as it continued to crystallize, further portions of the same more pegmatitic fraction continued to be given off and these forced or worked their way upward and out along lines of weakness, fractures or cracks in the rock mass above and produced, partly by the recrystallization of minerals already formed along the line or path of escape, and partly by further deposition of dissolved mineral material, the pegmatitic streaks or veins which, as we have noted, are characterized by irregular boundaries and lack of sharp and definite contacts with the containing rock.

Once a body of this pegmatitic fraction became individualized in the rock mass with crystallization not yet fully completed, we may conceive of its continuing the crystallization process without break in continuity but with such differences in sequence of crystallization and texture as were imposed by the somewhat different state of chemical and physical equilibrium which would necessarily obtain in such a fraction of the magma as we have pictured. The main characteristic of the outer portions of the pegmatite bodies, whether in the form of rounded masses or of dikes, appears to have been the simultaneous crystallization of the quartz and feldspar, either as fine granite or as an intergrowth. Why either one or both of these should have formed no one knows. They are undoubtedly very closely related and whatever the determining factor or factors are which produced one or the other, they are, we must suppose, small in magnitude. Some slight variation in physical condition, such as a brief period of supersaturation; some small variation in the chemical composition, perhaps the presence or absence of some constituent possessing extraordinary potency in its effect on crystallization may have been the cause or causes in question. It is perhaps not inconceivable that the fine granite was produced by the reaction of a highly active solution in the center of the mass on the original wall granite. At some period after the outer portions of the pegmatite were formed, substantially as we see them, there was a time during which coarse quartz and feldspar formed along with fayalite, lepidomelane and pyroxene, and this period seems to have ended, or come to a pause, with the exhaustion of the feldspar content and a very great, indeed almost exclusive, concentration of silica in the center of the pegmatite. That this silica solution contained a certain amount of other material, and that mineralizing activity continued after the surrounding rock had formed is evidenced by the fact that sulphides are found in the quartz and that they and other mineral compounds were introduced into the feldspar, etc., at a very late stage.

The fayalite became stable and crystallized at an early stage in the pegmatite as well as in the granite and nordmarkitic granite. It appears to have given place to pyroxene and this to hornblende and mica. Doubtless some part of it was reabsorbed, the other minerals appearing. Much of it was able to persist, although in part destroyed by late magmatic or postmagmatic reactions, forming the ferroanthophyllite, grünerite, and magnetite as has been described.

The sulphides, as noted, occur in the quartz centers, and are persistently associated with fluorite and black chlorite which penetrates fractures in the feldspar, and therefore these minerals are believed to belong to a late stage of formation. Evidently solutions, probably the last remaining liquor after most or all of the pegmatite material had crystallized, permeated the feldspar and aided by slight fracturing attacked the lepidomelane and converted it into chlorite and magnetite, at the same time depositing galena, danalite, and purple fluorite. Pyrite also belongs to this late stage, since it occurs in fractures in quartz and sphalerite.

The relation of these sulphides is of interest from the standpoint of ore deposition. In many descriptions of the successive generations of sulphides, the process is pictured as a series of invasions by hydrothermal solutions of differing composition. While such a process may have been followed in some localities, it appears that here, since there are no channels connecting the pegmatite bodies with deep or distant sources, the ingredients of the minerals of this later generation must have been concentrated in the pegmatitic fraction; for if sulphides passed through the almost completely solidified granite, either in gaseous or liquid solution, we should expect to find some alteration and deposition of sulphides in the surrounding rock. Hence it seems reasonable to suppose that the galena and pyrite, while evidently later than the sphalerite, were not imported at successive stages, but merely precipitated more tardily than that mineral.

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ILLUSTRATIONS.

PLATE I.

Generalized geologic map of eastern Massachusetts. (On this map the alkaline granites are called "Quincy granite.")

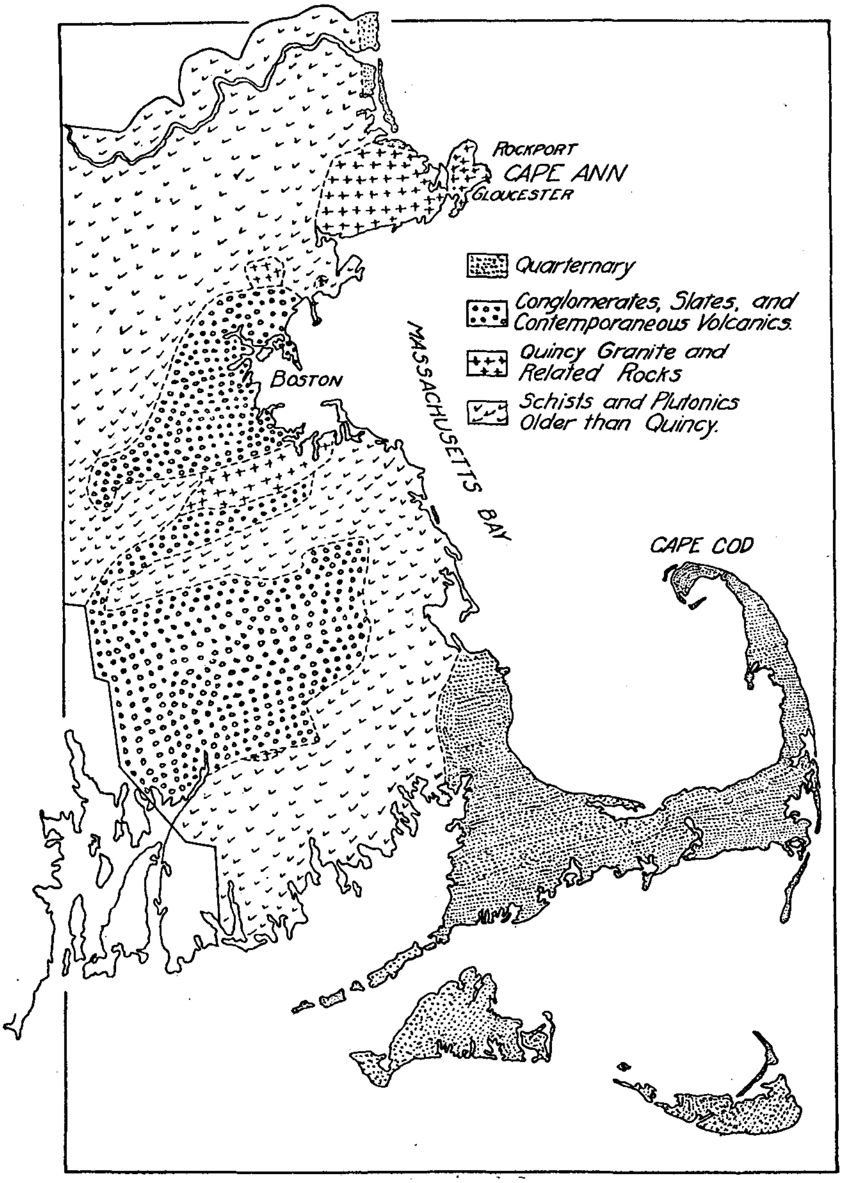
Rockport
Cape Ann
Gloucester
Boston
Massachusetts Bay
Cape Cod
Quarternary
Conglomerates, Slates, and Contemporaneous Volcanics.
Quincy Granite and Related Rocks
Schists and Plutonics Older than Quincy.

PLATE II.

Detailed geologic map of Andrew's Point, Cape Ann.

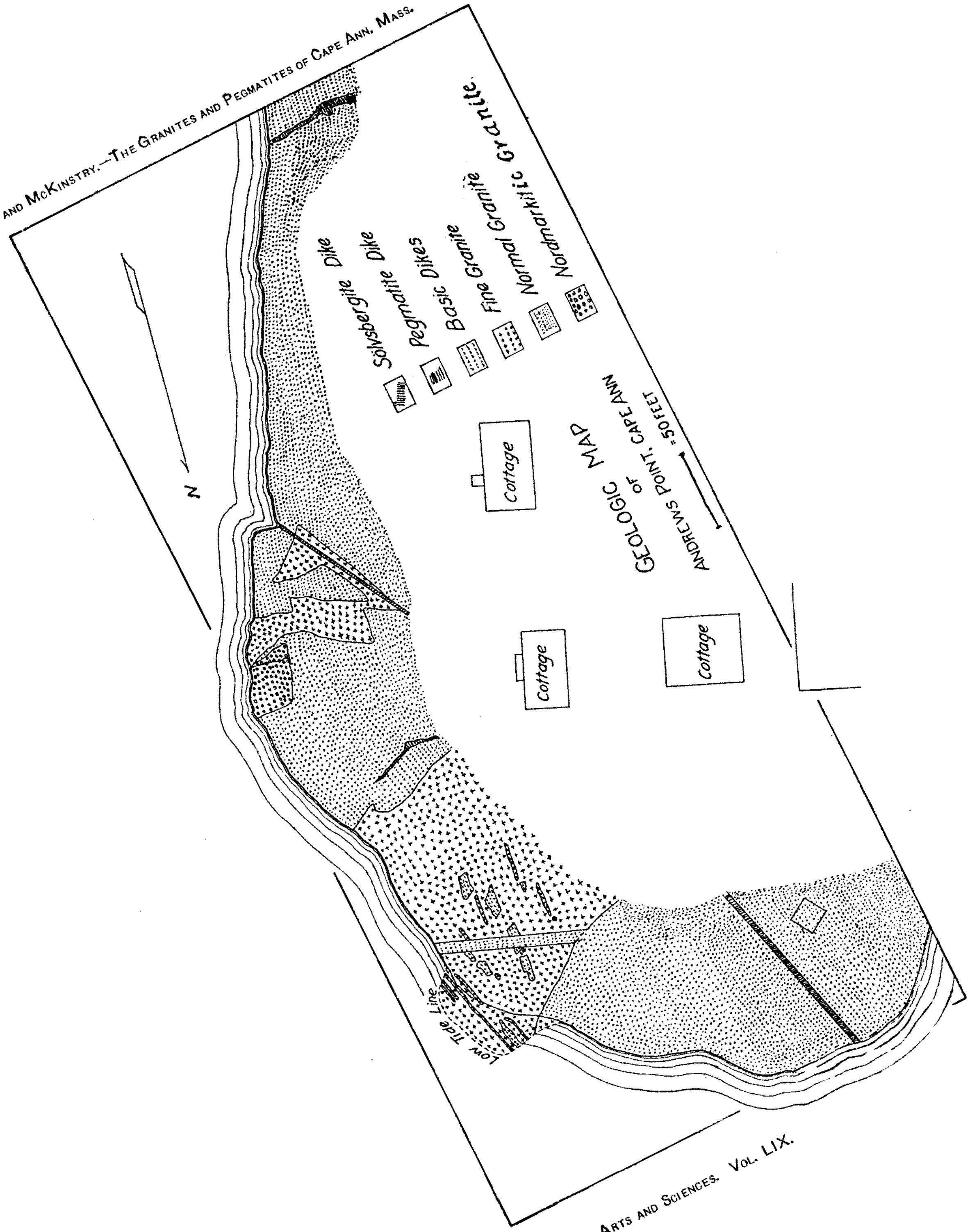
N
Sölvsbergite Dike
Pegmatite Dike
Basic Dikes
Fine Granite
Normal Granite
Nordmarkitic Granite
Cottage
Cottage
Cottage
Geologic Map of Andrews Point, Cape Ann
=50 feet
Low Tide Line

PLATE III.

FIGURE A. Fragments of nordmarkitic granite in normal granite.

FIGURE B. Pegmatite dike with quartz center and fine-granite margin cutting granite. Western coast near Bay View.

FIGURE C. Granite dike (light) cutting nordmarkitic granite near Bay View, west coast.

FIGURE D. Coarse pegmatite in fine-granite. Andrew's Point, Cape Ann.

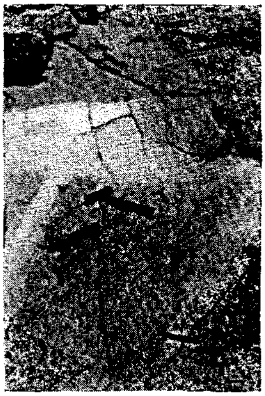

FIG. A

FIG. B

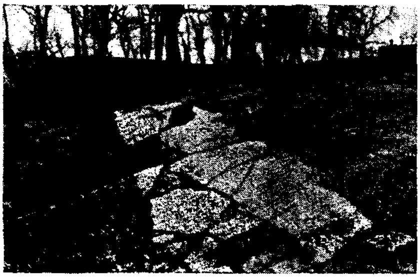

FIG. C

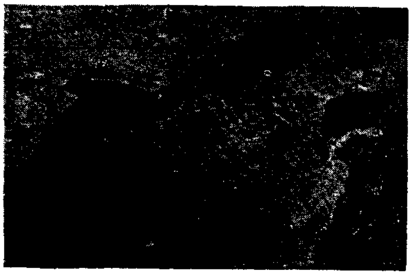

FIG. D

PLATE IV.

Figure A. Labradorite porphyry dike fragments included in nordmarkitic granite (light) near Bay View, west coast.

Figure B. Fine-granite and pegmatite dikes cutting nordmarkitic granite, ¼ miles southwest of Lanesville.

Figure C. Faulted and displaced trap dike in syenitic phase of fine-granite, Andrew's Point.

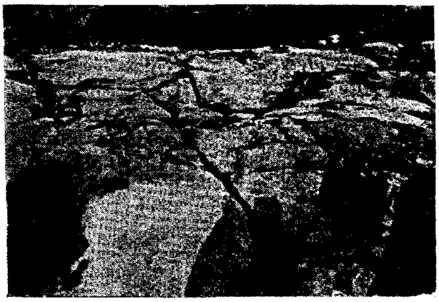

FIG. A

FIG. B

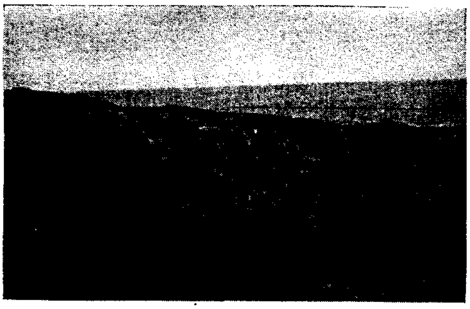

FIG. C

PLATE V.

Figure A. Fayalite (f) changed along contact with quartz, to ferroanthophyllite (a) and magnetite grains (black). From pegmatite, Rockport quarry. Photomicrograph, crossed Nicols, × 20.

Figure B. Basal section of microcline-microperthite of the "curved-plate habit." From large pegmatite block in granite, Flat Ledge quarry. Photomicrograph, crossed Nicols, × 20

Figure C. Thin section, Cape Ann granite. Shows fine-grained quartz and albite along the margins of the microperthite. Photomicrograph, crossed Nicols, × 30.

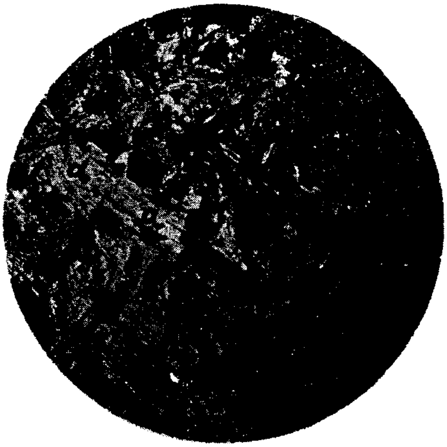

Fig. A

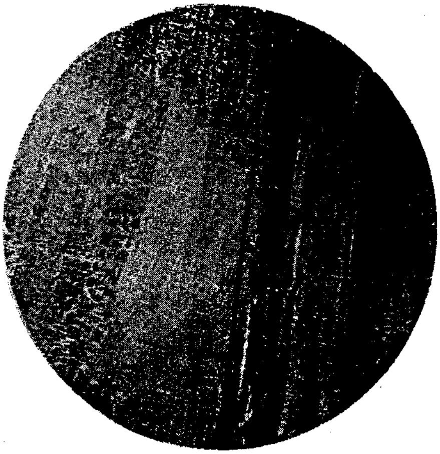

Fig. B

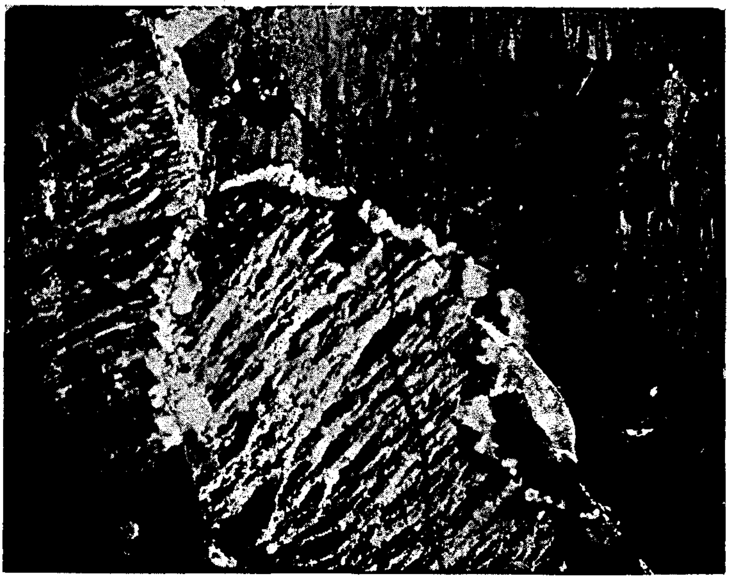

Fig. C

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OBSERVATIONS ON ANIMAL LIFE IN THE THERMAL WATERS OF YELLOWSTONE PARK, WITH A CONSIDERATION OF THE THERMAL ENVIRONMENT.

By Charles T. Brues.

With One Plate.

OBSERVATIONS ON ANIMAL LIFE IN THE THERMAL WATERS OF YELLOWSTONE PARK, WITH A CONSIDERATION OF THE THERMAL ENVIRONMENT.[1]

By Charles T. Brues.

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The following observations are of a rather fragmentary nature, and their presentation at the present time would demand an apology were it not for the great dearth of knowledge concerning the occurrence, distribution and habits of animal life in the many hot springs and thermal waters so generously scattered over our globe.

A few naturalists have from time to time given their attention to the animal life of certain hot springs, especially in Europe, but the accounts which have been published are so widely scattered that no recent attempt has been made to gather them together for comparison. Nevertheless two zoölogists, Issel ('00, '01, '06, '10) who made extensive studies of the thermal springs of Italy, and Vouk ('19, '23) in Croatia and Slavonia, have indicated several conclusions that may be drawn from a study of the thermal faunæ with which they have dealt. In the present account I have attempted to include a general consideration of the fauna of thermal springs, with particular reference to those groups of animals represented in Yellowstone Park.

My own observations were made during the summer of 1923 when I spent the month of July in Yellowstone Park, devoting a part of the time to a study of the thermal fauna of this exceedingly interesting region. For such a study, the Yellowstone National Park is a particularly favorable field on account of its great extent and the extreme variations presented by the thermal waters, not only in temperature, but in composition and in the ecological conditions which they offer.

For this opportunity to collect material, I feel deeply indebted to Mr. Stephen F. Mather, Director of the National Parks Service who expressed great interest in the biological studies which our party intended to make in the park. The resident naturalist, Dr. Frank E. A. Thone, also received us most cordially and we have him and the superintendent of the Park, Mr. H. M. Albright, to thank for many favors.

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[1] Contribution from the Entomological Laboratory of the Bussey Institution, Harvard University, No. 237.

The biological characteristics of thermal waters or "hot springs" depend upon several very evident factors. Of these the most striking is the high temperature of the water. Practically a continuous series with respect to temperature is to be found in Yellowstone Park. From the intermittent geysers water at the boiling temperature is poured forth and certain boiling springs or pools are equally hot at least in the parts where the water is in active ebullition. The outer margins of these pools are cooler and the overflow waters from both geysers and boiling springs gradually cool as they run off in small streams or more rapidly if they spread out over the surface of the soil. From these sources alone there is therefore a complete gradation from boiling water to that which has acquired a temperature as low as that of the surface waters of direct meteoric origin. Other pools and even ponds and small lakes contain water of more uniform temperature far below the boiling point, and often only slightly or imperceptibly warmer than pools or ponds of non-thermal origin. From the standpoint of any aquatic organisms living in such waters, the temperature of the medium is a most important consideration and this is the factor which has most generally attracted the attention of biologists. Before the beginning of the nineteenth century Saussure (1796) noticed in certain European hot springs that there were both plants and animals living in water at temperatures well above those which obtain in places ordinarily populated by living organisms, and since that time it has become more and more clearly apparent that the inhabitants of hot springs are in some way endowed with powers of resistance to heat that is fatal to the ordinary run of animals and plants. The exact nature of the physiological adjustments of this thermal fauna and flora have so far not been satisfactorily determined, although several theories have been advanced to account for the adaptations which undoubtedly exist.

Another common characteristic of thermal waters, particularly of those which emerge from the earth at very high temperatures, is the small amount of dissolved oxygen. As a result of this condition the fauna is further handicapped by the dearth of this essential element, except in the case of certain animals, *e. g.* the adults of aquatic beetles or the larvæ of certain Diptera which secure their oxygen not directly from the water, but from the atmosphere above its surface. This factor is one which has not received sufficient consideration.

Thermal water is also commonly impregnated with salts in solution, and the nature of these is naturally very dissimilar in different places, since it is dependent upon the constitution of the rocks with which the heated water has been in contact. In the Yellowstone Park springs there are two predominant types, one containing calcium carbonate and the other charged with silica. In the former case the water is acid in reaction due to the presence of much carbon dioxide, and in the latter the silica is dissolved in the presence of sodium carbonate. Other salts that are frequently present are sodium sulphate, sodium chloride and gypsum, with occasionally appreciable amounts of arsenical salts. The addition of such substances to the water is naturally an important environmental factor and presents conditions similar to those existing in alkaline lakes or ponds.

In addition to the carbon dioxide already mentioned hydrogen sulphide is frequently present in considerable quantities, and sulphur dioxide is probably emitted from certain steam vents. I can find no chemical analyses in support of this, but see no other way to account for the presence of charred wood at the openings of many vents. This wood has apparently been acted upon by sulphuric acid derived from the oxidation of sulphur dioxide. In connection with volcanic eruptions, it is of course known that appreciable amounts of sulphuric acid occur in rain water which falls near by.

Occasionally there are also noticeable amounts of finely divided mineral matter in suspension, especially in the tepid pools near the mud geysers or "paint pots."

The ecological peculiarities of the Yellowstone thermal waters are further enhanced by the cold climate of the park which is located near the 45th parallel of latitude. The hot springs are at an altitude of between 6,000 and 7,000 feet which thus places them within the Hudsonian life zone. In the thermal waters and also in the soil immediately surrounding them much higher temperatures of course prevail throughout the year than are normal for this zone and the rigors of the winter are so mitigated that certain animals and plants have found an opportunity greatly to extend their northern range, irrespective of those that seem to be specially adapted to life in or about the hot springs.

As is well known, certain plants occur in water of much higher temperature than that at which any forms of animal life are able to exist. Several botanists (Davis '97, Miss Tilden '98 and Setchell '03) have made observations of the temperatures at which living plants occur in the thermal waters of Yellowstone Park. Of these Setchell's account is the most complete and embodies a long series of observations on springs of different types. He found in the calcareous springs that chlorophyll-bearing algæ occur regularly in water at 60°–63° C. (140°–146° Fahr.) and that forms destitute of chlorophyll extend into still hotter water of 70°–71° C. (158°–160° Fahr.). These coincide almost exactly with temperatures of 145° and 162° Fahr. observed by us during the past summer and with a maximum of 160° reported years ago by Wood ('68) in Owen's Valley, California. In the alkaline silica-charged waters Setchell found that higher temperatures were compatible with plant life and that species with chlorophyll were growing in water of 75°–77° C. (167°–171° Fahr.) while non-chlorophyll-bearing forms inhabit water as high as 89° C. (194 Fahr.). At the altitude of the Park, the latter is practically the boiling temperature of water.

In the thermal springs of Europe many observations have been made, dating back to those of Saussure, who noted in 1796 the occurrence of plants in the waters at Aix at temperatures of from 35°–46° C. As early as 1854 De Laures ('54–'55) determined that higher algæ ("conferves") grew in the thermal waters of Néris at temperatures of from 42°–48° C. (107°–119° Fahr.) and he further made the interesting observation that the growth of these plants was augmented during the summer, which he interpreted as due to the influence of light over longer daily periods. In Italy Hoppe-Seyler ('75) found at Monte-Grotto near Abano in northeastern Italy algæ growing in water at 50° C. (122° Fahr.) and De Varigny ('93) refers to observations by Martens of Oscillatoria in a hot spring on the island of Lipari at a temperature of 55° C. (131° Fahr.). A slightly higher temperature of 60° C. (140° Fahr.) is reported for Oscillatoria by Schnetzler ('89) at Carlsbad. In the condensed water of steam caves in Ischia Hoppe-Seyler (*loc. cit.*) found green algæ at a temperature of 64.7° C. (149° Fahr.). From these data it would appear that green algæ have been found in Europe at temperatures approximately the same as in Yellowstone Park.

In Iceland several botanists have given accounts of the algæ which occur in the hot springs associated with the well-known Iceland geyser region. West ('02) reports filamentous algæ (Myxophyceæ) in water at 85° C. (185 Fahr.), which is approximately the same as the temperature given by Setchell for Yellowstone Park.

Concerning hot springs in other parts of the world, I have been unable to find many records that seem to be fully trustworthy. In the Himalayas, the eminent botanist Hooker ('54) reported "Leptothrix" in the waters of Thibetan hot springs where the temperature was as high as 75.5 C. (168° Fahr.). Strachey (*teste* De Varigny '93) found living algæ at Pugha in Thibet at 78.8° C. (173° Fahr.) and Dana refers to algæ obtained at 160° Fahr. (72° C.) on the island of

Luzon. In South America Dana speaks of vegetation in the hot springs at Las Trincheras in Venezuela occurring in water as hot as 71° C. (160° Fahr.).

In the Algerian hot springs at Hammam-Meskhoutine, Gervais ('49) found algæ in the second tier of basins of the cascade containing water at 57° C. (135° Fahr.), but the upper layer having a temperature of 63° C. (146° Fahr.) contained none.

Some of these records refer to green algæ, and others like the one at Las Trincheras undoubtedly relate to species without chlorophyll. With this in mind, it is evident that they agree closely with what is known of the algæ and other primitive plants in the thermal waters of Yellowstone Park, with the possible exception that no European species without chlorophyll appear to have been reported from such extremely high temperatures as in other parts of the world.

The temperature requirements for all animals seem to be more exacting than those for the plants just noted, and although I have considered several groups of animals more specifically in the sequel, it seems wise to make a few general statements at this point concerning the reactions of animals to high temperatures and their adaptation to thermal waters.

It is at once evident that practically all groups of animals that are represented at all commonly in fresh water contain a few species known to inhabit thermal waters, and of these a very small proportion seems to be restricted to this habitat. Compared with plants their powers of adjustment to high temperatures are considerably less and even the most resistant forms of animal life are unable to withstand the degree of heat at which certain plants thrive.

From the data furnished by experiments on certain marine animals, it appears very clearly that these forms which have always lived in the sea where they are never called upon to combat great fluctuations in the temperature of the water are far more susceptible to injury from excessive heat. Experiments like those of Richet ('85) and Frenzel ('85) upon a variety of marine animals illustrate this point. The latter author examined a series including a crinoid, actinians, worms, crustaceans, molluscs and a fish (Hippocampus) with respect to the temperatures they could withstand without immediate harm when the medium was slowly heated. In a general way he found that temperatures of from 25°–30° C. (77°–86° Fahr.) were the highest that could be borne even for a few hours, although a mollusc (Murex) (32° C.) and actinians (38° C.) exceeded these figures. According to De Varigny the pelagic *Beroë ovatus* displays a similar critical point of 28°–29° C.

(82°–84° Fahr.) and so far as I can ascertain these figures apply quite closely to all marine metazoa that have been examined. As there are no marine forms occurring in hot springs it is of course impossible to say what their powers of temperature adjustment might be after acclimatization over long periods, but when transferred from a uniformly cool environment it is evident that they are quite susceptible to heat and that forms like the actinian and snail just mentioned which must normally be subjected to excess heat in the littoral of the sea show the greatest resistance. The larvæ of certain marine littoral animals have been examined by Pelseneer ('01) with reference to their resistance to heat and he found that those of certain molluscs, annelids, trematodes and crustaceans were frequently killed by temperatures of 30°–31° C. (86°–88° Fahr.) while others might withstand 37° C. (97° Fahr.). On the other hand he found embryos of the fresh-water *Lymnæa ovata* able to survive a heating of the medium to 44° C. (111° Fahr.).

On account of the great fluctuations of temperature in all except the largest bodies of fresh water, the animals which inhabit them are periodically subjected to considerably higher temperatures than those encountered by marine forms, and we find that the fresh-water fauna is much better able to withstand extremes of heat. Observations cited on a later page by Davenport and Castle and others show that certain amphibians are unharmed by temporary heating of the medium to about 38° C. (102° Fahr.). In thermal springs the tadpoles of certain tailless batrachians have been found living at temperatures noticeably above this and as these are species known normally to inhabit waters of meteoric origin it is evident that they have become acclimatized to these higher temperatures by exposure extending over a number of generations. Fishes are more susceptible to heat, although there is evidence that certain forms have been acclimatized in a similar way. Even fresh-water turtles occur in hot springs according to Schmarda ('53) who cites the finding of these animals in Tunis in springs near the ancient Utica at 44° C. (111° Fahr.) and near Lenkoran on the Caspian Sea in sulphurous springs at 40° C. (104° Fahr.). Among invertebrates the molluscs and insects are most abundantly represented in thermal waters, and, as indicated in detail later, certain members of these groups extend into water of 40° C. (104° Fahr.) and several degrees higher in a few instances.

The range of temperature over which the process of acclimatization may extend is nevertheless very limited and moreover very uniform among quite diverse animals. This is beautifully illustrated by the classic observations of Plateau ('72) who determined experimentally the thermal death point of a number of fresh-water arthropods known to occur in thermal water as well as in ordinary ponds and streams. When we compare the temperatures at which these animals actually occur in hot springs with the highest temperature that can be successfully endured by unacclimatized individuals of the same species we find that the possible range of acclimatization is remarkably uniform. This is shown in the following table which is based on data furnished by two of the tables given by Plateau.

TABLE SHOWING RANGE OF TEMPERATURE ACCLIMATIZATION.

Species	Highest temperature at which unacclimatized individuals remain alive	Temperature at which individuals have been found in thermal waters	Location of thermal spring
INSECTS			
Culex pipiens (larva)	38° C.	40° C.; 39° C.	Néris; Luxeuil
Cloë diptera (nymph)	44° C. (?)	45°	Luxeuil
Notonecta glauca	37.5°	45°	Vichy
Nepa cinerea	43°	45°	Vichy
Agabus bipustulatus	36°	38°	Ems
Hydroporus dorsalis	39°	42°	Loèche-les-Bains
Hydaticus transversalis	38°	39; 39.2°	Luxeuil; Ems
Hydrous caraboides	38°	40°	Néris
ARACHNIDS			
Argyroneta aquatica	37°	38°	Ems
Hydrachna cruenta	44.5°	46°	Luxeuil
CRUSTACEANS			
Asellus aquaticus	38°	42.5°	Loèche-les-Bains
Gammarus roeselii	33°	34.4; 36°	Piemont; Gastein
Daphnia sima	32.5°	25°	Luxeuil
Cypris fuscata	35°	34.4; 36°	Piemont; Gastein
Cyclops quadricornis	35°	34.4; 36°	Piemont; Gastein

Practically all of the species listed above show that there has been an adjustment which enables them to live continuously at temperatures higher than those at which they are able to remain alive for more than a few minutes when taken directly from fresh water at ordinary temperature. The differences amount, however, to only a few degrees in each case and we may conclude that the range of temperature adjustment is ordinarily only from one to several degrees Centigrade.

The Protozoa are the most resistant of all animals and Issel ('00–'01) reports Rhizopods and flagellates from hot springs in Italy at 54.5° C. and 51° C. (130° and 124° Fahr.) respectively. Later (Issel '10) he found an Amoeba, apparently the common *A. limax*, in water at 50°–52° C. (122°–126° Fahr.). The same observer found ciliate infusorians extending into water at 46° C. (115° Fahr.). The last is, however, no higher than the temperature for small rotifers and for at least one species of beetles.

The exact relations of terrestrial and aerial animals toward the temperature of the soil and air about them are much more difficult to determine than is the case with aquatic forms. Several reasons for this are self-evident. The specific heat of water is much greater than that of air and soil and the closeness of contact with the soil is so much less than in the case of water that the body heat of cold-blooded animals responds far more quickly and accords very closely with that of the water. The cooling effect of evaporation can also greatly reduce the body heat of non-aquatic animals. Finally, the vast differences between sunlight and shade temperature are in turn modified by the external color of the animals themselves. With such a mass of variable factors, coupled with the fact that most animals may seek the sun or shade at pleasure and may choose between a period of diurnal or one of nocturnal activity, the actual temperatures to which such animals are subjected and the periods of time involved are to be determined only approximately.

In nature the most obvious field for comparison with the aquatic thermal environment is the desert. Desert animals are at times subjected to high temperatures which are quite comparable to those encountered by the inhabitants of hot springs. Here the most important difference lies in the opportunities enjoyed by the members of the desert fauna to escape the excessive heat (and also the dryness) of the surface of the soil by burrowing beneath it during the warmer parts of the day. According to Buxton ('23) the surface temperature of the soil of the desert may normally reach 50°–60° C. (122°–140° Fahr.) in parts of Palestine, while even higher readings of 78°–84° C. (173°–184° Fahr.) have been obtained in other desert regions. The same author records the occurrence of certain insects, two species of Mantis nymphs and adult grasshoppers, actively moving about on the ground where the superficial temperature had reached 50.8° C. (121° Fahr.). Still more recently Buxton ('24) has actually taken the bodily temperatures of certain desert insects by means of a minute thermocouple which can be inserted *per rectum* into the body of the insect. By this ingenious device he has been able to determine that the body temperature may approach closely to that of the heated soil, but that it always appears to remain slightly lower. Thus in the black tenebrionid beetle, *Ademsia ulcerosa*, body temperatures of from 36°–39.5° were found where the substratum registered 38°–44° C. and in several pale-colored insects the body was somewhat cooler in proportion, registering 33.5°–39.5° when the surface of the soil was 39.5°–45° C. As the opportunities for bodily temperature reduction through evaporation in such chitin-encased animals, possessed of no special regulatory apparatus for this purpose and lacking a readily available supply of water, must always be very scant, the conclusion appears inevitable that the temperature of their bodies cannot regularly be far below that of the soil-surface. Such being the case, it is evident that the temperatures tolerated by desert insects correspond closely to those endured by the inhabitants of thermal waters.

Under experimental conditions the discordant effects of certain factors may be eliminated and this has been done to some extent by several entomologists who have determined the reactions to excess heat and the thermal death point of certain unacclimatized insects. Many years ago Cantoni ('72) examined the behavior of the domesticated silk-worm at abnormally high temperatures and found that they would continue to feed at 47° C. (117° Fahr.). Various others (*v.* Bachmetjew '01) have performed similar experiments with many other caterpillars and found that temperatures of 40°–50° C. (104°–113° Fahr.) are not fatal over periods of many hours. More recently Dewitz ('06) obtained similar results with another series of caterpillars and the larvæ of muscid flies, which place the upper limits of temperature tolerance at 40°–41° C. (104°–106° Fahr.).

In connection with his studies upon the symbiotic intestinal protozoan parasites of termites, Cleveland ('23, '24) has examined with great care the reactions of both termites and protozoa to high temperatures. He found that the thermal death point of the intestinal protozoa is 36° C. (97° Fahr.) while that of practically all of the species of termites with which he worked was considerably higher. The host termites were entirely unaffected by a 24-hour exposure to 36° C. and were able to withstand without injury exposures of ten minutes to 42°–47° C. (108°–117° Fahr.). He thus regards 48° C. as the thermal death point of the termites, although with longer exposures this would no doubt be lowered to some extent.

The very general agreement of nearly all of the observations relating to the fauna of thermal springs with experimental data secured from other animals of the most diverse sorts shows that the tolerance of all animals to high temperatures coincides within quite narrow limits. These would appear to range between 40° and 45° C., or 104°–113° Fahr., usually nearer the lower figure, except in the case of marine species to which as already stated temperatures above 25°–30° C. (77°–86° Fahr.) quickly lead to death. There is also a close agreement between the limits above indicated for non-marine animals and the body temperatures of warm-blooded animals. As the latter are equipped with a thermoregulatory mechanism and are of course not subject to the fluctuations of the medium in which they live we must look for abnormal conditions to determine the upper limits of body temperature compatible with continued life. Such conditions are furnished by individuals, including those of our own species suffering from acute febrile diseases, where the failure of the regulatory apparatus allows the body temperature to rise. Under such conditions a rise of body temperature of a few degrees above the normal level of 37°–42° C. (99°–108° Fahr.) is fatal to the animal. This is in close agreement with the reactions of the lower animals toward heat, especially when we consider that the homoiothermal animal must be regarded as acclimatized to high temperature; or in other words that the warm-blooded vertebrate normally maintains a body temperature very close to the thermal death point.

This uniformity of reaction among the most diverse animals would seem to be due to the fact that high temperatures act directly upon their protoplasm, or at least upon cell metabolism, and this view finds support in the experiments of Vernon ('99) who found in a series of both vertebrates and invertebrates that the voluntary muscles pass into a condition of rigor at temperatures approximating the lethal temperature limit. Unfortunately no satisfactory understanding seems to have been reached concerning the physiology of this process, for upon it must depend any theories relating to the acclimatization of animals to high temperatures. Maurel and LaGriffe ('99) have discussed the matter, but reached no definite conclusions. Davenport and Castle ('95) advanced the theory that the increased resistance of organisms acclimatized to heat is due to a partial dehydration of the protoplasm whereby, on a purely physical basis, its coagulation point is raised in the temperature scale. This theory is based in part on the well-known fact that egg albumen coagulates at higher temperatures as its water content is reduced, and in part on the resistance of the spores of various microörganisms to temperatures far above those that can be withstood by their actively growing stages. That the same condition holds true in the encysted or desiccated infusoria and even in minute metazoa like the rotifers and tardigrades is also apparently well authenticated. In these animals the amount of exsiccation possible without loss of vitality is surprisingly large. Even in amphibians, *e. g.* Rana, it may proceed to a point where as much as 41% of the body weight may be lost (Hall '22) and subsequently regained without ill effects. In certain batrachians like the desert frogs of Australia such an occurrence normally follows the periodic changes of the seasons. Setchell ('03) from his study of the thermal flora is not inclined to accept this view of exsiccation, at least in regard to the thermal algæ. Applied to the thermal fauna this theory would imply a decreased water content in the blood and tissues and it seems probable that their saline content in most invertebrates is actually greater, due to the excess of salts in the medium which pass into the blood and tissues (Fredericq '04). There is the further evidence (referred to on a later page) that those animals which are able to adjust themselves to the highly saline water of brackish ponds or of the ocean itself are the types which most generally occur in thermal waters.

This question therefore involves physicochemical matters which I do not feel competent to discuss further than to consider the relationships between the thermal and brackish water faunæ. The similarity between these two faunæ and that of the littoral zone of the sea also is very striking and cannot fail to attract the attention of any zoölogist who cares to examine the thermal fauna in any detail. It is frequently referred to in the sequel in connection with various groups of animals, particularly the insects and molluscs. It may be said that a great majority of the animals which occur in thermal waters have close relatives which live in alkaline, saline or brackish water or even in the sea. As the only similarity between all such situations lies in the considerable amounts of soluble salts in the water, irrespective of their nature, it seems reasonably evident that the fresh-water groups of animals that have developed species adapted to thermal waters have done so through their ability to adjust their metabolism to the increased osmotic pressure of the medium. That the members of a number of small quite unrelated groups possess this ability to an unusual extent is equally evident. That much variation occurs among animals in this respect is of course well known (*cf. e. g.* Garrey '05, Sumner '06) and the cases cited indicate that it is not safe to draw broad conclusions regarding extensive groups of animals from experiments with members of isolated genera or families.

In practically all thermal waters there is undoubtedly a considerable deficiency in dissolved air and a dearth of oxygen compared with meteoric waters as gases are naturally in great part expelled from the highly heated waters when not under pressure. Thus in the overflow from geysers and in pools rapidly supplied by water from boiling springs the process of cooling to temperatures compatible with animal life takes place quickly and there is little opportunity for atmospheric air to be absorbed. The green thermal algæ growing in such water are a source of oxygen supply but the amounts which they are able to furnish to rapidly moving water must be slight even where they are present in a considerable quantity. In the hottest water the non-chlorophyll-bearing plants (*e. g.* Beggiatoa) of course furnish no oxygen and their occurrence indicates the presence of some oxygen in the water when it reaches them.

As already mentioned, many of the animals, *e. g.* many insects and snails, are not entirely dependent upon the dissolved oxygen as they obtain atmospheric air. Nevertheless some are entirely dependent upon what they extract directly from the water, *e. g.* Gammarus and the larvæ of Chironomus and water-beetles. In reference to the fauna of fresh-water lakes, it has been conclusively shown that the amount of dissolved oxygen which varies at different depths is an important factor in determining the distribution of fishes and certain water-breathing invertebrates (Juday '09, Juday and Wagner '09). It is interesting to note that at least one type which Juday found to be characteristic of the lower water layers of Wisconsin lakes occurs in what appears to be one of the most anaërobic situations where I found living animals in the thermal waters of Yellowstone Park. These are the larvæ of Chironomus containing a red hæmoglobin pigment, discussed in the latter part of the present paper.

The thermal fauna has, as we have already stated, undoubtedly been derived mainly from an assemblage of fresh-water animals that have found a suitable if not ideal environment in more or less saline waters of abnormally high and uniform temperature. We cannot, however, consider it at all homogeneous either in origin or in constitution. On the basis of our conception of the geological development of the earth we have only to go far enough back in the evolution of life to reach the point when all animals and plants belonged to a "thermal fauna." Just how elaborate and highly differentiated living organisms may have been at the time when all water on the earth was at a considerably higher temperature than at present is by no means to be stated. Our knowledge of fossil animals does not however suggest in any way that highly evolved types were present at that time and certainly no one would entertain for a moment the idea that arthropods or molluscs were then in existence. Even if other evidence did not preclude such a possibility it would therefore be fallacious to trace any great part of the present thermal fauna to a primordial one. Schnetzler ('89) would trace some of the highly adapted lower plants like Oscillaria and certain diatoms to a primordial flora, an assumption which is more plausible.

There is no doubt, however, that thermal springs like those now extant have existed in the past as there remain widespread evidences of their activity. It is well known that many fossil plants owe their preservation to the deposition within their tissues of silica derived from hot waters charged with silica in which they have been immersed or from which they may have imbibed the solution by capillary attraction. The "fossil forests" of tree stumps, logs and bits of wood have originated in this way, and in one instance at least it is evident that deposits of fossil insects are to be attributed to the same source (Cockerell '15). In the Gurnet Bay deposit, on the Isle of Wight, there is an accumulation of this sort which includes many insects in addition to numerous molluscs. Concerning the remarkable preservation of the insects Cockerell (*loc. cit.*) says: "There was absolutely no compression, and when the rock is fractured so as to bisect an insect longitudinally, a cast of its internal organs is preserved without losing their more delicate parts as is shown by a mosquito wing still carrying the scales. . . . There was perhaps a mud spring, with heated waters, into which the insects fell, overcome by gaseous emanations."

This Gurnet Bay formation is early Tertiary, probably lower Oligocene, and in connection with the present-day fauna of thermal springs it is interesting to compare the forms which have been found there. Thus, in his account of the insects, Cockerell finds two species of Ephydra and Stratiomyia, both of them genera known to contain modern species that breed in thermal as well as saline waters. In addition there are winged ants and other Hymenoptera, quite similar to the miscellaneous assortment of insects that fall into hot springs. Certain deposits of silicified wood are much older and the preservation of some Cretaceous and even Jurassic wood is so similar to that now in process of silicification that there can be little doubt that the hot springs which furnished the silica were so like those of the present day that they offered similar opportunities for animal life.

On the basis of these facts it may be stated that very probably there have always been hot springs and that they have presented quite similar conditions for a long time, but that their fauna has had its origin in animals from cool fresh water that have become adapted to life in the higher temperatures of thermal springs.

The present-day thermal fauna has been represented by Issel ('06) as composed of several series of animals which he groups as follows:

1. Animals derived from cold water, usually species which are much more abundant and widespread in various other environments and consequently with a greater ability to accustom themselves to extreme conditions (*e. g. Philodina roseola, Chilodon cucullus*).
2. Animals that come periodically from the sea (*e. g. Anguilla vulgaris*), or which migrate for short periods into fresh water from the sea (*e. g. Palaemonetes varians*).
3. Animals that although absent in the ordinary waters of the region, occur in that of warmer regions (*e. g. Hydroscapha gyrinoides*).
4. Animals at present restricted to the thermal waters of a region, but similar to fossil species of wider distribution in deposits from ordinary water in the same region (*e. g. Melanopsis etrusca*).
5. Animals up to the present time apparently restricted to thermal waters (*e. g. Laccobius sellæ*).

Quite recently Vouk ('23) has grouped the thermal fauna and flora on the basis of the temperature of the water and indicates three divisions as follows:

1. Hypothermophilous Formations, below 15° C. (58° Fahr.).
2. Mesothermophilous Formations, 15°–30° C. (58°–86° Fahr.).
3. Euthermophilous Formations, 30°–80° C. (86°–176° Fahr.).

This classification is essentially arbitrary and artificial as it fails to recognize the varying critical temperatures for different types of organisms, although in its gradually rising scale, it necessarily reflects to some extent the progress of adaptation. From what has already been said concerning the temperatures withstood by unacclimatized animals of most kinds it is evident that there is a clearly marked point in the neighborhood of 40° C. (104° Fahr.) which is highly significant, as it is the upper limit for all but the most resistant animals. Among plants on the other hand, as already mentioned, temperatures in the neighborhood of 60° C. (140° Fahr.) are critical as they mark the upper limits for chlorophyll-bearing vegetation.

Issel's classification is based on the probable evolutionary development of thermophily. The most recent accessions to the fauna are placed in the first category and those types which are undoubtedly the oldest are placed last, and the arrangement thus conforms to the probable origin and development of the thermal fauna as we have outlined them above. Applied to Yellowstone Park, all of the above categories are included, with the exception of the second which is precluded by the topography of the region, and the fourth for which I cannot at present pick out any well-authenticated case.

The first group is typified by a widespread species like *Gammarus limnæus;* the third by *Libellula saturata* and *Tropisternus californicus;* and the fifth by *Ambrysus heidemanni* and the Chironomus larva (sp. near *tentans* described in the sequel).

The remainder of the present paper contains a more detailed account of the findings in Yellowstone Park and in connection with these I have attempted to summarize the present knowledge relating to the occurrence in thermal waters of each group that is referred to. As has been mentioned at the outset, the literature is widely scattered and many references to the thermal fauna, or ones important in connection with it, are included in works not dealing primarily with this subject. For this reason, a bibliography has been appended which includes the titles to which specific reference is made in the text.

INSECTS.

ORDER DIPTERA.

FAMILY CHIRONOMIDÆ.

The larvæ of three species of this family were found in the thermal waters of the Park, one of these in quite hot water, the others in streams and pools where the water was only slightly warm, although of thermal origin. Adults were not reared, but as the early stages of this family have been so extensively studied it is possible to classify their larvæ with considerable accuracy.

Chironomid larvæ, usually "blood-worms" of the genus Chironomus, have been mentioned a number of times as inhabitants of thermal and alkaline waters and a number of genera are known to be either exclusively marine or to include marine species. Wood ('68) seems to have been the first to record the presence of Chironomus larvæ in hot springs; these were observed by one of his correspondents in Owen's Valley, California, who found them in water said to be at a temperature of 51° C. (124° Fahr.). Hubbard ('92) speaks of the larvæ of minute gnats in the tepid water of the terrace basins at Mammoth Hot Springs in Yellowstone Park and these undoubtedly were Chironomidæ. Schwarz ('14) found the larvæ of two minute species of Chironomidae in water between 38°–40° C. (100° and 105° Fahr.) at Hot Springs, Yavapai County, Arizona. Issel ('01) found Chironomus in one of the Italian hot springs at 36° C. (97° Fahr.).

Ceratopogon may also be represented in alkaline ponds by a species from the western states. Packard ('71) described a supposed Tanypus from Clear Lake, California, which Johannsen has been able to refer to Ceratopogon *sens. lat.* As these specimens were really from an alkaline pond in the vicinity of the fresh-water Clear Lake (Packard '83), it seems evident that at least one species of this highly polymorphic and widely distributed genus breeds in alkaline water. Larvæ of Culicoides were found by Ping ('21) in artificial saline pools near Ithaca, New York, having a slight salinity. The two closely related genera Eretmoptera Kellogg ('00) from California and Psamathiomyia Deby ('89) from western Europe are both maritime, their larvæ living in tide pools in the American form and on alga-covered rocks in the case of the European one which feeds on Enteromorpha. Both have vestigial wings in both sexes

Thalassomyia frauenfeldi Schiner of northern Europe is also marine. The larvæ have been found several times along the coast of England by Swainson (Theobald '92) attached to hydroids, and also among seaweed. On account of its unusual habitat it was mistaken for an annelid-worm by Johnston, who described it as *Campontia eruciformis.* Johannsen ('05) believed that Thalassomyia might be synonymous with Scopelodromus, but if the published figures are accurate it is evident that the larvæ, at least, differ considerably, as those of the former are represented as having four pairs of ventral blood-gills while those of the latter have none.

In Scopelodromus Chevrel ('03), represented by a single species, *S. isemerinus*, the larva is marine also, the types coming from seaweed collected on the coast of France. As mentioned above, this may be a synonym of Thalassomyia.

Still a third similar genus, Halirytis, from the remote antarctic, is known from Kerguelen Island. The type *H. amphibius* Eaton ('75), also figured by Verrall ('79), is also subapterous at least in the female which occurs along the shore where the larvæ probably feed on Enteromorpha. Another antarctic genus from Patagonia, Bergica Jacobs ('00), has greatly reduced wings in both sexes and the larvæ are probably marine also.

The European *Clunio marinus* Haliday is well known as a maritime midge, with a marine larva obtained by Carpenter ('94) who has made interesting observations on the habits of the species. The female only, in this case, is unable to fly and is carried about by the male.

Species of Chironomus (*sens. str.*) are also known to be truly marine, *e. g. C. thalassophilus,* bred and studied by Bequaert and Goetghebuer ('13) in Belgium, and an unidentified species found in salt water on the coast of France by Moniez ('90). Larvæ of an unidentified species were even found in the pools of slightly saline water near a salt works at Ithaca, New York, by Ping ('21).

In eastern North America several marine larvæ have been found, but they have never been adequately described and their identity is unfortunately more or less shrouded in mystery. The first references are by Packard in 1869 ('69a, '69b) and relate to the larva of *Chironomus oceanicus* Pack., placed with some doubt in the genus Orthocladius by Johannsen ('05). *C. oceanicus* was found in several localities on the Massachusetts coast.

From the preceding account, it is evident that many members of this family in the most diverse parts of the world show adaptations to life in thermal waters, alkaline ponds and to marine life in the littoral zone.

A more specific account of my own collections in Yellowstone Park follows.

Chironomus sp., near *tentans* Fabr.

(Fig. **1**, Fig. **2**, *a*.)

The peculiar larvæ of this species were taken in a warm pool about two miles east of the point where the Yellowstone River emerges from Yellowstone Lake. Near this point there is a small number of hot and warm pools and a few steam vents within several hundred feet of the lake shore, but separated from it by an irregular, low-lying ridge a few feet in height. One more or less circular pool, about fifty feet in diameter, bore bird-tracks in the soft ooze within a few feet of its border, beneath a layer of water several inches in depth, and numerous bright red larvæ were readily to be seen in the water at a temperature of about 49° C. (120° Fahr.). As the zone in which they occurred was of considerable width there was a corresponding variation in temperature but some larvæ at least were in water of this temperature.

These larvæ undoubtedly belong to some rather large species of Chironomus, and as they are strikingly different from those of any species so far described I have thought it worth while to describe and figure them.

The larvæ measure 9-11 mm. in length; of the usual slender form prevailing in the genus; bright red in life, white as preserved; head capsule light testaceous, darker on the sides, with the posterior margin black ventrally and laterally; teeth of mentum, tips of mandibles and hooks of anal prolegs black; bristles on prothoracic prolegs brownish yellow. Two eye-spots on each side, of about equal size, separated by about twice their own diameter. Long joint of antenna cylindrical, three times as long as thick; second joint small, as long as the width of the first and of about the same shape, bearing internally at the base a stout bristle somewhat longer than the joint; third joint minute, globose; fourth a minute cylinder with a short apical appendage. Mentum (Fig. 2, *a*) broad, with its dentate edge evenly curved; median tooth broad, but not very long; first lateral short, acute; second the longest, rather closely united to the third; teeth to the last (sixth) decreasing in size from the second lateral. Mandible with three strong teeth before apex, with a series of conspicuous hairs inwardly that reach beyond the tip of the mandible; apex bidentate, the upper tooth brown and shorter than the ventral black one; outer side with a bristle externally near the base. Prothoracic prolegs extending as far forward as the eyes; their tips clothed with simple, strongly curved bristles. Posterior part of body (Fig. 1) bearing three pairs of ventral blood-gills, a pair attached near the ventral lateral angles of the seventh segment, as long as the thickness of the body; a second pair laterally at the middle of the eighth segment, these long and sharply geniculate near the base; a third pair near the anterior ventral angles of the ninth segment, these spirally curled; tenth abdominal segment

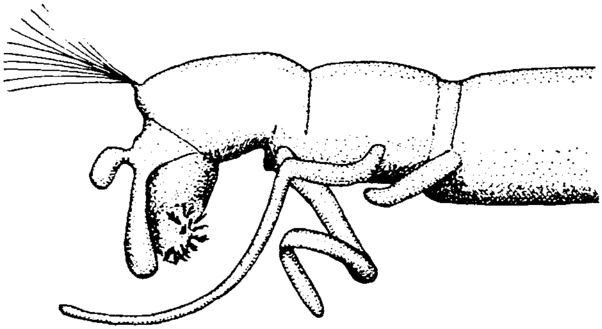

FIGURE 1. Chironomus sp., near *tentans* Fabr. Posterior part of abdomen of larva in lateral view.

with two dorsal pairs of blood-gills, the apical pair longest. Ninth segment dorsally at apex with a pair of median approximate nipple-shaped processes bearing together a cluster of about ten long bristles. Anal prolegs tipped with a crescent-shaped group of about a dozen bifid and trifid hooks.

This larva is remarkable in having three pairs of ventral abdominal gills. There are a few species known to have one or two pairs, but so far as I can ascertain positively, this is the first one to have a pair on the ninth segment. Whether this extra pair bears any relation to the undoubtedly low content of oxygen in the water which it inhabits, it is impossible to say, but this may quite possibly be the case. The supposed annelid, *Campontia eruciformis* (Johnston '65, Verrall '79, Theobald '92), which, as already mentioned, is probably the larva of Thalassomyia, appears to bear four pairs of ventral gills near the posterior part of the body, but the figure is so diagrammatic that this is open to question. The checkered taxonomic career of this

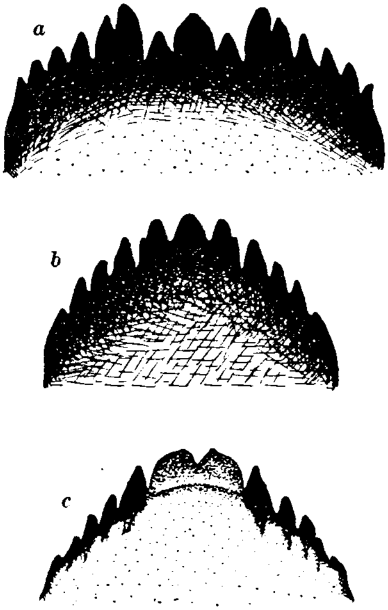

FIGURE 2. *a*, mentum of Chironomus sp., near *tentans* Fabr.; *b*, mentum of Chironomus sp., near *tenellus* Zett.; *c*, mentum of Orthocladius ? sp.

species and the dissensions which it caused are also related by Pavesi ('81). In structure the present larva agrees with those figured by Weyenbergh ('74), Johannsen ('05) and Malloch ('15) as *Chironomus tentans* Fabr. in the form of the mentum and mandibles, and were it not for the three pairs of gills, it might perhaps be considered as this species, although very similar to *C. viridicollis* V. d. Wulp, and to *C. decorus* Johannsen. Weyenbergh's figure (*loc. cit.*) does not show the gills.

Chironomus sp., near *tenellus* Zett.

Length 6 mm. Color as preserved in alcohol distinctly banded, the intersegmental bands pale, but each body segment brownish due to a very fine shagreen-like speckling of pigment. Head capsule very dark piceous, almost black; a pair of eye-spots at each side of the front, the anterior one only half as large as the posterior and separated from the latter by less than its own diameter. First joint of antenna long, cylindrical, three times as long as thick; second minute, but little longer than wide, as long as the width of the first; fourth and fifth barely visible, the terminal appendage obsolete; bristle-like appendage at base of second joint apparently not present. Mandible with three long teeth before apex and trace of a fourth basal to the others; its outer edge fluted (*i. e.* with a series of emarginations) and bearing a large bristle near base and another before the middle; inner surface without a tuft of hairs. Mentum (Fig. 2, *b*) narrow, the anterior margin strongly arched, deeply pigmented except for a hyaline setigerous spot near each hind angle; median tooth large, rounded at tip; first lateral tooth united with the second which shows only as a notch on its exterior side; third separated by a deep cleft; fourth to sixth decreasing in size. Prothoracic prolegs slender; their apical pads distinctly separated, clothed with dense brown curved bristles. Apical abdominal segments without ventral blood-gills and the two pairs of dorsal ones on the last segment extremely minute. Anal prolegs long and slender, their tips armed with the usual bifid hooks. Eighth abdominal segment with a pair of minute bristles near the middle on the ventral surface; tenth with a similar pair, one at the base of each proleg and above with a pair near the base of the blood-gills; ninth segment with a pair of approximate nipple-shaped tubercles bearing long hairs.

This species was taken together with the larvæ of Tabanus and dragon-fly nymphs mentioned on another page in an inactive pool of very turbid water at West Thumb. The temperature of this water is only slightly elevated and it is well aerated, due to the presence of growing Chara-like plants in abundance.

This larva resembles closely that of *Chironomus tenellus* Zett. as described and figured by Johannsen ('05) and may be that species.

Orthocladius? sp.

Length 8 mm. Color as preserved pure white, the head capsule pale luteous, with only the tips of mandibles, anterior margin of mentum and eye-spots black; bristles and hooks on prolegs pale brown. Antenna with first joint more than three times as long as broad and slightly tapered to the tip; second joint twice as thick and as long as the thickness of the first joint; third and fourth joints minute, apical appendage barely visible; second joint without secondary, ramus-like appendage. Mandible with four distinct, but rather short preapical teeth; outer edge smooth, with a very delicate hair before the middle; inner surface without hair-tuft. Mentum strongly arched along its anterior margin; no median tooth, but the middle pair large and broad (Fig. 2, *c*) and weakly pigmented; in addition to these there are five lateral teeth, gradually decreasing in size, the notches between them distinct, but not very deep. Lateral eye-spots very close together, the posterior one more than twice as long as the anterior. Prothoracic prolegs very small, but with their tips separated; apical pads clothed with extremely short, sharply curved bristles. Anal prolegs long and slender, tipped with pale brown bifid hooks. No ventral anal gills. Dorsal ones in two pairs; small, the upper pair somewhat larger than the lower; porrect and extending laterally. No bristles near apex of abdomen except for the two tufts at the apex of the penultimate segment; each consists of seven or eight hairs, twice as long as the diameter of the segment, and placed on a short cylindrical tubercle close to the dorsal median line.

This is probably an Orthocladius, as it agrees most closely with the known larvæ of this genus. It is easily recognizable by the pale, bidentate apex of the mentum. A pale median tooth or teeth seem to be more or less characteristic of Orthocladius, although not entirely restricted to it. Johannsen ('05) mentions (p. 249) a supposed Chironomus larva in which the median pair of labial teeth are paler than the others. It is of interest to note here that the marine Chironomid described many years ago ('69) by Packard as *Chironomus oceanicus* has been doubtfully referred to Orthocladius by Johannsen ('05); this species has been mentioned already on a previous page.

The present larva was collected at West Thumb near the shore of Yellowstone Lake in warm water which flows through coarsely spongiform siliceous deposits below the boiling springs. The temperature of this water was about 100° Fahr.

That the absence of ventral anal gills is not necessarily associated with an abundant supply of oxygen is evident from a note of Malloch's ('14) in which he records the occurrence of the giant blood-worm, *Chironomus ferrugineovittatus* Zett., in mud under 10-12 feet of water. This species has no ventral anal gills.

The presence of hæmoglobin in the larvæ of certain forms was originally interpreted as an adaptation to life under more or less anaërobic conditions by Miall ('91, '95) and this is no doubt true in a general way, although there are chironomid larvæ without this pigment which are able to exist in places where the oxygen supply is scanty and on the other hand true blood-worms often inhabit well-aërated waters. These discrepancies have often received comment (*cf.* also Gahan '12, Malloch '15, Muttkowski '18). The latter observer found some forms, *e. g. C. tentans* Fabr., practically confined to the bottom layer of Lake Mendota, Wisconsin. According to Miall's hypothesis they would have to come nearer to the surface periodically in order to lay up a store of oxygen. Such migration although apparently not regularly observed under such conditions would be obviously of no use in the case of larvæ living like the one described in the mud of shallow hot pools where the oxygen content can be no greater at the surface.

Leitch ('16) has studied experimentally the rôle of hæmoglobin in Chironomus and also in the snail, Planorbis, and comes to the conclusion that the function of hæmoglobin in these two animals, and among those invertebrates in general where it occurs, consists in making available, by its power of binding oxygen chemically, a quantity of oxygen sufficient for the needs of the animal at oxygen tensions so low that the necessary amount is not supplied by physical solution. He has shown also that the actual storage capacity for oxygen of the hæmoglobin in the Chironomus larva is sufficient for only a few minutes of anaërobic life. Thus hæmoglobin is characteristic of many mud-inhabiting invertebrates (Van der Hyde '17) that live where oxygen is present in only very small amounts. On the basis of Leitch's data we may thus interpret the ability of the larvæ of Chironomus to live under more or less completely anaërobic conditions, and we find them in the lower waters of lakes where the presence of oxygen cannot be detected by ordinary means. This is a common occurrence in fresh-water lakes (*cf.* also Immel '16) and in thermal waters (Ziegelmayer '24). In the alkaline water of silica-bearing hot springs there is the added fact that among fishes at least, an increased alkalinity serves in some way to increase the resistance of these animals to a lack of oxygen (Packard '05) and the same is probably true of Chironomus as the hæmoglobin appears to be identical in the two cases. There is one disturbing fact, however; Ping ('17) found that the larvæ of one species of Chironomus do not become red till they have attained a length of 5 mm. If this holds true for all the species the matter of respiration in these inhabiting thermal springs still remains obscure. It is true that the larvæ of certain related flies of the genus Corethra, (Juday '21) which live in the ooze at the bottom of lakes, do not enter the mud when very young, but this would not explain the case of the Chironomus larvæ in thermal waters even if they have the same habits, for they are confronted by an equally low tension of oxygen during their entire development and they cannot escape it by seeking water at a different level.

FAMILY CULICIDÆ.

Mosquito larvæ were found at only one locality breeding in the truly thermal waters. At West Thumb there are many inactive pools containing water which is only very slightly warm, although giving off gas in small amounts, and opaque from fine clay in suspension. These are really inactive mud volcanoes or "paint pots." In several of these the larvæ of *Culex tarsalis* Coquillett were extremely numerous, together with some small Dytiscid beetles. This species has been reported by Dyar ('23) as breeding in such pools and in the sulphurous marshes of the Park. He found *Culiseta inornata* Williston in similar places, but this species is not represented in the present collections. From their known habits and distribution it is evident that neither of these species is restricted to alkaline or thermal waters.

The larvæ of a rather large number of mosquitoes breed in brackish water or even in water of very high salinity, some regularly and others only occasionally. Howard, Dyar and Knab ('12) refer to three species — *Aëdes zammittii* of the Mediterranean region which breeds in rock pools containing concentrated sea water only; *Aëdes fluviatilis,* and a species of Stegoconops in the American tropics which breed also in the concentrated sea water of rock pools, but in fresh water as well. In the case of some of the many maritime species breeding in brackish water at least, the adjustment of the larvæ to certain concentrations of sea water is rather delicate, and their distribution is sometimes determined by the amount of salinity. This relation has been examined experimentally by Chidester ('15) who finds that the optimum salinity for the two abundant salt-marsh species (*Aëdes cantator* and *Aëdes sollicitans*) in New Jersey is quite different and also that *Culex pipiens* is unable to withstand salinities approaching those which the former species resist. The same author attempted to determine the relations of the several more abundant salts in sea water to the larvæ, but his results appear to be inconclusive in this respect.

Several species breed more or less regularly in alkaline water in the western states, and in the case of *Aëdes dorsalis* Meigen, a European species common to North America, the larvæ have been found in perfectly fresh water, alkaline pools, salt-marsh pools and even in the vats at salt works. In inland salt pools Ping ('21) finds that this species breeds only in those of low salinity (6% or less). The adults show remarkable variations in color, possibly associated with the different larval habitats. The ability among mosquitoes to breed occasionally in strongly alkaline water is recorded by Ballowe ('18), who found "both Anopheles and Culex" successfully developing in water containing sufficient caustic soda [probably transformed to sodium carbonate after standing for a long time] to give it a strongly alkaline taste. A lesser ability to withstand slightly concentrations of sea water has also been observed many times among a number of Anopheline mosquitoes in various parts of the world (De Vogel '07, Foley and Yvernault '08).

The occurrence of mosquito larvæ in actually thermal water seems to be very unusual, although two particularly interesting observations have come to my notice. Tamayo and Garcia ('06) obtained larvæ of an Anopheles at Lake Huacachina, near the coast in southern Peru. The temperature of this lake is about 75° Fahr. The species, which they named *Anopheles peruvianus*, is according to Knab ('15) the widespread neotropical *A. pseudopunctipennis* Theob. This species extends only sparingly into the warmest parts of the United States, but has been found in water perceptibly warmed by the influx of thermal water at Las Vegas Hot Springs, New Mexico, which is in the Transition Zone (Needham and Cockerell '03).

In Europe the larvæ of the common house mosquito, *Culex pipiens*, have been reported from thermal waters at Néris (40° C.) and again at Luxeuil (39° C.), according to Plateau ('72). Ziegelmayer ('24) found the larvæ of an undetermined mosquito abundant in the Italian sulphur springs known as Aque Albule.

FAMILY STRATIOMYIDÆ.

Larvæ of a species of this family were obtained at only one place, but as their habits serve to conceal them well, I do not doubt that their distribution may be quite extensive about the hot springs in the park.

In the canyon of the Yellowstone, about half a mile below the trail that leads down to the east bank of the river is a small cone-shaped geyser that sprouts more or less continuously to a height of a few inches close to the water's edge. Above it the canyon walls rise precipitately and are covered in streaks with an alga growing in water that trickles over the rock. Well hidden in this growth were numerous Stratiomyiid larvæ along the edges of the flow where the water had cooled sufficiently to feel only warm to the hand. As in the case of all flowing warm water of this type, the course of the flow is continually changing as algæ grow and the mineral deposits form. Consequently the station occupied by these larvæ is not only very limited in extent, but is continually shifting. In consequence they live a rather precarious existence, in danger of dessication if the water is diverted away and liable to still more sudden destruction should the hot water flow over them. The consequence of the latter alternative was seen in a number of larvæ which I recovered from a flow of very hot water several feet away. These had succumbed to the sudden rise in temperature and were already accumulating on the surface of their bodies a mineral deposit, becoming literally fossilized while still entangled in the mass of alga surrounding them. Viewed under the microscope the deposit is seen to consist of minute, nearly spherical bodies, which, as they are readily dissolved by hydrochloric acid, consist of calcium carbonate. They form in an irregular superficial coating and the larvæ are not destined to become recognizable fossils.

Following is a description of the larvæ.

Stratiomyia sp.

(Fig. 3.)

Length when well extended 23–28 mm.; greatest width about 3 mm. Head very dark above, pale beneath; body more or less distinctly longitudinally vittate above, with a median dark stripe and two on each side, one of which lies close to the lateral margin; apical segment much darker. Eyes placed close to the anterior corners of the head.

Body broadest at the base of the abdomen and more gradually tapering posteriorly than in front. Prothoracic spiracle behind the middle of the lateral margin, circular, with a radially fluted ring; posterior respiratory sac opening by an apical semicircular slit, fringed above and below with a series of plumose hairs, about 12 or 15 in each row. Body conspicuously hairy, but with the base and apex of each segment bare. Integument of the usual shagreened structure. Apical segment very slightly more than three times as long as broad and about one-half longer than the penultimate in moderately expanded specimens. Second abdominal segment below without hooks or stout bristles as are all segments except the penultimate which bears a pair of small, almost minute stout bristles behind near the median line;

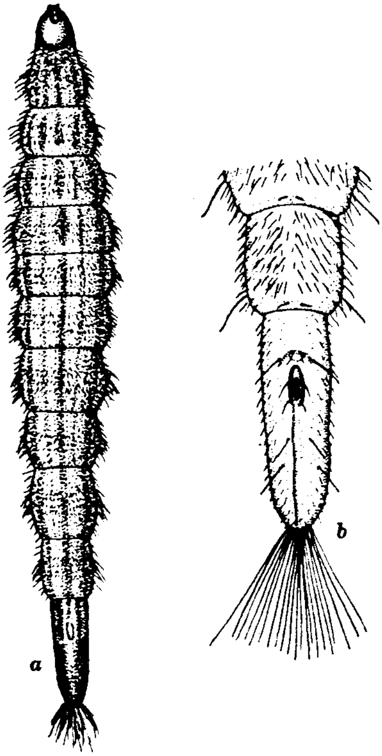

FIGURE 3. Larva of Stratiomyia sp. *a*, dorsal view; *b*, posterior portion in ventral view.

close examination reveals a similar pair, noticeable only under high magnification, on the antepenultimate segment. Anal slit terminating before the middle of the last segment.

According to the characters given by both Malloch ('17) and Johannsen ('22) these larvæ undoubtedly belong to the genus Stratiomyia and agree quite closely with the larva of S. *discalis* Loew according to Johannsen's key.

I obtained adults of three species of Stratiomyia in the Park where they frequent the flowers of *Heracleum lanatum* in company with many Hymenoptera and other Diptera, but no specimens were taken either by Professor Melander or myself in the neighborhood of the place where the larvæ were found.

The aquatic larvæ of this family are said to feed generally upon algæ and decaying matter, and possibly to some extent upon minute forms of animal life and the present species undoubtedly depends for food upon the algal growth in which it occurs.

The larvæ of the subfamily Stratiomyinæ are all aquatic, occurring in waters charged with lime at least in small quantities and regularly accumulate on the surface of the integument deposits of calcium carbonate from which they must be freed by immersion in dilute hydrochloric acid before they can be satisfactorily studied. Their occurrence in hot springs has been previously noted by Packard ('82) who gave a brief account of some larvæ collected by H. G. Griffith in a hot spring in Gunnison County, Colorado. The water in which they occurred is said to have been at 157° Fahr., but as this is so close to the coagulation point of insect protoplasm and barely below the highest temperature at which the most resistant algæ disappear I am inclined to believe that the larvæ must have been in adjacent water of lower temperature. From Packard's brief description it is probable that these larvæ represent a species of Stratiomyia, as he speaks of the anal segment as tapering and four times as long as broad, while no mention is made of the hooks on the penultimate segment characteristic of Odontomyia.

Another interesting case of the larvæ of some unidentified Stratiomyid is cited by Johnson in his study of this family ('95). The note, published in a newspaper (Evening Call, Lincoln, Neb., April 5, 1895), was written by Professor Lawrence Bruner, and the wording leaves little doubt that it has been printed essentially as written by the well-known entomologist. The specimens came from a thermal spring in Vinta County, near the extreme southwestern corner of Wyoming, collected by Mr. John C. Hamm. Johnson quotes this article as follows: "At the time I saw the larvæ in their native habitation I did not have a thermometer with which to take the temperature, but the water was so hot where I saw them I could not hold my hand in it. My best judgment is, and was at the time, that the water was not more than twenty or thirty degrees below the boiling point.... In color they are dull gray, in form much flattened, and in size the length of the line below the figure [15 mm.]. The two specimens examined by me are more or less encrusted with sulphur and other mineral substances."

According to this account the temperature of the water would have been 170–180° Fahr., unquestionably an impossible condition. It is of course easy to overestimate the temperature of hot water with the hand which can endure a temperature of about 140° Fahr. only with great discomfort.

In Europe Stratiomyiid larvæ are known from thermal waters. Issel ('00, '01) reports two cases in his paper on Italian hot springs, once at a temperature of 22.5° C. in the vicinity of Tivoli, near Rome, and again from the Valdieri springs in water of 37°–38° C.

Stratiomyid larvæ are known to occur in salt water also. In 1871 Packard reported the occurrence of a species in Clear Lake, California, as he supposed in salt water, but according to Aldrich ('12) this lake is fresh. It appears from a later note of Packard's ('83), however, that these specimens were really from Borax Lake and the species is therefore to be considered as an alkaline water form. Whether it may be a Stratiomyia or an Odontomyia is difficult to determine from the description. A truly marine larva, probably Stratiomyia sp., obtained by A. W. Pearson on the New England coast near the mouth of the Merrimac River is recorded by him ('83). This larva was living in a mass of sea wrack (Zostera).

Similarly, in Europe, larvæ of Stratiomyia have been found to develop in salt water. Florentin ('99) found the larvæ of *S. chameleo* Deg. living in excessively saline water with a salt content of 78 grams to the litre in the salt marshes of Lorraine. In Great Britain, Yerbury ('19) states that *Stratiomyia longicornis* Scop. has been bred from a larva found under rotting seaweed.

Larvæ of the related genus Odontomyia do not occur so generally in alkaline and saline pools, but Mr. C. W. Johnson tells me that he once reared a specimen of *O. cincta* Oliv. from a larva found in a salt marsh.

FAMILY TABANIDÆ.

The larvæ of a large species of Tabanus were found in thermal waters of the Park. Unfortunately the adults could not be bred out, and they belong to a species hitherto undescribed in the larval stage. Adults of at least six species of Tabanus were collected during our stay; the most abundant one, *T. osburni* Hine bothered us very frequently; the others (*T. affinis* Kirby, *T. phænops* O. S., *T. opacus* Coq., and *T. septentrionalis* Lw. and *T. punctifer* O. S.) were much less numerous, and all with the exception of *T. punctifer* are too small to associate with the larvæ collected. It is possible that adults of the latter were not flying at the time of our visit, or the larvæ may be those of *punctifer*, although this species has been rarely taken in localities that are far from thermal or alkaline waters.

A description of the larva follows.

Tabanus sp., probably *T. punctifer* O. S.[2]

(Fig. 4.)

Length in moderately extended condition 30–35 mm. Of the Of the usual pale buff or whitish color with dark brown markings as follows: stripes along median ventral and along each of the later prothoracic grooves; anterior margin of mesothorax, broader laterally above, narrowed medially above and obsolete below; the four lateral stripes extending nearly to the posterior margin of the segment and broadened apically; metathorax similarly marked, except that the band is not narrowed above nor below and the lateral stripes are much attenuated behind; abdominal annuli similar on the seven basal segments, each formed of a basal band, paler below and broken above by a white transverse band about half the width of the segment and one-third the width of the annuli; seventh abdominal segment also with a narrow longitudinal stripe along the lateral line; eighth with a similar broader one, as well as its complete apical margin and a curved band encircling the anal tubercle behind. Prothoracic spiracle, mandibles and posterior spiracle piceous or black. Mandibles with the inner edge very finely serrate, the teeth directed slightly backwards toward the base of the mandible. Second joint of antenna terminating in a short seta, very much more slender and about half as long as the first, articulated in an emargination before the apex of the first. Palpi three jointed, joints of about equal length, but the second and

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[2] See postscript on page 424.

third respectively are much more slender. Lateral striated area of prothorax extending over only its posterior third; the dorsal and ventral surfaces smooth except for a few very fine striæ on posterior fourth. Lateral areas of meso- and metathorax strongly striated between the longitudinal brown bands, the striæ extending from the basal anterior brown band almost to the posterior margin; upper and lower surface of mesothorax smooth, with a few weak striæ behind across the middle portion; metathorax, both above and below, finely reticulate striate, with an area at each side near the base, more nearly smooth. Pale parts of abdomen striate, more densely so on the sides and more sparsely so above and below where the surface is sometimes more or less reticulate. Body setæ few, very short and inconspicuous; each thoracic segment with a short bristle near the middle of each side of the ventral area; apex of abdomen with three pairs of very small ones along the sides of the spiracular opening.

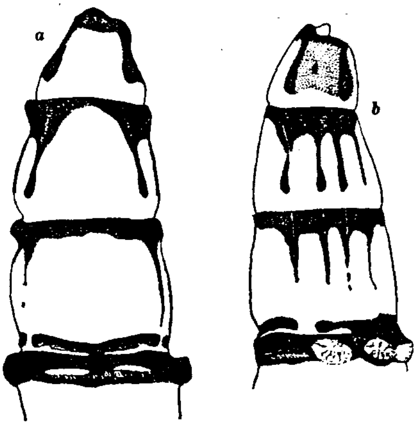

FIGURE 4. Larva of Tabanus sp., probably *T. punctifer* O. S. *a*, anterior part of body in dorsal view; *b*, same in lateral view.

The larvæ of this species were found in moderately warm water; in Bath Lake (33° C.; 91° Fahr.) July 7 and also in a small warm pool of about the same temperature at West Thumb on July 16. Another collection from the stream which serves as an outlet to Firehole Lake contains small larva which probably belongs to the same species; this water is also warm, about the same temperature at this point as at the other two stations, and very heavily charged with silica in solution.

These larvæ fall in the group with *Tabanus atratus* Fabr. as characterized by Hart ('95) on account of the reduced size of the lateral striate areas on the prothorax and the well-developed lateral longitudinal bands on all the thoracic segments.

I can find no records of the occurrence of larvæ of Tabanus in hot or warm springs, nor in alkaline waters. One European species, *T. autumnalis* L. has once been found in the salt lakes of Siebenbürgen (Friedenfels '80), and although no larvæ of the common "green-head" *T. nigrovittatus* Macq.) of the eastern Atlantic seaboard appear to have ever been taken, there can be little doubt that this species develops in the salt marshes along the coast. Schwarz ('91) in his account of the insects of Great Salt Lake, Utah, mentions the occurrence of larvæ of one or several species of Tabanus in the salt flats where the water of the shallow pools is more or less mixed with fresh or sulphurous water.

Johnson ('19) mentions the occurrence of two varieties of the common American *Tabanus atratus* Fabr. along the Atlantic coast, var. *nantuckensis* Hine in New England and var. *fulvopilosus* Johnson from New Jersey to Florida. These forms are so strictly confined to the marshy coast that he is convinced that they breed in the salt marshes. In this case, it is evident, as with the mosquitoes mentioned above, that the salinity of the breeding grounds is associated with discernible morphological characters in the adult flies.

No larvæ of Chrysops were found in any of the thermal waters, but several species were very annoying in the Park, usually in the woods. One species, however, never appeared in such situations, but was almost invariably noticed when we collected in the vicinity of the thermal pools. This is *Chrysops fulvaster* O. S., a form rather widely distributed in this region, and I think very probably associated with alkaline water and hot springs. Aldrich ('13) mentions its occurrence on the shores of Box Elder Lake, a brackish pond at Brigham City, Utah. Along the eastern coast of North America it is probable that *C. fuliginosus* Wied. (*plangens* Wied.) is restricted to the salt marshes (Johnson '19).

FAMILY EPHYDRIDÆ.

So far as their adaptations to alkaline and saline water are concerned, the larvæ of this family are perhaps the most remarkable of the order Diptera.

Both in Europe and in North America, several species of the genus

Ephydra have long been known to inhabit salt pools along the seacoast, as well as inland salt and alkaline lakes, and even the concentration vats of salt works. Nearly a century ago *Halmopota salinarum* was found by Bouché ('34) in brine pits, and a few years later the common European *Ephydra riparia* was found by von Heyden ('43–'44) to breed in similar situations. Later a number of other species of Ephydra, Dimeacenia, Cænia, Pelina and Parydra have been added to the list. In America the larva of Ephydra was first described by Packard ('69a, '69b, '71) from tidal pools on the New England coast, Great Salt Lake, Utah, and Borax Lake, California. Various other entomologists later interested themselves in these strange larvæ, but it remained for Aldrich ('12) to examine more closely into the habits and distribution of several of the species that occur in the salt and alkaline lakes and sea coast of the western United States. Their great abundance in Great Salt Lake has often been commented upon, and the early stages of the maritime species are frequently found locally abundant in both Europe and North America.

More recently Ping ('21) has investigated the occurrence and habits of *Ephydra subopaca* Loew, a common inhabitant of brine pools along the Atlantic seaboard, which he found to be abundant in the saline pools connected with salt works near Ithaca, New York. According to Ping, this species will develop readily in water having a salinity of 9%, or less, but in fresh water the larvæ are utterly unable to undergo their development as he found by attempting to rear them in this medium. The fully grown larvæ and puparia were, on the other hand, able to withstand the effect of fresh water.

The surprising adaptability of the Ephydrid larva is well illustrated by the Californian "petroleum fly," *Psilopa petrolei* Coq. whose larvæ inhabit pools of crude petroleum in large numbers where they probably feed upon flying insects that drop into the oil and are drowned. The habits of this remarkable fly were first described by Howard ('99) and the adult by Coquillett ('99), and the species has more recently been studied by Crawford ('12).

With these facts in mind I fully expected to find the larvæ of Ephydra breeding, perhaps quite commonly, in the thermal waters of Yellowstone Park. Much to my disappointment no larvæ were found during the first two weeks of our stay, but finally in the Norris Geyser Basin one pool yielded some larvæ and puparia, and later in the day they were found again about half a mile further south in the overflow of water running through a sulphurous marsh. In both places the water was only slightly tepid, and filled with a luxuriant growth of a filamentous alga. Both lots of larvæ were imbedded in the mass of alga from which they were later extricated only after considerable manipulation under the binocular microscope.

This species proves to be a true Ephydra and the adult has been kindly identified for me by E. T. Cresson, Jr., as *E. pectinulata* which he described (Cresson '16) from 40 miles east of Lusk in southeastern Wyoming. A description of the larva and puparium is appended.

Ephydra pectinulata Cresson.

(Fig. **5**.)

Larva. Total length 6 mm.; of anal tube 0.6–0.9 mm.; respiratory tube 0.4–0.6. Body pale yellowish or nearly white, without darker markings except those due to the varying density of the spiny vestiture. Thus in dorsal view there are darker bands where the integument is ridged, transverse dorsally and strongly bent and curved laterally; in lateral view these are strongly sinuous, maintaining in general a more or less vertical direction. Prolegs of about equal size, slightly larger posteriorly, with the last pair very much larger; each with about four strong curved hooks which curve forwards on all except the last leg, where the position is reversed and the spines curve strongly forward. Anal respiratory tubes slightly divergent, not sufficiently extruded in any specimens I have seen to show their basal union.

As in most members of the genus, the larva is entirely pale, without darker markings. The thorn-like body bristles are intensely black and appear to vary in size considerably, but this is due almost entirely to the deeply furrowed integument which causes the bristles to be viewed at different angles when the larva is examined under the microscope.

Puparium. Entire length 8.5–9 mm.; apical part from last proleg to fork 1.7–2.0 mm., prong of respiratory fork 0.6–0.9 mm. Very dark brown, the posterior tubular portion yellow; respiratory fork pale, with black tips. Body densely clothed with very minute black thorn-like bristles, those on the upper surface larger than those below and those on the posterior part of the body coarser, especially on the apical tube. On each of the two segments preceding the tube there is on the dorsal surface a small patch of somewhat larger bristles. First to fourth prolegs small, scarcely elevated, each armed with about four well-developed small hooks surrounded by a few thorns noticeably larger than those covering the body; fifth and sixth prolegs increasingly much larger but with the hooks no larger; seventh slightly smaller than the sixth; eighth the largest and overhanging the seventh, its hooks curved forwards while those on all the other legs curve backwards. Behind the prolegs near the base of the tube is a small tubercle on each side below the lateral line.

The puparium resembles closely those of other species of the genus described by Aldrich ('12). As will be apparent from the measurements given above there is little variation in size, except in the length of the respiratory prongs which are extruded much further in some specimens than in others.

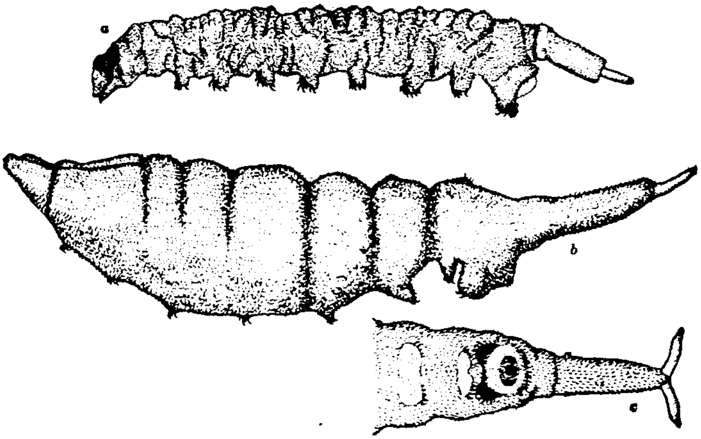

FIGURE 5. *Ephydra pectinulata Cress.* *a*, larva in lateral view; *b*, puparium in lateral view; *c*, posterior part of puparium in ventral view.

I have been able to find only a single reference to the occurrence of any other Ephydridæ in thermal waters. In Japan Matsumara ('15) found a species of Scatella (*S. calida*) in a hot spring.

On account of the long anal tube, these Ephydra larvæ and puparia recall those of the Syrphid fly, Eristalis which occurs also, though rarely, in sea water. Packard ('69b) years ago reported a species of this genus from salt pits and recently Lamb ('11) has reared *E.* (*Lathryophthalmus*) *æneus* Scop. from larvæ found in rock pools on the English coast. Ping ('21) also mentions the occurrence of rat-tailed maggots, which are no doubt a species of Eristalis in artificial salt pools near Ithaca, New York, having a salinity as high as 9%.

ORDER COLEOPTERA.

The members of this order occur more generally and abundantly in thermal waters than those of any other group of insects, and moreover, there appear to be some species that are restricted to warm springs and able to thrive at somewhat higher temperatures than those generally suitable for animal life.

Quite naturally a very large part of the references to the Coleoptera of thermal waters relate to the European fauna. In his extensive studies of the animal life in the various Italian hot springs, Issel ('01, '10) lists a considerable series of beetles distributed among four families. These include three species of Dytiscidæ, *Hydroporus tessellatus*, living in water at 30°–35° C., *Bidessus thermalis* (45°) and *Bidessus geminus* (30°–42°); two species of Gyrinidæ *Gyrinus elongatus* (30°–35°) and *G. distinctus* (30°–37°); seven species of Hydrophilidæ belonging to six widely separated genera, *Hydroscapha gyrinoides* (30°–46°), *Helophorus alternans* (39°), *Berosus affinis* (30°–35°), *Helochares dilutus* (39°–40°), *Laccobius gracilis* (35°–40°), *L. gracilis*, var. *sardous* (= *sellæ*) (40°–45°), and *Cœlostoma* (*Cyclonotum*) *hispanicum* (37°–38°); and one Dryopid, *Dryops luridus* (37°–38°). From the hot springs of Fordongianus on Sardinia Krausse ('13) lists one Dytiscid, *Hydroporus pubescens* and a number of Hydrophilids, *Octhebius impressus*, *O. pusillus*, *O. nanus*, *Limnebius evanescens* and *Helochares thermicola* (?). Four other water beetles are listed by Plateau ('72) as occurring in various European hot springs: *Agabus bipustulatus* at Ems (38° C.), *Hydroporus transversalis* at Loèche-les-Bains (39°), *Hydaticus transversalis* at Luxeuil and Ems (39°), and *Hydrous caraboides* at Néris (40°).

From these citations it is evident that a great many palæarctic beetles have taken up their abode in warm springs, some of them apparently being restricted to this habitat (*Laccobius gracilis* (= *sellæ*)), and others, like Hydroscapha, extending their tropical range very considerably by availing themselves of thermal water. It will be noticed also that these two occur at the warmest temperatures, 45°–46° C., several degrees higher than can be endured by unacclimatized animals. Issel ('08) calls attention also to the very sporadic and localized occurrence of these two species.

Similar observations have been made in many localities, and even in India the botanist Hooker, many years ago ('54), found a water-beetle provisionally identified as Colymbetes in water at 44°–45° C. (112° Fahr.) in the hot springs at Soorujkoond in the Behar Hills.

In America Schwarz ('14) has published a rather extensive list of

Coleoptera collected by himself and Mr. H. S. Barber at Hot Springs, Castle Creek Canyon, Arizona, in the pools and small stream containing warm water flowing from the springs. In the warmest water (42°–46° C.; 110°–115° Fahr.) they found two Hydrophilids, *Hydroscapha natans* and *Helochares normatus*, the latter also in cold water near by. Along a streamlet where the water had cooled to about 38° C. (100° Fahr.) eight species were encountered in addition to these: two Dytiscidæ—*Laccophilus pictus* and *Bidessus subtilis;* five Hydrophilidæ—*Helophorus obscurus*, *Epimetopus thermarum*,[3] *Berosus* sp., *Paracymus ellipsis*[4] and *Chætarthria minor*. In addition, the larva of a Scirtes (?) of the family Helodidæ was present. Of this latter series only one, the Bidessus, was also in the cold water near by, although all except *Paracymus ellipsis* are known from waters of normal temperature. Concerning the latter, Mr. Fall tells me that it is most closely allied to *P. elegans* and *P. lodingi*, the former of which has occurred only in salt or brackish water.

A most interesting fact appears from these observations; that Hydroscapha has taken up its abode in certain hot springs of both hemispheres, and that in both places it inhabits water of unusually high temperature.

According to Sherman ('13), *Deronectes striatellus*, a Dytiscid occurring in the southwestern United States, has been found by Schwarz in water of 40.5 C. (105° Fahr.).

Relating to the coleopterous fauna of the hot springs of Yellowstone Park, there is a very interesting account by Hubbard ('92), published in the form of a letter which contains a number of references to species observed by its writer. These include two Hydrophilidæ, a species of Enochrus (Philhydrus) living in tepid water at Mammoth Hot Springs and an Ochthebius and its larva inhabiting water which is very much warmer. It is probable that the Octhebius was *O. interruptus* and the Enochrus, *E. carinatus*. Hubbard found also two Helmidæ, which he refers to the genus Helmis, one in running tepid water and the other in the Firehole River below the overflow from the Excelsior Geyser at a temperature which he estimated as 21°–24° C. (70°–75° Fahr.). He also refers to several beetles found in the warm soil about the geysers and hot pools—Nebria, Bembidium and several other Carabids (Amara, Pterostichus and Patrobus). None of these appeared however to be peculiar to the warm springs.

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[3] Not described till the following year by Schwarz and Barber ('17).

[4] This was listed by Schwarz as *P. subcupreus*, but later described by Fall ('10) as a new species.

My own collections contain six species of aquatic Coleoptera from the thermal waters of Yellowstone Park, three species of Dytiscidæ and three Hydrophilidæ. For their identification I am deeply indebted to Mr. H. S. Barber who kindly examined the collection, as well as to Mr. H. C. Fall who determined two species which were not in the collection of the U. S. National Museum and further gave me a number of interesting notes on other water-beetles which have been incorporated in the present account.

Bath Lake, near the Mammoth Hot Springs, proved to be the most lucrative spot as five species were found there, beneath the fragments of travertine which line the edge of this pond. The most abundant form is *Tropisternus californicus,* a species that has evidently extended its range in this instance. Another smaller Hydrophilid, *Enochrus carinatus,* was obtained in smaller numbers; this is a species characteristic of the Pacific slope, where it has been reported from Oregon to southern California.

Three Dytiscids were taken in Bath Lake also: *Hydroporus humeralis,* a widespread boreal species; *Deronectes griseostratus,* another boreal form; and *Coelambus sellatus,* originally described from the Dakotas. The temperature of Bath Lake is 33° C. (91° Fahr.)

At West Thumb two species of beetles were obtained. In a slightly warm sulphurous pool containing great quantities of a Chara-like plant the same Tropisternus (*T. californicus*) was present in smaller numbers than in Bath Lake. In the running water which forms the overflow from hot springs at this place another small Hydrophilid not seen elsewhere was abundant, hidden beneath the sheets of highly porous travertine under which the water flows. At this point the temperature of the water ranges about 38° C. (100° Fahr.).

Larvæ of one Hydrophilid were numerous both at Bath Lake and West Thumb. These agree closely with the descriptions and figures of species of Tropisternus recently published by Wilson ('23) which, together with their large size, indicates without doubt that they are the larvæ of *Tropisternus californicus.*

In his account of the aquatic invertebrate fauna of Yellowstone Park Forbes ('93) mentions the occurrence of a species of Coelambus in the overflow water that fills a lagoon on the west side of Lewis Lake. This is no doubt the *Coelambus sellatus* mentioned above. At the same place Forbes obtained several Hydrophilid larvæ which appear never to have been identified.

In the case of the Coleoptera inhabiting thermal waters, it is less satisfactory to draw conclusions concerning their relationships to marine, brackish-water or alkaline-water forms than it is in the orders of insects previously considered. In the first place there seem to be almost no truly marine coleoptera, and the aquatic habit among the larvæ of beetles is restricted to only a few families. In these, however, it occurs almost universally, as for example in the Dytiscidæ, Gyrinidæ, Hydrophilidæ, Dryopidæ and Helmidæ. Nevertheless it is evident that several of the genera mentioned above as inhabitants of thermal waters are represented in the saline and brackish-water fauna. Thus forms of Coelambus and Hydroporus are characteristic of the maritime and submaritime fauna of England (Keys '18), and this is true also of Gyrinus (*e. g. G. marinus*). Similarly, a species of Hydroporus is a characteristic inhabitant of the salt marshes near Odessa on the Black Sea (Butschinsky '00). In America Packard ('73) reported *Laccophilus decipiens* from the saline pools near Great Salt Lake, Utah, and Sherman ('13) mentions six genera of Dytiscidæ that are represented in salt marshes: Coelambus, Coptotomus, Laccophilus, Thermonectes, Agabus and Copelatus, and of these it will be noticed that two, Coelambus and Laccophilus, occur in thermal waters, of which they are, moreover, particularly characteristic. Several names have been given to the Laccophilus which occurs so generally in European hot springs, but according to Issel ('10) all are to be referred to a single species, *L. gracilis* (= *L. subtilis*) and its variety *L. gracilis* var. *sardous*, of which *L. sellæ* and *L. thermarius* are synonyms. Sherman believes that no North American species are restricted to a saline environment. Among the Hydrophilidæ, however, Leng ('13) speaks of *Enochrus* (*Philhydrus*) *hamiltoni* as restricted to salt marshes, and it will be noted that this genus occurs in thermal waters also. Other species that may live in brackish water he believes occur in greater numbers in fresh water. It is of interest to note, however, that Packard ('73) reported *Berosus punctatissimus* from Great Salt Lake and Fall ('01) described *B. salinus*, a salt-water species from Lower California; and that *Berosus affinis* occurs, as noted above, in hot springs in Europe. *Tropisternus salsamentus* (Fall '01) is also a salt-water species, and Mr. Fall tells me that he has taken it together with *Berosus salinus* and *Paracymus elegans* in a small salt lake or pond with a saline content greatly in excess of the ocean itself.

As to the exact concentration of salt which may be endured by Hydrophilid larvæ and adults, Florentin ('99) records Hydrophilus larvæ in water containing 123 grams of salt per litre and *Acanthoberosus spinosus* in water containing 100 grams per litre.

ORDER TRICHOPTERA.

Caddis-fly larvæ also occur in the sulphurous ponds, although not so abundantly as in the clear cool water of the mountain streams. In one large pool on the south side of the road to Soda Butte, about one mile east of Tower Falls, the cases of four species were taken in numbers, deserted however, as the larvæ had already completed their transformations. From a comparison of the cases (Pl. 1, figs. 3, 4, 5) with the figures in Lloyd's recent paper on caddis-fly larvæ ('21), it seems quite probable that all may be referred to the family Limnophilidæ.

The first published reference to the occurrence of Trichopterous larvæ in any of the thermal waters or sulphurous pools of Yellowstone Park is contained in Peale's account ('83) which forms a part of Hayden's geological report for 1882. There he notes the discovery of larvæ of a species of Helicopsyche by Mr. W. R. Taggart in water having a temperature of 42° C. (108° Fahr.). Forbes ('93) refers to the occurrence of trichopteran larvæ ("case-worms") in a warm spring near the western side of Lewis Lake at 65° C. (150° Fahr.) but undoubtedly the larvæ were in a cooler part of the pool. Helicopsyche has never been reported from such waters in other parts of the world, nor is it ever found in salt or brackish water.

In Europe Perty ('41) found larvæ of an undetermined Phryganeid in the hot springs of Lenk, Switzerland. The temperature was not recorded, but the maximum in these waters is 51° C. (123° Fahr.)

One member of the order Trichoptera lives in tidal waters in Europe. This is *Agrypnetes crassicornis* McLachl. According to Rousseau ('21) the larvæ of this insect are met with mainly on the sea-shore in brackish water, where they live among Fucus and other marine algæ, although they are able to develop equally well in fresh water. In Great Britain Parke ('63) found the larvæ of *Limnephilus pellucidus* living in brackish water. Miall ('95) refers to observations of the Rev. A. E. Eaton who found an abundance of small Hydroptilidæ developing in salt or brackish water near the borders of the African Sahara. Even in New Zealand McLachlan describes the habits of a caddis-worm which occurs between the tide marks, where it utilizes bits of coralline seaweed for the construction of its case, behavior which at once stamps it as a truly marine insect.

ORDER ODONATA.

The occurrence of nymphs of Odonata in brackish water is well known and their behavior in water of different salinities has been examined experimentally by Osburn ('06). He found the nymphs of various species in brackish water on both the northern Pacific coast of America and in New England, but found none restricted to the salt marshes. Similar habits have been observed among European species, Indian ones (Annandale '07) and in Australia by Tillyard ('17). Osburn determined that a density of about 1.01 (less than half that of sea water) did not prevent the hatching of the eggs of several species, nor the development of the nymphs, but that they were unable to develop in water of greater salinity. One record of a nymph of a Libellula in Europe is given by Florentin ('99), who found it in water containing 18 grams of salt per litre.

In alkaline or sulphurous waters fewer observations have been made, but Schwarz ('91) records the presence of dragon-fly nymphs in the salt flats bordering Great Salt Lake, Utah, living in brackish and sulphurous water. These are possibly the Argia referred to in the next paragraph. In a highly alkaline lake at Roswell, New Mexico, and also in a strongly alkaline stream in the Arroyo Pecos, near Las Vegas in the same state, Needham and Cockerell ('03) report the presence of nymphs of several species.

In thermal waters Odonata do not appear to occur very generally, although there are a number of cases on record. As some of these show quite clearly how the opportunity to breed in warm water has extended the range of tropical or subtropical forms, I have been tempted to refer to them in detail. Cockerell found nymphs of the Agrionid *Hyponeura lugens* Selys present in the Gallinas River at Las Vegas, New Mexico, at a point where the water is perceptibly warmed by the addition of water entering from the hot springs at that place (Needham and Cockerell '03). This species belongs to the upper Sonoran fauna and Las Vegas is in the Transition Zone. At the same place Cockerell obtained nymphs of an undetermined species of Argia which was later figured by Needham ('04), who records its occurrence also in White Sulphur Springs near Great Salt Lake, Utah, and in Yellowstone Park. Possibly the nymphs from the Yellowstone were living in warm water there also. Needham (*loc. cit.*) records *Libellula saturata* Uhler, a Sonoran species, from Yellowstone on the basis of nymphs obtained there, presumably in warm water, by S. A. Forbes, and an adult taken by O. S. Westcott. These nymphs are possibly the "libellulid larvæ" mentioned by Forbes ('93) which he found in a lagoon filled with the overflow waters of a geyser at the western side of Lewis Lake. This locality is the northernmost record for the species.

The occurrence of unidentified Odonate nymphs in hot springs in California has been reported by Kellicott ('97). The nymphs were obtained in a pool in Lassen County, California, said to be near the boiling point at one end and of blood heat at the other, the insects occurring in the warmer parts of the pool as well as at the cooler end. His figures show them to be of the broad libelluloid type.

In the slightly warm alkaline thermal waters, probably about 75° Fahr., of Lake Huacachina in Peru, Tamayo and Garcia ('06) have reported the occurrence of Libellulid nymphs in large numbers.

ORDER PLECTOPTERA.

The nymphs of an undetermined may-fly have been recorded from Yellowstone Park by Forbes ('93). They were collected in a warm spring near the west shore of Lewis Lake, at a temperature said to have been 66° C. (150° Fahr.), although it would seem almost certain that this is an error and that the temperature must have been considerably lower.

In Europe the nymphs of one species, *Chloë diptera,* have been found living in the hot springs of Luxeuil in water of 45° C. (113° Fahr.) according to Plateau ('72). This is an unusually high temperature for an insect.

ORDER HEMIPTERA.

The only Hemiptera collected were numerous specimens, both nymphs and adults, of *Ambrysus heidemanni* Montandon. These were kindly identified for me by Professor H. M. Parshley. The bugs were in warm running water (96° Fahr.) in the overflow from Firehole Lake. This water is very heavily charged with silica, which is deposited in a thick layer on all objects in the stream. This species was originally collected in thermal water in Yellowstone Park and as it does not appear to have been taken elsewhere is probably a member of the true thermal fauna.

Members of another family *Corixa hieriglyphica* Duf. have been found in thermal waters in Europe. Issel ('00) found this species living in water at 40° C. In Yellowstone Park Hubbard ('92) saw an unidentified Corixa in a small tepid stream which was depositing silica rapidly upon submerged objects, and Forbes ('93) found a member of the same genus in the warm overflow waters of a geyser that form a lagoon on the west side of Lewis Lake. The European *Notonecta glauca* is said by Plateau ('72) to occur at Vichy in eastern France in water at 19°–45° C. (57°–113° Fahr.) in company with *Nepa cinerea.*

ARACHNIDA.

A few specimens of an undetermined species of Pirata were obtained in the porous siliceous crust over running warm water in the overflow from boiling springs at West Thumb. Dr. R. V. Chamberlin tells me that he has observed certain species of this genus associated with other hot springs in the western United States.

Spiders have been reported several times at least as associated with hot springs. One said to be a Lycosa, but quite possibly also a Pirata, occurs about the boiling pools at Hammam Meskoutine in Algeria where Gervais ('49) observed them running about over the hot soil and even venturing onto the surface of the hot water in small craters. The occurrence of spiders was also noted years ago on the hot soil about a thermal spring in Owen's Valley, California, by Wood ('68).

Another Arachnid, the mite, *Hydrachna cruenta*, has been found according to Plateau ('72) at Luxeuil in water of 46° C. (115° Fahr.), an unusually high temperature for an Arthropod.

CRUSTACEA.

No special search was made for Crustaceans in the thermal waters of the Park and any extremely minute forms which may exist there were not collected. One species of Amphipoda was, however, so abundant in several places that numerous specimens were preserved. This proves to be *Gammarus limnæus* Smith, a widespread form, whose distribution is so interesting that I have been led to consider it in the present account.

It was described by Smith ('75) from material collected by the naturalists accompanying the Hayden exploration party in a cool spring in the Firehole Basin, Yellowstone Park, and in a lake near Long's Peak, Colorado. Smith described also from the Wasatch Mountains, Utah, *Gammarus robustus*, now regarded as a synonym of *G. limnæus*. It was later reported as *robustus* by Forbes as present about Lewis Lake, in a warm lagoon on the west shore and in a spring near by. The temperature of the water in the latter is given as 150° Fahr., but this is evidently in error. My specimens agree perfectly with the description and figures given by Weckel ('07) who records the species also from localities in Maine, Michigan, Wisconsin, New Mexico, Idaho, Utah and Montana. Later, Pearse ('14) found it in material from northeastern Nevada at elevations of about 5,000 feet, and Shoemaker ('20) found it well represented in the collections of the Canadian Arctic expedition made in Alaska and the Canadian northwest territories. In one case specimens were secured on June 23 under two metres of ice in an inland lake near Bernard Harbour, and in another from "Warm Creek" near Camden, Alaska. Whether this creek actually contains warm water is not stated, but in his report on the algæ Lowe ('23) says that this creek comes from some warm springs further inland, and that it contains calcareous sinter with imbedded algæ.

The present specimens were collected in the swiftly running overflow from boiling springs at West Thumb and in the outlet to Firehole Lake where the water had cooled to about 100° Fahr., as well as in a sulphurous pond east of Tower Falls, where they were especially abundant.

This species occurs therefore not only across the entire northern part of North America, but thrives beneath the ice as well as in warm water. It is one of a number of fresh-water species of the genus which includes also typical marine forms. In view of the classical case of the Phyllopod Crustacean, Artemia, and other no less interesting experiments on other genera of this group of animals (Cyclops, Daphnia, *et al.*) made by Schmankewitsch ('72, '75, '77), subsequently substantiated in additional forms of Artemia in other parts of the world (v. Günther '99 and Kellogg '06) where certain morphological differences depend upon the salinity of the water,[5] it is natural to inquire whether such variations occur in a species like the Gammarus here considered. So far as I can ascertain such variations do not exist, at least in relation to habitat. Smith did erect two species to include the present *G. limnæus* but his forms *robustus* and *limnæus*, although differing in slight details, appear to bear no relation to habitat. Certainly my specimens do not approach in any way related species, *e. g.* *G. pribilofensis* Pearse.

We do find, however, as with most of the groups of insects, where those including marine species have also forms adpated to life in alkaline or thermal waters, that the genus Gammarus is no exception. Observations have been made showing the ability of a fresh-water species to withstand saline water. Chevreux and Guerne ('92) found *Gammarus rhipidiophorus* Costa living in pools close to the shore of the Mediterranean where the water becomes brackish during the summer, due to a temporary connection with the sea. *Cyclops bicuspidatus* Claus, a fresh-water copepod of Europe, also occurs in salt pools and in this species Florentin ('99) states that those in brackish water actually acquire varietal differences like the forms studied by Schmankewitsch. This genus is also represented in thermal waters (Pavesi '81).

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[5] Günther (*loc. cit.*) states that the morphological characteristics of Artemia are so closely correlated with the density of the water that they furnish a determination of the salinity with an error of less than .03 in specific gravity.

Gammarus locusta of Europe occurs similarly in thermal waters, and has been found in the warm waters that discharge from the hot springs at Abano in northern Italy (Martens '24) and *G. roeselii* is known from two localities in hot springs (Plateau '72), Piemont at 34.4° C. and Gastein at 36° C.

Another Amphipod, *Hyalella knickerbockeri* Bate, has been reported from Yellowstone Park. This is the *Allorchestes dentatus* discovered by Forbes ('93) in thermal water at two stations near the west shore of Lewis Lake. He found this species in a lagoon which is filled with the overflow waters from a geyser and also in a warm spring. The temperature of the latter is given as 65° C. (150° Fahr.), but this certainly cannot refer to the part of the pool in which the crustaceans were living. This species enjoys a very wide distribution across North America from Maine to California and extends into the American tropics. It varies considerably and has received several names (*Hyalella dentata* Smith ('75), *Hyalella inermis* Smith ('75), and *Hyalella faxoni* Stebbing ('03)) since it was originally described by Bate ('62), but Weckel ('07) considers the several forms under one name. From a search of the literature I have been unable to find any association between habitat and the several variations, although it is quite possible that some such correlation exists. Professor T. D. A. Cockerell writes me that he collected this species in hot water at Las Vegas Hot Springs, New Mexico, so that it is undoubtedly a common inhabitant of thermal waters.

Cypris balnearia Moniez occurs in the well-known Algerian thermal waters at Hammam-Meskhoutine, where Moniez ('93) found this species so abundant in water at 45°–50.5° C. that it formed a sort of continuous chocolate-colored zone along the edge of the water. An undetermined Cypris was found there previously by Gervais ('48) in water at 40° C., probably the same species, although Moniez does not mention it, having probably overlooked Gervais' short note. At Ischia in Italy, Pavesi ('81) observed *Cypris thermalis* Costa already known by its describer to be an inhabitant of hot springs. Another member of the genus, *C. fuscata*, enters the thermal fauna at Néris (34.4° C.) and in the Hungarian springs at Gastein (36° C.). Gervais (*loc. cit.*) also reports the European fresh-water crab, *Telphusa fluviatilis*, at Hammam-Meskhoutine in the overflow water having a temperature of 36°–40° C., and Studer ('89) reported a Papuan crab of the genus Ocypoda on the coast of New Guinea in water heated to 50° C. by hot springs that discharge into the sea.

Even Isopod crustaceans have been found in thermal waters, perhaps not so surprising in an essentially marine group of highly adaptive animals that have developed fresh-water, terrestrial and parasitic forms. Thus in the family Sphæromidæ, typically marine, there are a number of fresh-water species and two that have been found so far only in thermal waters. They are *Exosphæroma dugesi* Dollfus ('93) from Aguascalientes, Mexico, and *E. thermophilum* Richardson ('97, '04, '05), collected by Cockerell in a warm spring near Socorro, New Mexico. The palæarctic *Sphæroma rugicanda* Leach occurs in brackish water and two species of Cæcophæroma (*C. virei* Dollfus and *C. burgundum*) inhabit the waters of the Grottos of the Jura.

Many years ago Woodward ('79) described two fossil species of Eosphæroma from the Gurnet Bay deposits in England. These beds, as noted elsewhere in the present paper, were undoubtedly laid down in connection with hot springs and the finding of these species shows that the occurrence of Sphæromidæ in thermal waters dates back at least as far as the Oligocene.

The common fresh-water *Asellus aquaticus* of Europe is sometimes found in hot springs. Plateau ('72) records it from Loèche-les-Bains in water at 42.5° C. (108.5 Fahr.).

The same author records the small copepod, *Cyclops quadricornis*, from Néris and Gastein in water at 34.4° C. and 36° respectively.

Ziegelmayer ('24) reports the occurrence of *Cyclops serrulatus* or a closely related species in the sulphur springs, Aque Albule, in Italy at a temperature of 21°–23° C. (70°–73° Fahr.). By analysis this water appeared to be entirely devoid of dissolved oxygen and thus presents a similar condition to that referred to in connection with the blood-worms of the genus Chironomus mentioned on a previous page, except that in this case the Cyclops possesses no hæmoglobin which might aid it in utilizing such minute quantities of oxygen as may exist in this water. Ziegelmayer found these crustaceans covered with tufts of filaments of sulphur bacteria which are attached to certain parts of the body where a pitted structure of the chitinous cuticula affords them a foothold, and he would interpret the association as a symbiotic one.

MOLLUSCA.

Molluscs do not appear to be so abundant in the thermal waters of Yellowstone Park as might be expected, to judge from the considerable variety that has been reported from European thermal springs. Those which I obtained are snails belonging to three genera, Physa, Limnæa and Amnicola. Previously Forbes ('93) has reported "a small Physa" in the overflow water from a geyser at the western shore of Lewis Lake.

In the small stream which forms the outlet to Firehole Lake there were large numbers of a small snail which Dr. H. A. Pilsbry has kindly identified as a dwarf form of *Physa heterostropha* Say. They were in company with the larvæ of Tabanus, the water-bug Ambrysus and the Crustacean, Gammarus, discussed in another part of the present account. This water was at a temperature of 96° Fahr., and is so heavily charged with silica that all objects in the water become coated with a thick blackish coating of this mineral. This dwarf form is, according to Dr. Pilsbry, the one described by Lea ('39) as *Physa aurea* from a hot spring in Bath County, Virginia. Later Binney ('65) suspected this to be a form of the common *P. heterostropha* and quotes Lea's account of its habitat, which is so striking that I have been tempted to transcribe it again: "Mr. Nicklin informed me that he found the *Physa aurea* in a little water course by which a hot and a cold spring discharge their mingled waters. The former exhibits a temperature of 106° and the latter of about 56° of the scale of Fahrenheit. Near the meeting of the waters, one side of the little stream is cold and the other side hot; and multitudes of these beautiful Physæ are to be found on both sides of the line of junction, availing themselves of the power which the locality affords them of changing their climate according to their fancy." *Physa heterostropha* var. *elliptica* has been recorded by Call (*teste* Stearns '01) in Warm Spring Lake, near Salt Lake City, Utah, and the variety *gyrina* from Hot Springs, Panamint Co., California (Stearns '01). In connection with these forms and others referred to in his paper, Stearns found no indication of any dwarfing of the individuals living in thermal waters, although he believes that depauperation is association with brackish water. Still another Physa (*P. heterostropha* subsp. *cupreonitens*) has been described by Cockerell ('89) from hot springs at Wellsville, Arkansas.

Aside from the above, there are references to the occurrence of Physas in European thermal waters. Dubalen ('73) found *Physa* *acuta* Drap. in the warm springs of Dax in southern France at temperatures ranging from 88°–91° Fahr., but Issel ('01) does not include the genus in his rather extensive list of molluscs inhabiting the Italian warm springs. A variety of *P. acuta* is mentioned by De Varigny from thermal water at 77°–86 Fahr. near La Preste in the eastern Pyrenees and a form of this species was reported by Dupuy ('77) and Fischer ('81) at Barbotan in southern France at 86° Fahr. This variety is no doubt *Physa acuta thermalis* Dupuy known together with *P. acuta gibbosa* Massòt and *P. ataxiaca* Fagot to occur in the Pyrenees only in thermal waters (Gineste '85), where they invade water up to 27°–30° C. *P. acuta* is also recorded from artesian water at Rochefort at a temperature of 32.5°–33° C. (Regelsperger '83, '85).

It is possible that an old reference to a strongly ribbed Physa (*P. costata* Newc.) in Clear Lake, California, by Newcomb ('63, *v.* also Stearns '01), may refer to a species in a near-by alkaline pond (Borax Lake) as there has been considerable confusion in the past concerning the provenience of material from these two lakes, one of which is fresh; the strongly costate shell suggests brackish or alkaline water. In the palaearctic region *Physa fontinalis* occurs in brackish water (Parke '63) and *P. acuta* in salty pools (Coutagne '82).

Lymnæa palustris Müller (identified by Dr. H. A. Pilsbry) was found in large numbers in a warm sulphurous pool formed by the overflow from a hot spring near Mary's Bay, close to the northern shore of Yellowstone Lake. The snails did not extend into the warmer part of the stream where the tadpoles mentioned on another page were found, but were numerous in water of between 80° and 96° Fahr. which contains large quantities of filamentous green algæ. This widespread holarctic species is reported by Issel ('01) from thermal waters in Italy at a temperature of 61° Fahr.

Another species of Lymnæa, unfortunately not represented by any mature individuals, was taken in warm flowing water in the Yellowstone Canyon, near the level of the river and not far below the lower falls.

Issel ('01, '06) found *Lymnæa peregra* and *L. truncatula*, which is probably only a variety of the former, in water ranging from 77°–90° Fahr. (*truncatula*) and at 72.5° (*peregra*) in Italy. *L. peregra* is a common inhabitant of thermal springs in other parts of Europe, occurring in the springs at Barbotan at 86° (Fischer '87) and 88° (Dupuy '77), at Aix at 45° C. (Gineste '85), at Baden at 46° C. (Martens), at Bagnaralo at 37°–38° C. (Issel '06) and in Staffordshire in England at 36.7° C. (Masefield '96). The varieties *truncatula* and *geisericola* are known from the sulphurous thermal waters of Iceland at 43° C. (Mörch '68, Sikes '13). *Lymnæa vulgaris* occurs in abundance in thermal waters near Lyon according to Locard, and De Varigny records the same species in a creek arising from these springs in water at 77° Fahr. but the individuals were about half the usual size, thus exhibiting the dwarfing noticed elsewhere among many of the molluscs that inhabit thermal waters. This is probably due to some extent to the greater amount of dissolved salts, since fresh-water molluscs which occur in brackish water are generally dwarfed (De Folin '79, Pelseneer '20, Stearns '01). According to Gineste ('85), in the Pyrenees *Lymnæa thermalis* requires warm water, while *L. peregra* is indifferent in this respect. He did not find the former in water above 27°, however, which is far below the temperature at which *peregra* occurs. *Lymnæa peregra* occurs even in very cold water, as its variety *glacialis* is an inhabitant of cold mountain springs. It is also rather widely distributed in brackish and salt pools such as the salt marshes of the Baltic (Pelseneer '20), in the Berre salt pool near Marseille (Gourret '97), in brackish ponds along the Aral Sea (Pelseneer '20), and in the north of England (Cooper '10). In America Dall ('19) found *Lymnæa caperata* Say in the collections made by Mr. Fritz Johansen in northern Alaska. These specimens came from Hot Spring Creek, tributary to the Sadlerochit River, 25 miles inland from Camden Bay on the Arctic coast. This is true thermal water, but the temperature is not indicated.

Amnicola is represented in the sulphurous pools of Yellowstone Park, as a somewhat imperfect shell of a member of this genus forms a part of a caddis-worm case (see Pl. 1, fig. 3) which I obtained in a highly sulphurous pond near Junction Butte, a short distance east of Tower Falls. A species of this genus has been reported from Hot Springs, Bath Co., Virginia (Gould '45), which is possibly *Bithynella nickliniana* Lea (*Paludina* Lea; *Amnicola* Haldeman) (Binney '65b).

The common *Paludina* (*Viviparа*) *muriatica* Lam. of southern Europe occurs generally in fresh water and is known from thermal waters. Linnæus applied the name of *Turbo thermalis* to a species common in hot springs, *Bythinella thermalis*, now known (Issel '06) to occur in water up to 46° C.

In addition to the above, Issel ('01) records the following molluscs from hot springs in Italy: *Melanopsis etrusca* (22°–41° C.), *Theodoxia prevostiana* (32° C.), *T. fluviatilis* (16°–26° C.), *Bythinia tentaculata* (25° C.), *Bythinella etrusca* (25° C.), *Planorbis lævis* (?) (25° C.), *Valvata piscinalis* (16° C.), *Ancylus lacustris* (25° C.).

Among the Pelecypoda, representatives of two genera have been found in thermal waters in Europe, *Pisidium cazertanum*, var. *thermale* by De Varigny at Evaux (24° C.) and by Dupuy ('77) at Cauterets (39° C.). *Unio requieni* has been reported at Barbotan by Dupuy ('77) in water at 31° C. In America Forbes ('93) mentions the occurrence of an undetermined Pisidium in a warm spring near the west shore of Lewis Lake. These are said to have been in water at 150° Fahr. (65° C.) but this must be a mistake.

From the foregoing it will be seen that very high temperatures have been reported for some European molluscs. De Varigny ('93) mentions the occurrence of *Bythinella thermalis* at 37.7° C., but Issel reported the exceptional occurrence of the same species at 46° and its regular appearance at 42° C. Fischer ('87) believed that 42° was the upper limit for molluscs, based on the presence of a species of Hydrobia in the Algerian springs near Bône. This agrees closely with Issel's findings and with observations on American species. Only one apparently authentic record greatly exceeds these. Studer ('90) observed *Neritina thermophila* in New Britain in water at 52° C., a record accepted without comment by Pelseneer ('20). Studer also reported *Ostrea cucullata* growing in brackish water at the same place, heated to 45° C. by water from hot springs that flows into the ocean

FISHES.

The present writer made no observations on the occurrence of fish in the thermal waters of Yellowstone Park, but Jordan ('89) mentioned three species in his report on the fishes of Yellowstone Park. He found the chub (*Leuciscus atrarius*) in the warm water of Witch Creek at 85° (29° C.) and the sucker (*Catostomus griseus*) in the Gardiner River below the mouth of Hot River, at Mammoth Hot Springs where the water reached a temperature of 88° Fahr. (31° C.). He also found young trout in water of 75° Fahr. (24° C.). Exactly similar observations are reported by Doflein ('14), apparently quoted from Jordan's paper.

The many records of fish in thermal waters in Europe, Africa and other parts of the world indicate that their tolerance extends to about 40° C. (104° Fahr.). Thus Gervais ('49) found barbels (*Barbus setivimensis*) in water of 36°–40° C. in the outlet to the Hammam Meskoutine springs in Algeria. Desfountaines (De Varigny '93) found *Sparus desfountainesii* in the hot springs at Cafsa at 38° C.

Rochet d'Héricourt found in hot saline springs in Eritrea small fish at 44° C. Several observers in Ceylon have observed fish in the thermal waters which occur there. The temperature of the pools in which these fish were living did not exceed 32° or 33° C.

In Europe Hoppe-Seyler ('75) made careful observations near Battaglia in the Euganean Hills west of Venice and found fish living in a stream having a temperature of 44°–45° C. at the surface, but with cooler water at lower depths (25°). Here the fish remained in the cooler lower strata and occasionally individuals that came to the surface at too hot a place were overcome by heat rigor from which they did not always recover. Similarly, Knauthe ('95) found experimentally that the death temperature for a series of fishes ranged from 23°–37° C.

It would appear, therefore, for fish that the critical point of heat endurance lies about 40° C. (104° Fahr.). Observations attributing greatly higher temperatures to the medium in which they occur are obviously due to erroneous notes or faulty transcription, as for example the oft-quoted account of Sonnerat (1776), who gives a lengthy and highly embellished account of fish living in hot springs in the Philippines at 86° C.

So far as permanent existence and reproduction are concerned, it seems probable that thermophilous fish live only in waters not exceeding 25° or 30° C.; the lower figure is the one set by De Varigny ('93), and agrees quite closely with observations in Italy by Issel ('01), who found *Leuciscus aula* Bonap. and several other species (Blennius, Anguilla,[6] Squalius, Telestes) in water at from 25°–26° C. (77°–79° Fahr.). Experiments by Maurel and Lagriffe ('99) showed that some species may live at 30°–32°, but that ordinarily above 25 degrees respiratory disturbances set in, usually passing into a "delirium" at 30° C.

AMPHIBIA.

Large numbers of tadpoles were observed in one locality near the north shore of Yellowstone lake in a very slowly moving stream that has its origin in a hot sulphurous spring. This little stream is very hot at its source, but after a distance of perhaps a hundred feet it gradually broadens out, becomes stagnant and cools rapidly. The coolest parts (50°–70° Fahr.) are well stocked with masses of green filamentous algæ and contained many tadpoles. Further up the stream where the water was warmer, their numbers diminished, but numerous individuals were actively swimming about where the temperature of the water ranged between 104° and 106° Fahr. They did not, however, appear to enter the warmer water farther up stream, although adult frogs frequented the thick vegetation along the sides where the water was still warmer. A number which I attempted to catch in my insect net escaped and appeared to enter the water or soft mud, although observation was difficult on account of the presence of steam vents here and there near the edge.

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[6] An old mention of "anguilles" in the thermal waters at Aix by Saussure (1796) evidently does not refer to eels, although it has often thus been quoted. At that early date Saussure had evidently taken great pains to search for living things and speaks of "des rotifères, des anguilles et d'autres animaux des infusions" in these waters.

Dr. Thomas Barbour has kindly examined the tadpoles and assures me that they undoubtedly belong to the genus Rana, and that they are quite probably *Rana temporaria pretiosa* Baird and Girard. This species has been reported from Yellowstone Park by Cope ('89) on the basis of specimens collected by Merriam in the Upper Firehole Basin, and Cope (*loc. cit.*) believes that the *Rana septentrionalis* mentioned by him ('72) may be the same form. It was from Carrinton's Lake.

Of special interest in connection with these tadpoles are some experiments made a number of years ago by Davenport and Castle ('96) on tadpoles of the common toad of the eastern United States, *Bufo lentiginosus*.

They found that tadpoles reared in water at an ordinary temperature of about 60° Fahr. (15° C.) were unable to withstand temperatures approximating 104° Fahr. (40° C.) as they passed into heat rigor at about that point (from 39.5° to 41° C.). They then reared the tadpoles from the egg at somewhat higher temperatures, 76° Fahr. for a time (three or four weeks), followed by a temperature of about 92° Fahr. for a varying period of five days to three weeks. After this treatment the tadpoles showed a markedly increased ability to withstand heat and went into heat rigor at from 106°–111° Fahr.

It is evident therefore that the tadpoles from the thermal waters of the Park were living in water at practically the lethal temperature for unacclimatized toad tadpoles, and only a few degrees below that which proved fatal for those acclimatized by prolonged exposure to abnormally high temperatures. There is every reason to suppose that the heat reactions of frog and toad tadpoles differ in no essential way. This is also borne out by experiments made by Maurel and Lagriffe ('00), who found that frogs could not withstand a water temperature above 36°–38° C. There is, as a matter of fact, probably only a slight range of temperature at the higher end of the scale over which these amphibians might become acclimatized, since Vernon ('99) has shown experimentally that the heat rigor is due to an actual physiological state of the muscular tissue. This rigor he obtained in frog and toad muscle at between 38° and 40° C.

Davenport and Castle (*loc. cit.*) cite an old statement, made by Spallanzani in 1777, to the effect that he had observed frogs in thermal water of 46° C. (115° Fahr.), but the accuracy of this is very doubtful. Nevertheless more recent observers have noted batrachians in the thermal waters of the Old World. Gervais ('48) collected adults and tadpoles of *Rana esculenta* Linn. in the overflow from the springs at Hammam-Meskhoutine, in Algeria, at from 36°–40° C. Issel ('01) obtained the same frog from several Italian springs at temperatures ranging from 25° to 40° C., and mentions a toad, *Bufo viridis* Laur., from one spring at 25° C. Hooker ('54) found frogs at the Sooruj-koond hot springs, in India, in water at 32° C. (90° Fahr.).

Tadpoles are, of course, like other amphibians, unable to live in sea water, although it has been shown that they may be gradually accustomed to a very considerable salinity. Thus Yung ('85) was able to bring them into water containing 10 grams of salt per litre without ill effects. That tadpoles may actually develop in brackish water in nature is attested by a statement by Schwartz ('95). In speaking of the saline pools in the flats near the shore of Great Salt Lake, Utah, he says: "Here in the midst of the pools covered with the Ephydras, a commotion was occasionally observed as if a fish of considerable size had risen to the surface of the water. Mr. Hubbard succeeded in capturing one of these mysterious creatures, which proved to be the larva of a toad. It appeared that this tadpole comes to the surface of the water right among the Ephydras, with the dexterous motion of its tail sweeps a goodly number of flies into its wide mouth, and retires again to the bottom of its ill-smelling abode." Coming from two such accurate observers as Schwarz and Hubbard there can be no doubt that tadpoles inhabit these pools, but unfortunately no statement is made as to the probable amount of salt in the particular pools, and the supposition that they are toad tadpoles is probably a mistake as toads transform into adults when very small whereas the larvæ of our common frogs attain much greater size before transformation.

European observers have occasionally found a similar association of palæarctic batrachians with brackish water. Thus Florentin ('99) found tadpoles in water containing 8 grams of salt to the litre and

Boulenger ('97) speaks of the common *Rana esculenta* occurring in water of somewhat higher density (11 grams per litre). He mentions also (Boulenger '99) a young specimen of *Hyla arborea*, collected in salt water by Günther, at Lake Urmi in northwestern Persia. Dr. Thomas Barbour tells me that *Rana virescens sphenocephala* Cope of the southeastern United States lives occasionally in slightly brackish water, although he doubts whether the tadpoles could develop in this medium. Experimental attempts to hatch the eggs of frogs in weak salt water accord with this, as Hertwig ('97) found a concentration of 7 grams of salt per litre soon arrested their development.

INSECTS IN A STEAM VENT.

Scattered over the ground in various parts of the Park, especially where boiling pools or geysers occur, are numerous openings in the soil. Some of these are quite cool, others warm or hot, and from some steam escapes. The cavities into which they lead may in the case of those having a low temperature contain such large quantities of carbon dioxide that small mammals are sometimes overcome and suffocated; the warmer ones emit malodorous gases, probably consisting in great part of hydrogen sulphide, with perhaps the addition of some sulphur dioxide. Those which are actually steaming usually have an unpleasant odor and frequently there is such a copious escape of steam that it is dangerous to attempt to insert the hand or to place the face too close to the opening. In a small one of these extremely hot cavities near Firehole Lake we were surprised to see the ground just inside the opening covered with a mass of insects which had been killed by the heat. By means of a stick we succeeded in extricating a handful of these from the accumulation which appeared to contain a quart or more, and were able to get an idea of what it was composed. The majority were blow-flies (Calliphora), with a number of another smaller blue-green Muscid, but there were numerous insects of other kinds, including two species of wasps (Tachytes and Bembex), a bee, a Hesperiid butterfly, two species of Tipulidæ and a Cerambycid beetle. What had attracted this miscellaneous assemblage is difficult to surmise. The fetid odor may have served as a lure to the blow-flies, and the wasps which store their nests with such flies may have come in pursuit of the latter, but the crane-flies, bees and longicorn beetles could hardly find more than warmth to attract them; once enveloped in the steam, escape is of course impossible.

We noticed many other instances of insects floating on the surface or drifting about in the water of the boiling pools. Many are caught thus and of course instantly killed by contact with the extremely hot water. Like sticks, leaves and other objects in these pools, the insects soon begin to accumulate deposits of calcium carbonate or silica on the surface of the body and sink to the bottom to aid in the formation of the deposits of travertine and sinter. In the calcareous springs the mineral matter forms in irregular granules or flocculent masses and such organic inclusions are not actually fossilized in the true sense. The accompanying photograph (Pl. 1, figs. 1, 2) of a caterpillar taken from a calcareous hot spring shows clearly the process of incrustation.

A published reference to a similar occurrence of dead insects in fumaroles is contained in Moseley's account of the Challenger expedition ('79). He found on the slopes of the volcano on Banta Island large numbers of flying insects dead in the cracks in the hot soil, killed, as he says, by the poisonous volcanic vapors. As these observations were made near the top of the mountain, the number of insects caught was undoubtedly augmented by the abundance of flying species, which he noticed, in conformity with the usual tendency for insects to fly in great numbers to the tops of mountain peaks.

The great numbers of dead insects in the boiling pools of Yellowstone Park has been commented upon by Hubbard ('92) and by Schwartz in the official minutes of the meeting of the entomological club of the American Association for the Advancement of Science for 1891 (Canadian Entom., vol. 23, p. 230).

Postscript

While this paper was in press Böving (Bull. U.S. Dept. Agric. No. 1218, p. 13. (1924)) has published a description of the larva of *Tabanus punctifer* and it appears that I was correct in the surmise that the larva described on page 399 belongs to this species. Unfortunately Böving does not mention the taxonomic characters used by Hart in describing the larva of Tabanus, but there seems to be no question as to the identity of the two series.

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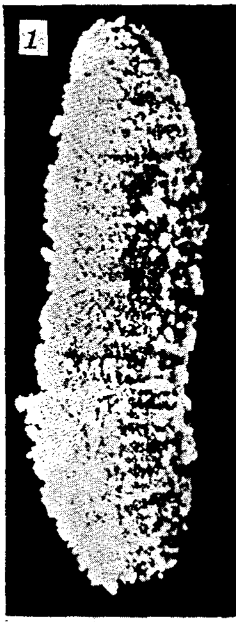
EXPLANATION OF PLATE.

FIGURES 1 and 2. Two views of caterpillar drowned in hot calcareous pool, showing manner in which immersed objects become encrusted with calcium carbonate.

FIGURES 3, 4 and 5. Cases of three species of caddis-worms from sulphurous pond. One of those in Figure 3 has a minute shell (Amnicola) attached.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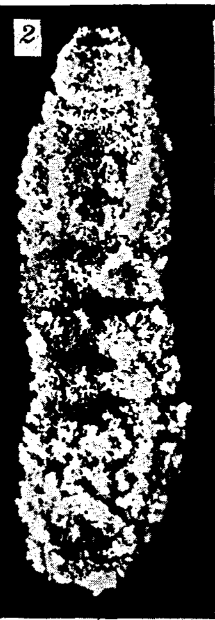
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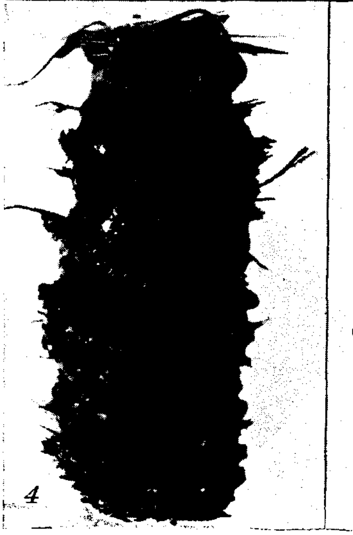
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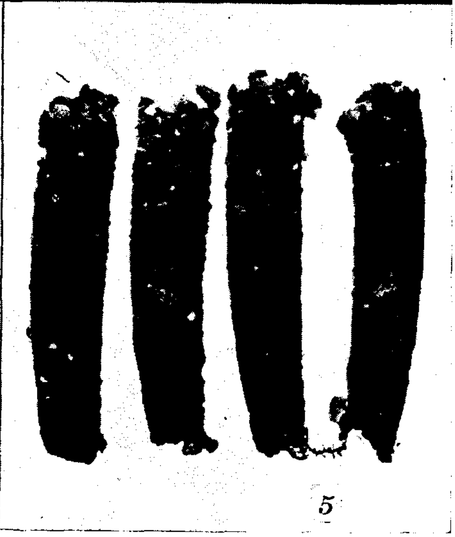
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THE BODILY PROPORTIONS OF WOMEN IN THE UNITED STATES; BASED UPON MEASUREMENTS TAKEN FROM ONE HUNDRED SMITH COLLEGE STUDENTS.

By Harris Hawthorne Wilder, Ph.D., and
Margaret Washington Pfeiffer, M.A.

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CONTENTS.

		Page.
Introduction		441
I	Head Measurements	455
II	Trunk Measurements	482
III	Arm Measurements	532
IV	Leg Measurements	558
V	Intermembral Correlations	580
VI	Breadths	581
VII	Individual Measurements	593
VIII	Summary of Averages	593
Bibliography		599

Introduction.

When White in 1794, basing his assertions upon observation of both skeletons and living men, made the statement that the forearm of negroes, in proportion to the upper arm, was longer than in white men, he inaugurated the science of comparative anthropometry, and showed that there were differences in the bodily proportions of the different human races. Differences of this sort seem to have been unrecognized before this, even by artists and sculptors, who from the time of the Egyptian carvings had elaborated and even emphasized the racial characteristics of the face and head, but had given no heed to differences in the other parts of the body. It is thus possible that the classic sculptors of Greece and Rome used as models indifferently their own people and African slaves, which may explain the asserted negroid proportions of the Apollo Belvidere.

Thus the assertion of White was an advance upon new ground, and although it was accompanied by neither detailed measurements, calculation of averages, or indices, it was yet of great value in the development of the subject. After this, however, the work rested for forty-four years, when, in 1838, Humphrey made careful measurements, not only of humerus and radius, but also of femur and tibia, of twenty-five mounted skeletons of negroes and the same number of skeletons of white men. He compared each individual length with the total height of the skeleton belonging to it, taken as 100, thus obtaining indices which could be directly compared. In this he corroborated White in regard to the long forearm (radius) of negroes, and found a corresponding greater length in the lower leg (tibia) of this race, hitherto unsuspected. He found, for example, that the average humerus in the two races bore practically the same proportion to the total height, 19.52% in negroes and 19.54% in whites, while the figures for the radius, also expressed in percentage of the total height, were respectively 15.16 and 14.15%. In the same way the figures for the femur were 27.40 and 27.51%, taken in the above order, while those for the tibia were respectively 23.23 and 22.15%.

But by this time other anthropologists had become interested in racial differences in proportions, and under the critical scrutiny of Broca this subject received still more careful treatment. He pointed out the fact that a total height obtained from an articulated skeleton depended too much upon the preparator who put it together, and hence substituted for Humphrey's standard the lengths of the long bones themselves, comparing them now with the length of the femur, now with that of the radius, and again with humerus + radius, or femur + tibia, yet with the same final result.

The next great advance in treating the general subject of racial anthropometry was the realization that many of the bones could be measured practically as well in the living subject by ascertaining certain superficial landmarks; also that certain external points, such as the umbilicus, and the nipples, were also of value in the study of individual proportions. This line of work, the anthropometry of the living, developed naturally in the field, as osteometry had developed in the museum, and was the direct result of the series of great scientific voyages, like those of the *Novarra* and the *Challenger*, characteristic of the last third of the nineteenth century. Naturally in the development of physical ethnology the facial features had long received much attention, and had become the subject of careful measurements with averages and indices, and the extension of this work to the rest of the body naturally followed.

During this epoch, in 1882 to be precise, a young anthropologist, M. Alphonse Bertillon, noticing the individual character of bodily measurements, saw in them important data for the solution of the difficulty which had up to this time confronted the judicial arm of the government, that of the personal identification of criminals, and inaugurated the famous system of "Bertillonage," based upon eleven easily taken bodily measurements, a system that has since yielded the most surprising results and has been adopted by all civilized countries.

The investigators of this period began by measuring the distances between landmarks, that is mainly the lengths of the long bones, directly, as had already been done with the shorter distances involved in the head and face; but it was soon seen that if the subject were standing erect, with heels together, in military position, it was necessary only to ascertain the exact height from the floor of the various landmarks, and obtain the lengths and distances desired by subsequent subtractions, thus sparing both the subject and the mensurator in the active work of measuring. The humerus length, for instance, could be accurately measured by subtracting the total height of the head of the radius from the floor from the similar figure for the height of the acromium, while in cases involving paired landmarks, such as the distance from the nipple level to that of the umbilicus, subtraction of heights is the simplest method of obtaining the distance desired.

Thus, by the first years of the present century, the anthropometry of the living body had become an active branch of the subject, and although by the nature of the case similar measurements were taken by investigators, there were numerous minor differences in the landmarks used, the instruments employed, and the measurements emphasized. To meet this difficulty and so standardize the work of all investigators so that the data from all sources could be directly compared, a committee was appointed by the Fourteenth International Congress of Anthropology and Archeology, meeting in Geneva, Switzerland, in September, 1912. Their report, which appeared in English, German, French and Italian, prescribes for general use 49 separate data, of which 39 are linear distances (including heights from floor), 11 are girths (perimeters), and two are contours (outlines of hand and foot).[1]

Up to the present time, however, although anthropological data of living male subjects are now abundant, and include many races, measurements of females, other than such easily obtained ones as total height, arm stretch, and the facial features, are extremely rare, and are practically limited to three papers: that of Frau Jochelson-Brodsky, on certain of the Siberian tribes, collected when on the Jesup expedition during the long and enforced rest of the Arctic night; that of Koganei on Japanese and Ainu women; and that of Fräulein Sara Teumin on her fellow-students in the University of Zürich, Russians and Russian Jewesses. Fischer of Freiburg had the data concerning over a hundred young women from Baden just before the war, and was getting the notes ready to publish, but we cannot tell at present when these important data will be available.

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[1] This committee consisted of 23 members: 6 were French, 4 Italian, 4 English, 3 Russian, 2 from the United States, and one each from Germany, Spain and Hungary. The committee was presided over by the distinguished Roman anthropologist, Giuseppe Sergi, and the American members were Hrdlička and MacCurdy.

Aside from such full records, covering the entire body, there are three classes of papers from which some data may be obtained, but as the object differs in each case, they are hardly to be included in the present field. These are (1) obstetrical measurements of the pelvic proportions, (2) the measurements made by the departments of physical education in colleges and other gymnasiums, and (3) the measurements of school children, for the study of growth. These all differ materially both in ground covered, in subjects, and in the purpose of the investigation. Mentioning but a few of these papers, there are those of Osawa on the pelvis of Ainus and Japanese, of Prochownik on that of Germans, of Schröter on that of Poles, Polish Jews and Russians, and of Stratz on that of the Javanese. On school children, where the data are generally limited to such measurements as height, arm stretch, trunk length, and proportions of head and face, there are the studies of Bowditch on the school children of Boston, and of West on those of Worcester; later in this field there is the paper of Fräulein Olga Spitzer on 1000 school girls from Krakau. From the gymnasiums of colleges and universities we have the paper of Hall on students from Haverford, and that of Sargent from the Hemenway Gymnasium of Harvard.

Of strictly anthropometric value, dealing with bodily measurements other than those of the head and face, we have two papers: that of Rodes (1905) on the thoracic index of the negro, and that of Miss Marjorie Johnson (1911) on the position of the umbilicus, taken upon students of the University of California.

The field is thus peculiarly clear for such papers as the one here presented, and it was for the purpose of supplying an important series of data which were actually non-existent that we set ourselves to work to collect them.

The scope and treatment of this paper will be seen to closely resemble that of Fräulein Sara Teumin, although it differs in the nationality of the subjects, being here mainly British in origin, or at least from a western European ancestry, while hers were mostly either Jewish or Slavic, and arose originally from eastern Europe, or from southwestern Asia.

In addition to the data presented by Fräulein Teumin, we have introduced several new features of considerable value, such as the Standard Deviation, which gives the amount of deviation from the average, divided up among the items, and the Coefficient of Variation, which allows the amount of variation in different measurements to be directly compared. Thus by the aid of these figures we can definitely assert that the femur, with a coefficient of length variation of 5.85, is less variable than the tibia, with a length coefficient of 6.26. On the other hand, the total length of the leg, from trochanter to floor, including these two lengths, together with the height of the foot, varies less than either alone, 4.42, and suggests that in some way a shortness of one of these three elements of which the entire leg length is composed, is compensated for by an extra length in one of the others. The total height, although distributed in the 100 subjects between 1504 and 1760 mm., taking into consideration the great length dealt with, is not so variable as would seem at first sight (3.33), and is thus seen to be proportionately less variable than the lengths of any of the long bones. Again, while the length of the arm stretch is very variable (9.70), such is the correlation between this and the total height, taking individuals by themselves, that the coefficient of variation of the index between these two, arm stretch and total height, varies very little indeed (2.2). As would be expected, extreme variations are found between points which are independently movable, such as the distance between the level of the vertebra prominens and that of the incisura sterni (sitting), which is 20.5.

Certain of the possible correspondences suggested by these figures show the desirability of calculating correlations for these, and these, too, we have attempted in a few cases, and show (1) the correlation of the length of arm to the length of leg, (2) the correlation of the length of humerus to that of femur, (3) that of arm length to total stature, and (4) that of hand to foot. In some of these the correlation is striking; in other cases there is little relation between the measurements taken, and again, in the humerus length and femur length there seems to be no correlation. This is a comparatively new direction of work and may be looked upon to give important results in the future.

In making acknowledgments to those who have definitely furthered the completion of this paper, we would place first and foremost the

100 Smith College students who have offered their services as material for this study. Each subject has given us an hour or two out of a full schedule, and submitted herself to an ordeal at least as tedious as that of the dressmaker, and, in this case, without any benefit to herself. Aside from our associates in the Department of Zoölogy, we remember with special gratefulness the work of Miss Mary Head, former Curator of the Department, who aided in the taking of measurements. We wish also to acknowledge the assistance rendered us by Dr. Stuart Chapin, Professor of Sociology, for advising us in the matter of compiling our statistics, and to Professor Ralph Wheaton Whipple, Professor of Geology at Marietta College, who drew for us the graphs and charts and put them in form suitable for publication.

MATERIAL AND METHODS.

The measurements that were taken were in general those recommended in the "Manual of Anthropometry" (Blakiston, 1920) of the senior author, especially Nos. 1–40. These consist mainly of the height from the floor of the chief landmarks of trunk and limbs, and various breadths, mainly of head, face, and trunk. Girths, measured by tape measure, were not included, since they are variable in the same individual and vary with the state of health. From the measures of heights from the floor, or from a table-top in a sitting subject, various other important measurements may readily be obtained by subtractions, thus saving the time of both subject and measurer. The detailed list of measurements, taken directly from our printed card designed for such use, is as follows:

1 age
2 stature
3 arm stretch
4 ht. of tragus
5 ht. vertebra prominens
6 ht. incisura sterni
7 ht. acromion
8 ht. nipple
9 ht. umbilicus
10 ht. ant. ventral spine of ilium
11 ht. trochanter
12 ht. symphysis
13 ht. capitellum of radius
14 ht. styloid of radius
15 ht. tip of middle finger
16 ht. knee
17 ht. internal malleolus
18 biacromial breadth
19 cristal breadth
20 trochanteric breadth
21 bimammillary breadth
22 spinal breadth
23 thoracic breadth, xiphoid level
24 thoracic depth, xiphoid level
25 foot length
26 sitting ht.
27 ht. vert. prom., sitting
28 ht. incis. sterni, sitting

29 ht. acromion, sitting
30 head length
31 head breadth
32 least frontal breadth
33 biocular breadth
34 bizygomatic breadth
35 bigonial breadth (jaw)
36 chin to hair
37 chin to nasion
38 nasion to prosthion
39 nasal length
40 nasal breadth
41 ear length
42 ear breadth
43 color of eyes
44 color of hair
45 color of skin

Placed in tabular form, in the order presented in this paper, these measurements and those computed from them are as follows:

PAGE.

I. HEAD MEASUREMENTS.

Table 1. Length of Head . . . 455
Table 2. Breadth of Head . . . 456
Table 3. Cephalic Index (length-breadth index) . . . 457
Results of Other Authors: Cephalic Index . . . 458
Table 4. Height of Head . . . 459
Table 5. Index of Height of Head to Trunk Length . . . 460
Table 6. Index of Height of Head to Total Stature . . . 461
Table 7. Height-length Index . . . 462
Table 8. Physiognomic Facial Length . . . 463
Table 9. Anatomical Facial Length . . . 464
Table 10. Superior Facial Length . . . 465
Table 11. Anatomical Facial Index . . . 466
Table 12. Frontal Breadth . . . 467
Table 13. Distance between the Eyes (interorbital breadth) . . . 468
Table 14. Zygomatic Breadth (bizygomatic) . . . 469
Table 15. Mandibular Breadth (bigonial) . . . 470
Table 16. Mandibulo-zygomatic Index . . . 471
Table 17. Nasal Length . . . 472
Table 18. Nasal Breadth . . . 473
Table 19. Nasal Index . . . 474
Results of Other Authors: Nasal Index . . . 475
Table 20. Length of Ear . . . 475
Table 21. Breadth of Ear . . . 476
Table 22. Aural Index . . . 477
Table 23. Color Distribution: Skin, Hair, and Eyes . . . 478
Head Measurements: Summary . . . 480

II. TRUNK MEASUREMENTS.

Table 24. Total Height . . . 482
Results of Other Authors: Total Height . . . 483
Table 25. Height of Tragus . . . 489

PAGE.
Table 26. Height of Vertebra Prominens 490
Table 27. Height of Incisura Sterni 491
Table 28. Distance from Height of Vertex to Height of Vertebra Prominens 492
Table 28a. Comparison of the Above Measurements: Standing and Sitting 492
Table 29. Distance from Height of Tragus to Height of Vertebra Prominens 493
Table 30. Distance from Height of Vertebra Prominens to Height of Acromion 494
Table 30a. Comparison of the Above Measurements: Standing and Sitting 494
Table 31. Distance from Vertex to Incisura Sterni 495
Table 32. Distance from Height of Vertebra Prominens to Height of Incisura Sterni 496
Table 33. Distance from Height of Tragus to Height of Incisura Sterni (subject standing) 497
Table 34. Distance from Height of Incisura Sterni to Height of Acromion 498
Table 35. Height of Nipple 500
Table 36. Index of Height of Nipple to Total Height 501
Table 37. Height of Umbilicus 502
Table 38. Index of Height of Umbilicus to Total Height . . . 503
Results of Other Authors: Height of Umbilicus . . . 503
Table 39. Height of Anterior Ventral Spine of Ilium 505
Results of Other Authors: Height of Anterior Ventral Spine of Ilium 505
Table 40. Height of Symphysis Pubis 506
Results of Other Authors: Index of Height of Symphysis Pubis to Total Height 507
Table 41. Trunk Length (tragus to symphysis pubis) 508
Table 42. Index of Trunk Length to Total Height 509
Table 43. Distance from Incisura Sterni to Symphysis 510
Table 44. Index of this Latter Distance to Total Height . . . 511
Table 45. Distance from Height of Incisura Sterni to Height of Nipple 513
Table 46. Distance from Height of Nipple to Height of Symphysis 514
Table 47. Index of this Latter Distance to Distance from Incisura Sterni to Symphysis 515
Table 48. Distance from Umbilicus to Symphysis 516
Table 49. Index of this Latter Distance to Distance from Incisura Sterni to Symphysis 517
Table 50. Distance from Incisura to Umbilicus 518
Table 51. Height of Mid-Point of Body (when standing erect) 519

PAGE.
Table 52. Difference between Height of Umbilicus and Mid-Point of Body 520
Table 53. Distance from Height of Umbilicus to Height of Anterior Ventral Spine of Ilium 521
Table 54. Distance from Height of Anterior Ventral Spine of Ilium to Height of Symphysis 522
Table 55. Distance from Height of Trochanter to Height of Symphysis 523
Table 56. Distance from Height of Anterior Ventral Spine of Ilium to Height of Trochanter 523
Checks to Certain of the Preceding Tables 524
Table 57. Total Sitting Height 525
Results of Other Authors: Index of Sitting Height to Total Height 526
Table 58. Height of Vertebra Prominens (subject sitting) . . . 528
Table 59. Height of Incisura Sterni (subject sitting) 529
Table 60. Height of Acromion (subject sitting) 530

III. ARM MEASUREMENTS.

Table 61. Height of Acromion 532
Table 62. Height of Head of Radius 532
Table 63. Height of Styloid Process of Radius 533
Table 64. Height of Tip of Middle Finger 534
Table 65. Length of Arm (acromion to tip of middle finger) . . 535
Table 66. Index of Arm Length to Total Height 536
Table 67. Index of Arm Length to Trunk Length 537
Results of Other Authors: Index of Arm Length to Total Height 538
Results of Other Authors: Intermembral Index . . . 541
Table 68. Arm Stretch 542
Table 69. Index of Arm Stretch to Total Height 543
Results of Other Authors: Index of Arm Stretch to Total Height 544
Table 70. Length of Humerus 546
Table 71. Index of Humerus Length to Trunk Length 547
Results of Other Authors: Index of Humerus Length to Total Height 547
Results of Other Authors: Femoro-humeral Index . . 549
Table 72. Length of Radius 550
Table 73. Index of Radius Length to Trunk Length 551
Results of Other Authors: Index of Radius Length to Total Height 551
Results of Other Authors: Index of Arm Length without Hand to Total Height 553
Results of Other Authors: Tibio-radial Index . . . 553

PAGE.
Table 74. Length of Hand 554
Table 75. Index of Hand Length to Trunk Length 555
Results of Other Authors: Index of Hand Length to Total Height 556

IV. LEG MEASUREMENTS.

Table 76. Length of Leg (height of trochanter) 558
Table 77. Index of Leg Length to Total Height 559
Table 78. Index of Leg Length to Trunk Length 560
Results of Other Authors: Index of Leg Length to Total Height 561
Table 79. Height of Knee (articulation of femur and tibia on *inner side of leg*) 565
Table 80. Height of Inner Malleolus 565
Table 81. Length of Femur 566
Table 82. Index of Femur Length to Trunk Length 567
Results of Other Authors: Index of Femur Length to Total Height 568
Table 83. Length of Tibia 570
Table 84. Index of Tibia Length to Trunk Length 571
Results of Other Authors: Index of Tibia Length to Total Height 571
Results of Other Authors: Index of Tibia Length to Femur Length (crural or tibio-femoral index) . . . 573
Table 85. Length of Foot 574
Table 86. Index of Foot Length to Trunk Length 574
Results of Other Authors: Index of Foot Length to Total Height 575

V. INTERMEMBRAL CORRELATIONS 580

VI. BREADTHS.

Table 87. Biacromial Breadth 581
Results of Other Authors: Index of Biacromial Breadth to Total Height 582
Table 88. Thoracic Breadth 584
Table 89. Thoracic Depth 585
Table 90. Thoracic Index (depth to breadth) 586
Results of Other Authors: Thoracic Index 586
Table 91. Bimammillary Breadth 587
Results of Other Authors: Bimammillary Breadth . . 587
Table 92. Spinal Breadth 588
Table 93. Cristal Breadth 589
Table 94. Trochanteric Breadth 590
Results of Other Authors: Spinal, Cristal, and Trochanteric Breadths (females only) 590

PAGE.
VII. INDIVIDUAL MEASUREMENTS OF THE ONE HUNDRED SUBJECTS USED IN THIS PAPER. Cf. Wilder: "Manual of Anthropometry," Appendix B, pp. 171–185. 593

VIII. SUMMARY OF AVERAGES AND INDICES 593

The instruments are mainly those obtained from P. Hermann, Zürich, and designed by Dr. Rudolf Martin. These instruments are the following:

Anthropometer; it can also serve as a slide compass.
Craniometer.
Pelvimeter.
Slide compass.

For the various cases in which color comes into consideration, namely, eyes, hair and skin, we used the three color tables, also manufactured by Hermann, as follows:

eye-color table, prepared by Dr. Rudolf Martin.
hair-color table, prepared by Dr. Eugen Fischer.
skin-color table, prepared by Dr. von Luschan.

An osteometric board, designed primarily for measuring the lengths of bones by Dr. Paul Broca, was found useful in the measurement of foot lengths. This was made locally. A Mannheim slide rule was found especially useful in the computations involved in the work. This was manufactured by Keuffel and Esser, New York.

The statistical formulae that were used are the following:

In all the tables the *arithmetic mean*[1] was calculated according to the formula

$$M = A + \frac{1}{N}\Sigma\,(f.\,\xi)$$

N = total number of observations.
M = the arithmetic mean.
A = the arbitrarily chosen value of the mean.
Σ = Summation.
ξ = the deviation of a given class from A.
f = the frequency of a given class.

The difference from A of M is obtained by this method in class intervals and must be reduced to units of measurement before the mean is found.

---

[1] Cf. Yule, p. 110.

For the measurement of dispersion the *standard deviation*[2] was used, the formula being

$$s^2 = \sigma^2 + d^2$$

σ = the standard deviation.
d = the difference between M and A.
s = the root-mean-square deviation from A and is defined by the equation[3]

$$s^2 = \frac{1}{N} \Sigma (\xi^2)$$

For a measure of relative dispersion the *coefficient of variation*[4] was used, the formula of which is

$$v = 100 \frac{\sigma}{M}$$

v = the coefficient of variation.

The points used in the measurements are those described by the senior author in his "Manual of Anthropometry," previously mentioned. The most of these are obvious and superficial, like the anterior ventral spine of the ilium, or the incisura sterni. Others, like the vertebra prominens, are brought into prominence through certain motions, by which they may be easily located and the skin marked, after which the subject may resume her conventional position, and the measurement may be easily effected. Some cases require finding the separation between two bones, like that between radius and humerus in the elbow, or that at the knee on the inside of the leg, and these, after a little experience, can be readily and accurately found. The majority of the subjects were themselves students of anatomy and took a lively interest in the work.

In cases where the breasts showed any tendency to sag, the nipple measurements were omitted. The determination of the point on the trochanter to be used (trochanterion) is at best somewhat uncertain, and the measurements that involve this point are felt to be a little unreliable. The junior author, who took charge of the measurements, sought in ascertaining this point "the deepest point in the extended depression ectad of the great trochanter"; Fräulein Teumin says "Die Trochanterhöhe direct zu messen, ist wegen des durch die starke Muskulatur sehr erschwerten Abtastens des Trochanters unbequem."

---

[2] Yule, p. 135.

[3] Ibid., p. 134.

[4] Ibid., p. 149.

The point may always be felt approximately by seeking the bottom of the lateral depression often so conspicuous a feature of muscular individuals, and the establishment of this point is often assisted by the subject's abducting the leg, while standing on the other. In spite of all this it is a very uncertain point, and not to be too completely relied upon.

Theoretically, for anthropometrical purposes the subjects should present themselves before the measurer unclothed, but certain loose garments which have no binding influence on the body or its parts may be allowed. We have found by actual measurements that with subjects clothed with a modern bathing suit and again without clothing the difference is negligible, and this use of bathing suits often renders feasible an extensive study of anthropometry involving both sexes.

The one hundred girls used as subjects were students at Smith College. They were thus in the modern sense "Americans," which means that in every case their ancestry was derived from the nations of western Europe. The three Jewesses probably could trace their ancestry into western Asia, but their number was negligible. An endeavor was made to ascertain the ancestry in each case, but this knowledge was ill-defined except in those instances whose families had immigrated recently. There were representatives of the principal European nationalities, but the large majority came from families that had been in this country for a number of generations, and would be spoken of as "Anglo-Saxons," a phrase with very little meaning.

List of nationalities	*Number of individuals*
English	43
English-Scotch	8
English-Irish	3
English-Irish-Welsh	3
English-Irish-Scotch	2
Irish	2
Scotch-Irish	6
English-French	4
English-German	5
English-Dutch	6
Scotch-Dutch	2
Irish-German	1
Welsh-French	1
German	6

List of nationalities	*Number of individuals*
German-French	3
Russian (Jewish)	3
German-Dutch	1
German-Italian	1
	100

The subjects were all young adult females between 17 and 27 years of age, but with an average of twenty or twenty-one years. Their age distribution is shown by the following table:

Age in years	*Number of individuals*
17	2
18	13
19	13
20	28
21	27
22	8
23	4
24	3
25	1
26	0
27	1
	100

The mean is 20.4 years.

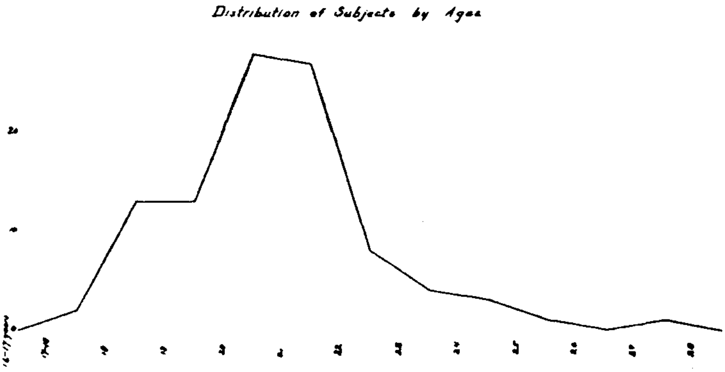


FIGURE 1.

I. HEAD MEASUREMENTS.

TABLE 1.

LENGTH OF HEAD.

Class	*Frequency*
173 to 173.99 mm.	2
174	2
175	1
176	1
177	1
178	3
179	0
180	6
181	5
182	2
183	5
184	5
185	8
186	7
187	11
188	9
189	4
190	4
191	5
192	4
193	4
194	5
195	0
196	3
197	1
198	0
199	0
200	1
201	0
202	1
	100

The mean is 186.88 mm.
The extremes are 173 mm.
202 mm.
The standard deviation is 5.81 mm.
The coefficient of variation is 3.10

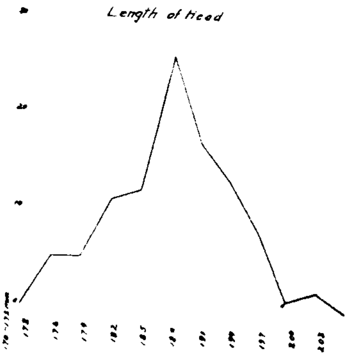


FIGURE 2.

TABLE 2.

Breadth of Head.

Class	*Frequency*
137 to 137.99 mm.	1
138	1
139	1
140	4
141	2
142	4
143	8
144	6
145	8
146	13
147	9
148	7
149	6
150	12
151	5
152	4
153	2
154	2
155	3
156	1
157	1
	100

The mean is 147.11 mm.
The extremes are 137 mm.
157 mm.
The standard deviation is 4.19 mm.
The coefficient of variation is 2.80

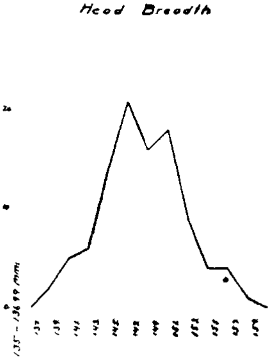


Figure 3.

TABLE 3.

CEPHALIC INDEX.
(Length-breadth index.)

Class	*Frequency*
71.50 to 71.99	1
72.0	1
72.5	0
73.0	0
73.5	5
74.0	1
74.5	3
75.0	3
75.5	2
76.0	3
76.5	4
77.0	5
77.5	3
78.0	7
78.5	12
79.0	8
79.5	5
80.0	11
80.5	4
81.0	2
81.5	7
82.0	3
82.5	4
83.0	1
83.5	2
84.0	1
84.5	0
85.0	0
85.5	0
86.0	0
86.5	2
	100

The mean is 78.96 (mesocephalic)
The extremes are 71.5
86.5
The standard deviation is 2.90
The coefficient of variation is 3.70

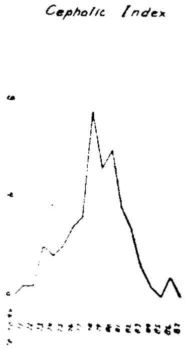


FIGURE 4.

RESULTS OF OTHER AUTHORS: CEPHALIC INDEX.

Race	*Author*	♂	♀
Ainu	Koganei	77.3	78.04
Ainu Yezo	Deniker	77.8	
Korjaks Kamtschatka	Brodsky	78.1	78.0
Kamtschatkans	"	78.5	77.4
Tunguses Kolyna	"	78.5	79.4
Japanese	Deniker	78.5	
Tunguses Gischiga	Brodsky	78.7	79.3
Ostjaks	Iwanowsky	79.23	
Korjaks Gischiga	Brodsky	80.3	80.0
Jukagirs	"	80.4	80.0
Tunguses Anadyr	"	80.8	80.3
Asiatic Eskimos	"	80.8	79.7
North Tunguses	Mainoff	81.39	
Tschuktsches	Olsufyeff	81.79	
"	Brodsky	82.0	81.8
Koreans	Deniker	82.06	
Kasan Tatars	Iwanowsky	82.08	81.84
Transbaikal Tunguses	Talko-Grinzewitch	82.23	
Manchurians	Pojarkoff	82.32	
Jakuts	Witaschewsky	82.33	
Baschkirs	Iwanowsky	82.55	
Kalmucks	"	82.57	
Jakuts	Mainoff	82.66	
South Tunguses	"	82.69	
Orotsches	Iwanowsky	82.81	
Jakuts	Hecker	83.01	
Sojots	Iwanowsky	83.03	
Jakuts	Brodsky	83.1	83.3
Samoyedes	Iwanowsky	83.95	
Lapps	"	84.00	
Torgouts	"	84.73	
Manchus	Ujfalvy	84.91	
Kara Kirghiz	Iwanowsky	85.17	
Buriats	"	85.87	
Giljaks	Deniker	86.03	
Kirghiz	Iwanowsky	87.10	

TABLE 4.

HEIGHT OF HEAD.

Class	*Frequency*
100 to 104.99 mm.	1
105	4
110	5
115	9
120	15
125	23
130	21
135	8
140	10
145	3
150	0
155	1
	100

The mean is 128.14 mm.
The extremes are 100 mm.
155 mm.
The standard deviation is 10.13 mm.
The coefficient of variation is 7.90

TABLE 5.

INDEX OF HEIGHT OF HEAD TO TRUNK LENGTH.

Class	*Frequency*
15 to 15.49	1
15.5	2
16.0	3
16.5	5
17.0	4
17.5	10
18.0	13
18.5	14
19.0	11
19.5	7
20.0	9
20.5	4
21.0	9
21.5	2
22.0	4
22.5	0
23.0	2
	100

The mean is 18.61
The extremes are 15.1
23.0
The standard deviation is 1.66
The coefficient of variation is 8.90

TABLE 6.

INDEX OF HEIGHT OF HEAD TO TOTAL STATURE.

Class	*Frequency*
6.0 to 6.49	1
6.5	5
7.0	18
7.5	35
8.0	26
8.5	14
9.0	1
	100

The mean is 7.88
The extremes are 6.0
9.4
The standard deviation is 0.60
The coefficient of variation is 7.62

TABLE 7.

HEIGHT-LENGTH INDEX.

Class	*Frequency*
55 to 55.99	2
56	0
57	0
58	0
59	3
60	2
61	4
62	4
63	5
64	4
65	6
66	6
67	9
68	7
69	10
70	8
71	4
72	4
73	4
74	7
75	1
76	2
77	4
78	0
79	3
80	0
81	0
82	1
	100

The mean is 68.70
The extremes are 55.0
82.0
The standard deviation is 5.37
The coefficient of variation is 7.80

TABLE 8.

PHYSIOGNOMIC FACIAL LENGTH.

Class	*Frequency*
160 mm.	2
162	5
164	4
166	3
168	8
170	9
172	10
174	14
176	7
178	3
180	9
182	7
184	9
186	5
188	3
190	1
192	0
194	0
196	0
198	1
	100

The mean is 175.59 mm.
The extremes are 160 mm.
190 mm.
The standard deviation is 7.92 mm.
The coefficient of variation is 4.50

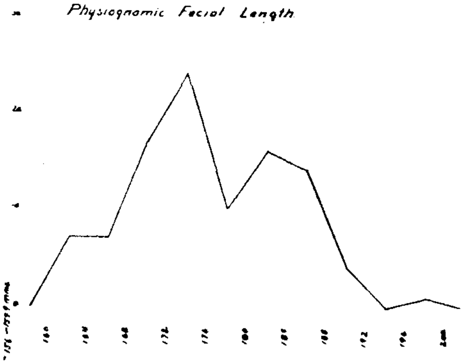


FIGURE 5.

TABLE 9.

ANATOMICAL FACIAL LENGTH.

Class	*Frequency*
98 to 99.99 mm.	1
100	1
102	0
104	3
106	1
108	9
110	16
112	11
114	10
116	6
118	17
120	7
122	10
124	5
126	2
128	0
130	0
132	1
	100

The mean is 115.35 mm.
The extremes are 98 mm.
132 mm.
The standard deviation is 6.06 mm.
The coefficient of variation is 5.30

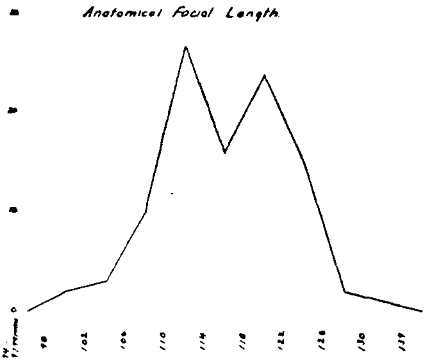


FIGURE 6.

TABLE 10.

SUPERIOR FACIAL LENGTH.

Class	*Frequency*
62 to 63.99 mm.	4
64	2
66	8
68	12
70	12
72	25
74	19
76	12
78	4
80	0
82	2
	100

The mean is 72.07 mm.
The extremes are 62 mm.
82 mm.
The standard deviation is 3.92 mm.
The coefficient of variation is 5.45

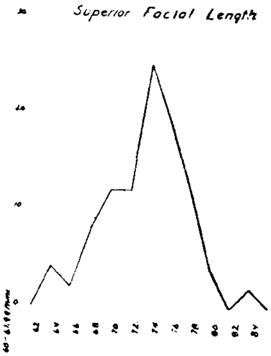


FIGURE 7.

TABLE 11.

Anatomical Facial Index.

Class	*Frequency*
83 to 83.99	1
84	1
85	0
86	0
87	0
88	2
89	2
90	2
91	2
92	3
93	4
94	0
95	4
96	6
97	4
98	4
99	6
100	13
101	6
102	6
103	4
104	8
105	3
106	2
107	3
108	3
109	4
110	2
111	1
112	1
113	0
114	3
	100

The mean is 100.65
The extremes are 83
114
The standard deviation is 6.50
The coefficient of variation is 6.50

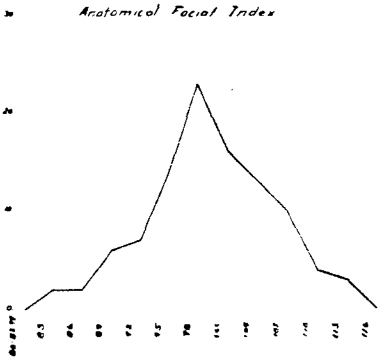


FIGURE 8.

TABLE 12.

FRONTAL BREADTH.

Class	*Frequency*
92 to 92.99 mm.	1
93	0
94	1
95	2
96	4
97	7
98	4
99	8
100	8
101	8
102	9
103	11
104	11
105	5
106	6
107	6
108	3
109	1
110	1
111	2
112	0
113	0
114	1
115	1
	100

The mean is 102.72 mm.
The extremes are 92 mm.
115 mm.
The standard deviation is 4.23 mm.
The coefficient of variation is 4.18
The frontal breadth is *69%* of the breadth of the head.

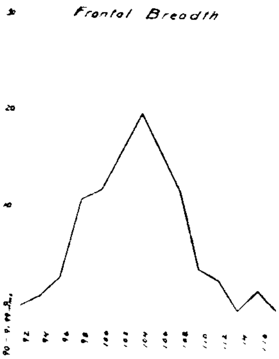


FIGURE 9.

TABLE 13.

DISTANCE BETWEEN THE EYES.

(Interorbital breadth.)

Class	*Frequency*
24 to 24.99 mm.	1
25	1
26	1
27	5
28	11
29	12
30	21
31	13
32	12
33	8
34	7
35	5
36	3
	100

The mean is 31.17 mm.
The extremes are 24 mm.
36 mm.
The standard deviation is 2.55 mm.
The coefficient of variation is 8.10

TABLE 14.

ZYGOMATIC BREADTH.

(Bizygomatic.)

Class	*Frequency*
90 to 91.99 mm.	1
92	0
94	0
96	0
98	0
100	0
102	0
104	2
106	4
108	6
110	7
112	8
114	13
116	14
118	12
120	11
122	10
124	9
126	1
128	0
130	1
132	1
	100

The mean is 116.35 mm.
The extremes are 90 mm.
133 mm.
The standard deviation is 6.10 mm.
The coefficient of variation is 5.29
The zygomatic breadth is 79% of the breadth of the head.

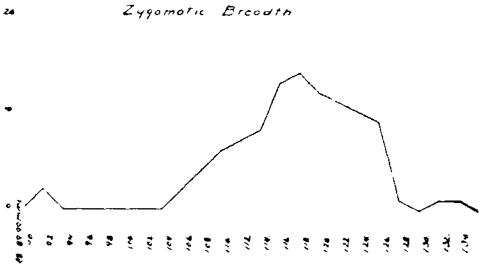


FIGURE 10.

TABLE 15.

MANDIBULAR BREADTH.

(Bigonial.)

Class	*Frequency*
84 to 84.99 mm.	2
85	1
86	2
87	0
88	1
89	2
90	4
91	1
92	0
93	3
94	8
95	3
96	6
97	12
98	9
99	12
100	5
101	6
102	4
103	4
104	4
105	4
106	3
107	1
108	3
	100

The mean is 98.27 mm.
The extremes are 84 mm.
108 mm.
The standard deviation is 5.20 mm.
The coefficient of variation is 5.30

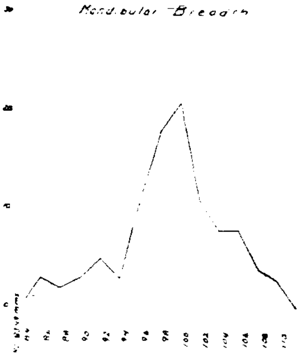


FIGURE 11.

TABLE 16.

MANDIBULO-ZYGOMATIC INDEX.

Class	*Frequency*
72 to 72.99	1
73	0
74	0
75	3
76	3
77	2
78	2
79	2
80	5
81	8
82	11
83	12
84	11
85	17
86	4
87	7
88	2
89	2
90	1
91	2
92	1
93	0
94	1
95	1
96	2
	100

The mean is 84.09
The extremes are 72
96
The standard deviation is 4.41
The coefficient of variation is 5.20

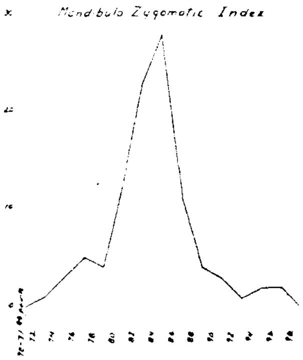


FIGURE 12.

TABLE 17.

NASAL LENGTH.

Class	*Frequency*
42 to 42.99 mm.	1
43	0
44	2
45	0
46	1
47	1
48	6
49	7
50	4
51	8
52	9
53	12
54	4
55	19
56	12
57	5
58	5
59	2
60	2
	100

The mean is 53.61 mm.
The extremes are 42 mm.
60 mm.
The standard deviation is 3.84 mm.
The coefficient of variation is 7.15

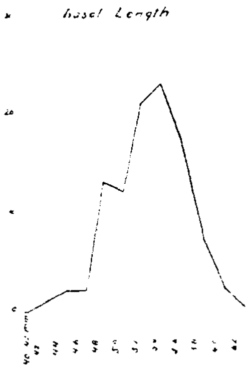


FIGURE 13.

TABLE 18.

NASAL BREADTH.

Class	*Frequency*
25 to 25.99 mm.	1
26	1
27	0
28	7
29	9
30	13
31	16
32	14
33	19
34	8
35	6
36	3
37	2
38	1
	100

The mean is 32.18 mm.
The extremes are 25 mm.
38 mm.
The standard deviation is 2.37 mm.
The coefficient of variation is 7.49

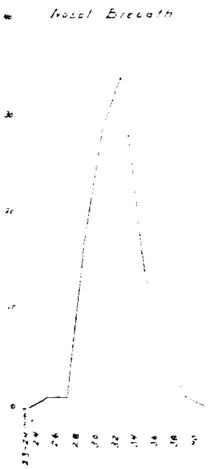


FIGURE 14.

TABLE 19.

NASAL INDEX.

Class	*Frequency*
45 to 45.99	1
46	1
47	0
48	2
49	1
50	2
51	4
52	3
53	2
54	8
55	3
56	3
57	4
58	9
59	5
60	8
61	10
62	5
63	9
64	2
65	3
66	3
67	2
68	1
69	1
70	3
71	4
72	0
73	0
74	0
75	1
	100

The mean is 59.89
The extremes are 45
75
The standard deviation is 6.04
The coefficient of variation is 10.10

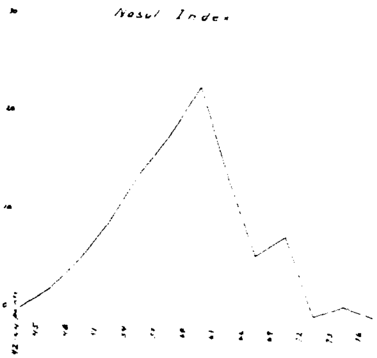


FIGURE 15.

RESULTS OF OTHER AUTHORS: NASAL INDEX.

Race	*Author*	♂	♀
Sojots	Gorostschenko	76.20	69.50
Ainu	Koganei		66.70
Kalmucks Don	Iwanowsky	73.90	
" Volga	Deniker	70.57	
Torgouts	Iwanowsky	60.46	
Nunatagmiut Eskimos	Boas		63.00
Koukpagmiut "	"		62.87
Tahltan Indians	"		62.87
Jakuts	Brodsky		64.6
Tunguses	"		66.0

TABLE 20.

LENGTH OF EAR.

Class	*Frequency*
49 to 49.99 mm.	1
50	0
51	0
52	2
53	1
54	6
55	8
56	13
57	9
58	14
59	12
60	10
61	5
62	8
63	2
64	65
65	3
66	0
67	1
	100

The mean is 58.89 mm.
The extremes are 49 mm.
67 mm.
The standard deviation is 3.16 mm.
The coefficient of variation is 5.40

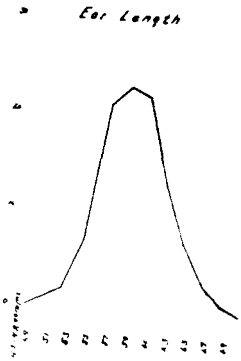


FIGURE 16.

TABLE 21.

BREADTH OF EAR.

Class	*Frequency*
25 to 25.99 mm.	1
26	2
27	7
28	5
29	14
30	27
31	12
32	14
33	8
34	6
35	4
	100

The mean is 30.99 mm.
The extremes are 25 mm.
35 mm.
The standard deviation is 2.16 mm.
The coefficient of variation is 6.90

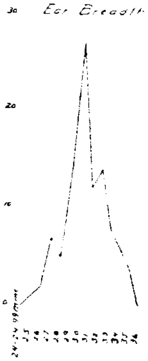


FIGURE 17.

TABLE 22.

AURAL INDEX.

Class	*Frequency*
41 to 41.99	1
42	0
43	2
44	3
45	3
46	9
47	8
48	13
49	4
50	18
51	9
52	5
53	7
54	6
55	3
56	0
57	3
58	1
59	2
60	1
61	1
62	1
	100

The mean is 50.63
The extremes are 41.94
62.56
The standard deviation is 3.91
The coefficient of variation is 7.72

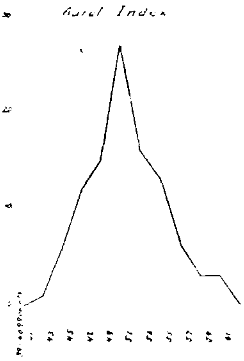


FIGURE 18.

TABLE 23.

COLOR DISTRIBUTION.

The skin, hair, and eye colors are classified according to the numbers of the color tables used

Class	EYES *Frequency*	HAIR *Frequency*	SKIN *Frequency*
1	0	0	0
2	0	0	2
3	5	2	22
4	9	8	0
5	11	24	0
6	6	3	0
7	11	13	16
8	7	26	6
9	2	10	10
10	3	3	15
11	3	1	15
12	9	3	7
13	12	2	5
14	4	0	2
15	15	0	—
16	3	1	100
17	—	0	
18	100	0	
19		0	
20		0	
21		0	
22		1	
23		0	
24		0	
25		1	
26		2	
		—	
		100	

Hair. Practically one half have light brown hair, included in the numbers 7, 8, 9. There are more with hair much darker than this intermediate color than with lighter hair. The numbers running down from 6 to 3 are all darker, 3 being an inky black. The decided

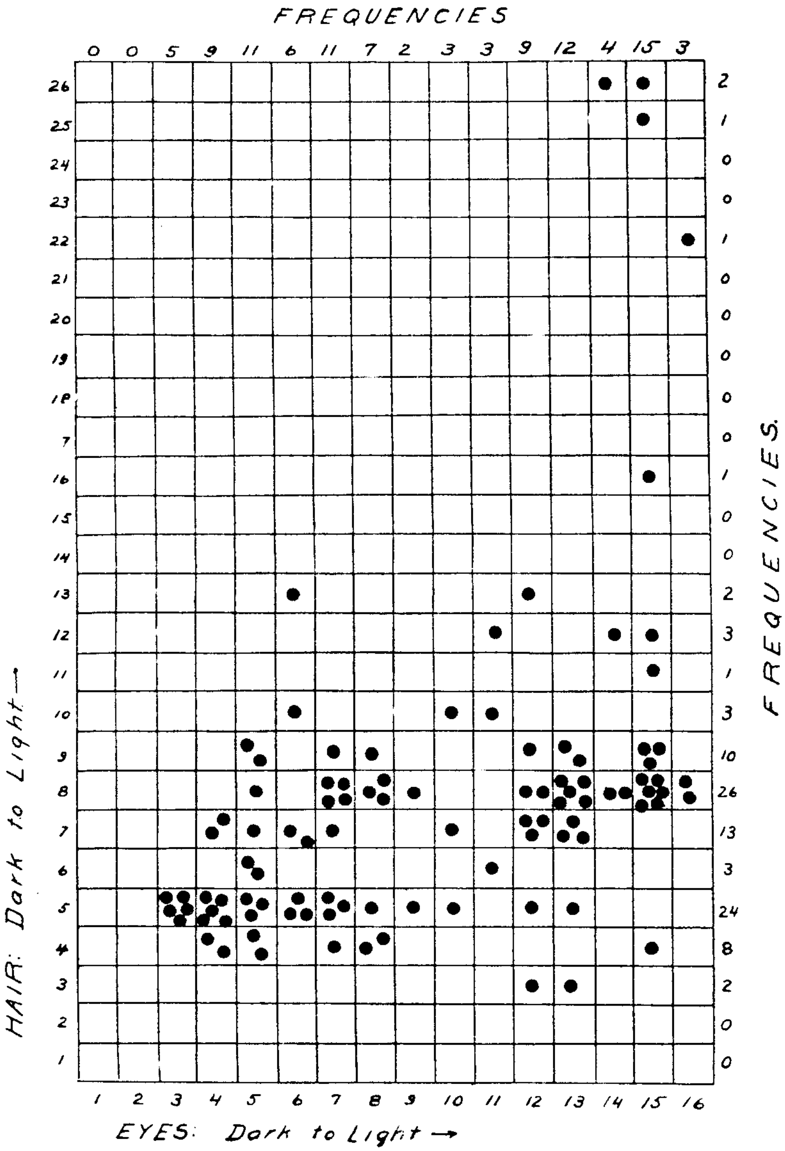

FREQUENCIES
0 0 5 9 11 6 11 7 2 3 3 9 12 4 15 3
26 25 24 23 22 21 20 19 18 7 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
2 1 0 0 1 0 0 0 0 0 1 0 0 2 3 1 3 10 26 13 3 24 8 2 0 0
FREQUENCIES
HAIR: Dark to Light →
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
EYES: Dark to Light →

FIGURE 19.

blondes are few in number. The ordinary blonde colors are included in the numbers 11 to 16. Those with extremely light hair or hair tinged with a greenish cast are in the numbers from 22 to 26.

Eyes. There are more blue eyes than any other one color, these being however neither very deep blue nor very washed-out. There are nearly as many of a gray-blue, of a fairly dark brown and of a medium hazel. The very dark browns, the deep blues and the very light blues and the yellow hazels are few in number. The numbers from 3 to 8 include the very dark, the dark browns and the darker hazels. The numbers from 12 to 16 include the grays, the gray-blues and the different shades of blue.

Skin. Nearly a fourth of all the cases had number 3, which is a light skin with a decided rosy tint. Number 7 is light but lacks the rosy tint while the numbers from 8 to 12 are darker and generally sallow.

There is a decided correlation between the eye and hair color as is shown by the following table. Generally those with brown or dark hazel eyes have dark and medium dark hair, while those with light hazel or blue eyes have medium light and light hair.

HEAD MEASUREMENTS: SUMMARY.

The head is mesocephalic with a small coefficient of variation for both the length and breadth measurements as well as for the cephalic index. There is much more variation in the height of the head and in the indices between this, the trunk length and the total height. There is a moderate amount of variation in all the facial lengths, the zygomatic breadth and the mandibular breadth. The face is mesoprosopic. The range of deviation in the facial index is very large but the standard deviation and the coefficient of variation are not large. The mandibular-zygomatic index which indicates the shape of the face has a small coefficient of variation although the range is large. The majority have a decided median slant from the zygomatic arches to the angles of the mandibles. The average nose is leptorrhine but there is a very large coefficient of variation in the nasal index. The coefficients of variation of the length and of the breadth of the nose are somewhat smaller. Rather curiously the coefficients of the length and of the breadth of the ear and of the aural index are much smaller than those of the nose.

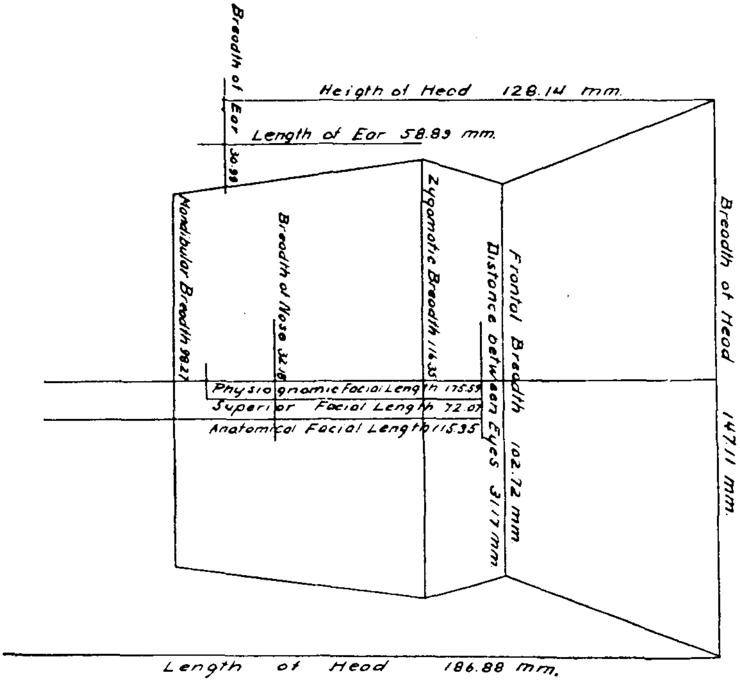

Breadth of Ear 30.99
Heigth of Head 128.14 mm.
Length of Ear 58.83 mm.
Mandibular Breadth 98.27
Breadth of Nose 32.18
Zygomatic Breadth 116.35
Distance between Eyes 31.17 mm
Frontal Breadth 102.72 mm
Breadth of Head 147.11 mm.
Physiognomic Facial Length 175.59
Superior Facial Length 72.07
Anatomical Facial Length 115.35
Length of Head 186.88 mm.

FIGURE 20.

II. TRUNK MEASUREMENTS.

TABLE 24.

TOTAL HEIGHT.

Class	*Frequency*
1500 to 1519.99 mm.	2
1520	1
1540	7
1560	6
1580	8
1600	11
1620	15
1640	15
1660	13
1680	15
1700	2
1720	2
1740	2
1760	1
	100

The mean is 1638 mm.
The extremes are 1504 mm.
1760 mm.
The standard deviation is 54.60 mm
The coefficient of variation is 3.33

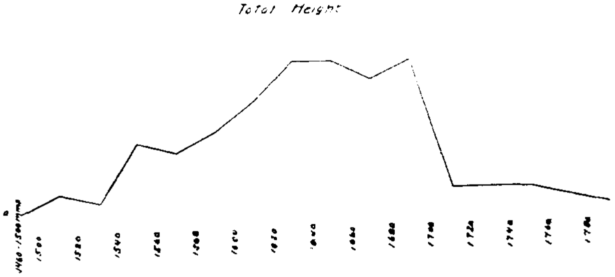


FIGURE 21.

RESULTS OF OTHER AUTHORS: TOTAL HEIGHT.

Europe

Race	*Author*	♂	♀
Lapps (Norway)	Mantegazza	1523	1450
Lapps (Russia)	Anutschin	1558	
Woguls	Deniker	1591	
Ruthenians	Magierowski	1595	
Polish Jews	Elkind	1610	1506
Sardinians	Livi	1619	
Spaniards	Oloriz	1620	
Polish Jews	Sneijirew	1622	
Little Russians	Talko-Grinzewitch	1625	
Tatars Volga	Deniker	1628	
Galician Jews	Maier and Kopernitzky	1628	
Roumanians	Pittard	1635	
Poles	Elkind	1639	
Italians	Livi	1640	
French	Bertillon	1641	1570
Portuguese and Spaniards	“	1642	
Russians	Rozdestwensky	1644	1528
Galician Poles	Magierowski	1645	
Mingrels	Deniker	1646	
Little Russians	Weissenberg	1648	
Gypsies	Pittard	1649	1532
White Russians	Eichholz	1650	
Poles	Teumin		1620
Little Russians	“		1540
Polish Jews	“		1550
Russians	“		1610
Roumanians	Felix and Pittard	1650	
Russians	Anutschin	1650	
South Russian Jews	Weissenberg	1651	1536
White Russians	Schedrowizky	1654	
Belgians	Houzé	1655	
Lithuanians	Boronas	1656	1546
Bavarians	Ranke	1656	
Tatars in Balkans	Pittard	1657	1547
Russians in Twer	Galai	1660	1540
People of Baden	Ammon and Fischer	1662	1560
Bulgarians	Wateff	1665	1557
Finns	Kolmogaroff	1666	1545
Little Russians	Talko-Grinzewitch	1667	1548
Roumanians (Bukowina)	Pittard	1671	
Dutch	Deniker	1675	

Race	*Author*	♂	♀
Poles	Danitowiky	1679	
Finns	Retzius	1679	1552
Fäero Islanders	Soren Hansen	1691	1584
Danes	Makeprang and Hansen	1691	1592
Danes Bornholm	Ribbing	1697	1588
Serbians	Lazarevitch	1699	
Little Russians	Tscherpourkowsky	1701	
Lithuanians (Letts)	Waeber	1705	1563
Swedes	Retzius and Fürst	1709	
Norwegians	Daae	1721	
Irish	Beddoe	1726	
English	Pearson	1728	1599
Livonians	Topinard	1736	
	Asia		
Negritos of Zambales	Reed	1463	1378
South Andaman Islanders	Census of India	1482	1402
North " "	" " "	1486	1385
Senoi	Martin	1520	1420
Semangs	Annandale	1520	
Veddahs	Sarasin	1533	1433
Kanikars	Schmidt	1536	
Igorots	Bean	1549	1467
Ostjaks	Deniker	1540	
Orotsches	Margaritoff	1545	1443
North Tunguses	Mainoff	1548	
Samoyedes	Sommier and Sograff	1550	1430
Miao-tse	Ryūzō Torii	1550	1450
Irula	Schmidt	1554	
Jukagirs	Brodsky	1560	1470
Tunguses	"	1565	1465
Jakuts	"		1488
Tóala	Sarasin	1561	1454
Ostjaks	Sommier	1563	1441
Ainu	Koganei	1567	1468
Kasia	Census of India	1569	
Menangkabau Malays	Kleiweg de Swaan	1570	
Kayan	Haddon	1572	1440
Tunguses Anadyr	Brodsky	1574	1488
Kadirs	Thurston	1577	1430
Cochin Chinese	Deniker	1578	
Ostjaks	Iwanowsky	1579	1441
Moi	Deniker	1579	
Palaung	Census of India	1581	

Race	*Author*	♂	♀
Tonkinese	Deniker and Bonifacy	1584	
Japanese	Bälz	1585	1450
South Chinese	Census of India	1585	
Tunguses Kolyma	Brodsky	1588	
Karagasses	Salessky	1589	1445
Cambodians	Mondierre and Bonifacy		1451
Anamites	“ “ “	1587	1464
Japanese	Miwa and Martin	1593	1472
Sundanese	Kohlbrügge	1594	1494
Schan	Census of India	1594	
Koriaks	Bogoras	1596	1491
Koriaks	Martin	1596	
Kota	Schmidt	1597	
Siamese	Annandale	1599	
Kamtschatkans	Bogoras	1601	
Tenggereses (Java)	Kohlbrügge	1604	
Battaks	Hagen	1605	
Formosans	Torii	1605	
Jakuts	Witaschewsky	1607	1498
Setchuan Chinese	Legendre	1611	
Upper Burmese	Ethnol. Survey	1619	
South Chinese	Hagen	1622	1498
Deli-Malays	“	1622	
Tschuktsches	Bogoras	1622	1520
Bugi	Sarasin	1622	
Asiatic Eskimos	Bogoras	1623	1518
Jakuts	Mainoff	1624	1512
Singalese	Census of India	1625	
Tunguses	Mainoff	1627	
Permiaks	Khomiakow	1629	1524
Bhil	Census of India	1629	
Tatars	Sneijerew (Sucharew)	1630	1517
Buriats	Talko-Grinzewitch (Hryncewicz)	1631	1530
Buriats	Iwanowsky	1631	
Torgouts	“	1631	
South Tunguses	Mainoff	1631	1530
Koreans	Jijima	1631	1530
Kalmucks	Deniker	1634	1498
Malăalis	Thurston	1634	
Jarans	Kohlbrügge and Stratz	1635	1540
Tunguses	Talko-Grinzewitch	1638	
Kalmucks	Iwanowsky	1640	1504
Kirghiz	“	1640	1511
Baschkirs	Nikolsky	1641	

Race	*Author*	♂	♀
Tatars	Varuschkin	1645	1521
Minahasses	Sarasin	1647	1544
Tschuktses	Deniker	1649	
Cossacks (Kirghiz)	Iwanowsky	1651	1503
Chinese	"	1653	
Baschkirs	"	1655	
Tschuktsches	Olsufjeff	1660	
Kara Kirghiz	Iwanowsky	1673	
Armenians	Delisle	1673	1510
Lolo	Legendre	1675	
Manchurian Tunguses	Deniker	1675	
North Chinese	Koganei	1676	
Balocks (Iranians)	Census of India	1678	
Ossets	Pittard	1680	
Armenians	Chantre	1685	1529
Kurds	Tschepourkowsky	1686	
Ossets	Riskine	1690	1573
Kurds	Pittard	1701	
Sikhs	Census of India	1709	
Rajputs	" " "	1749	
Malays (Sumatra)	Kleiweg de Swaan and Mass	1755	
	Africa		
Mawambi pygmies	Czekanowski	1408	1356
Lukumba-Kumba pygmies	"	1414	
Bushmen	Fritsch, von Luschan and Pock	1440	
Ba-Binga	Poutrin	1510	1450
Ba-Tua	"	1522	1437
Ba Tschua	Starr	1542	1437
Kung Bushmen	Seiner	1564	1482
Kagoro	Tremearne	1600	
Dschagga	Widemann	1625	
Niam-Niam	Tucker and Myers	1627	
Yoruba	Deniker	1630	1561
Mandscha	Hösemann	1631	1557
Duala	von Luschan	1633	
Ewe	Deniker	1637	1574
Egyptians	Hrdlička	1638	
Yakoma	Girard	1640	
Kuari and Miusa	Hösemann	1646	
Furawi	Tucker and Myers	1653	
Fiot	Poutrin	1657	
Banda	"	1660	1530
Togo	von Luschan	1661	

Race	*Author*	♂	♀
Suaheli	von Luschan	1669	
M'Baka	Poutrin	1671	1568
Bedjah	Chantre	1676	
Berbers (Algeria)	Deniker	1680	
Negroes	Gould	1681	
Betschuans	Fritsch	1684	
South African Bastards	Fischer	1684	1570
Bugu	Girard	1687	
Aschanti	Deniker	1693	1554
Fans	Poutrin	1698	
Mandingo	Deniker	1705	
Bornus	Tucker and Myers	1710	
Bantus	Fritsch	1718	
Dinkas	Tucker and Myers	1727	1572
Nubians	Tucker and Seligmann	1735	1572
Massia	von Luschan and Widemann	1747	
Lobi	Ruelle	1754	1659
Buchuma	Poutrin	1755	1620
Kabba	"	1770	
Dinkas	Mochi	1773	1657
Schilluks	Tucker and Myers	1777	
Baghirmi and Wadai	" " "	1780	
Dinkas	" " "	1800	
Saras	Poutrin	1817	1676
	America		
Polar Eskimos	Steensby	1570	1454
Karibs from Guiana and Venezuela	Deniker	1572	
Labrador Eskimos	Boas	1575	1480
Terra del Fuegians	Hyades and Deniker	1577	1473
Eskimos	Duckworth	1577	1497
Tupi	Deniker	1588	
Juki (Central America)	Boas	1590	1488
Otomi	Hrdlička	1593	1474
Trumai	Ranke	1595	1487
Chileans	Outes	1603	
Bakairi	Ehrenreich	1608	1516
Aztecs	Hrdlička	1610	1489
Salishans (Harrison Lake)	Boas	1613	
Salishans	"	1618	
Zuñi	"	1623	1511
Greenland Eskimos	Deniker	1621	
Chiriguano	Lehmann-Nitschi	1634	1517

Race	*Author*	♂	♀
Mataco	Lehmann-Nitschi	1638	1529
Hopi Pueblos	Hrdlička	1638	1507
Kwakiutl Indians	Boas	1639	
Maidu (California)	"	1644	1533
Pueblos	Hrdlička	1645	1523
Alaska Eskimos	Boas	1658	1551
Shoshoni Indians	"	1661	1528
Bella Coola Indians	Deniker	1661	
Southern Utes	Hrdlička	1668	1537
Tshimshian Indians	Deniker	1666	
Shuswap Indians	Hrdlička	1673	1557
Eastern Cherokees	Boas	1677	1549
Chinook Indians	"	1691	
Apaches	Hrdlička	1691	1568
Toba Guaicuru	Lehmann-Nitschi	1698	1555
Southern Californian Indians	Boas	1700	1580
Navajo Indians	Hrdlička	1713	1573
Pima Indians	"	1718	1574
Eastern Ojibways	Boas	1723	1574
Sioux	"	1726	1595
Iroquois	"	1727	1585
Ona	Lehmann-Nitschi	1736	1596
Bororó	Ehrenreich	1737	1606
Maricopa	Hrdlička	1749	1604
Patagonians	Moreno and Lister	1750	
	Oceanic Islands		
Tapiro people	Wollaston	1449	
Pygmies from Kaiser Wilhelmsland	Neuhauss	1460	1380
Pygmies from Goliathberg	Van den Broek	1492	
Toricelligebirge people	Schlaginhaufen	1519	
Kai (New Guinea)	Pöch	1525	1435
Poum people	Schellong	1543	1498
New Mecklenburgers	Schlaginhaufen	1574	
Jakumul (N. New Guinea)	"	1583	
Arup " " "	"	1600	
Jabim people	Schellong	1606	1530
Solomon Islanders	Deniker	1616	
Australians and New South Wales	Schlaginhaufen	1636	
Tanga Islanders	"	1651	1540
Meraukê	Koch	1672	1567
Central Australians	Deniker	1670	

Race	*Author*	♂	♀
Fak-Fak	Koch	1677	
Maoris	Deniker	1680	
Motu-Motu (New Guinea)	Chalmers	1702	
Samoans	Deniker	1726	
Tahitans Pomotu Islanders Tubuai Islanders	"	1733	
Marquesas Islanders	"	1743	

TABLE 25.

HEIGHT OF TRAGUS.

Class	*Frequency*
1380 to 1399.99 mm.	2
1400	2
1420	6
1440	7
1460	11
1480	9
1500	19
1520	19
1540	10
1560	7
1580	4
1600	2
1620	2
	100

The mean is 1510.00 mm.
The extremes are 1393 mm.
1627 mm.
The standard deviation is 52.00 mm.
The coefficient of variation is 3.42
The index of height of tragus to total height is *92.0*

TABLE 26.

HEIGHT OF VERTEBRA PROMINENS.

Class	*Frequency*	
1260 to 1279.99 mm.	1	The mean is 1399.59 mm. The extremes are 1274 mm. 1515 mm. The standard deviation is 49.00 mm. The coefficient of variation is 3.50 The index of height of vertebra prominens to total height is *85.5*
1280	2	
1300	4	
1320	6	
1340	7	
1360	11	
1380	15	
1400	19	
1420	16	
1440	11	
1460	4	
1480	1	
1500	3	
	100	

TABLE 27.

HEIGHT OF INCISURA STERNI.

Class	*Frequency*
1200 to 1219.99 mm.	1
1220	2
1240	4
1260	3
1280	6
1300	15
1320	15
1340	13
1360	22
1380	7
1400	10
1420	1
1440	1
	100

The mean is 1342.19 mm.
The extremes are 1213 mm.
1450 mm.
The standard deviation is 47.6 mm.
The coefficient of variation is 3.54
The index of height of incisura to total height is *81.7*

TABLE 28.

DISTANCE FROM HEIGHT OF VERTEX TO HEIGHT OF VERTEBRA PROMINENS.

Standing		Sitting	
Class	*Frequency*	*Class*	*Frequency*
210 to 214.99 mm.	6	210 to 214.99 mm.	3
215	2	215	1
220	6	220	9
225	12	225	11
230	21	230	16
235	12	235	16
240	11	240	14
245	7	245	20
250	14	250	5
255	2	255	3
260	5	260	2
265	2		100
	100		
The mean is 238.04 mm. The extremes are 210 mm. 266 mm. The standard deviation is 13.20 mm. The coefficient of variation is 5.55		The mean is 237.84 mm. The extremes are 210 mm. 264 mm. The standard deviation is 10.80 mm. The coefficient of variation is 4.55	

TABLE 28a.

COMPARISON OF THE ABOVE MEASUREMENTS: STANDING AND SITTING.

Class	*Frequency*	
−20 to −15.01 mm.	3	The mean is .39 mm. shorter in the sitting measurements. In subtraction of the averages this difference is .20 mm. The extremes are −17 mm. 21 mm. This distance is somewhat shorter in sitting. This may be due to the fact that the spine is pushed up slightly by the pressure exerted upon it by the sitting posture.
−15	3	
−10	20	
− 5 to 0	24	
0 to 4.99 mm.	24	
5	15	
10	6	
15	4	
20	1	
	100	

TABLE 29.

Distance from Height of Tragus to Height of Vertebra Prominens.

Class	*Frequency*
80 to 81.99 mm.	1
82	0
84	1
86	1
88	2
90	4
92	0
94	3
96	4
98	6
100	2
102	7
104	8
106	4
108	4
110	5
112	6
114	3
116	7
118	7
120	5
122	10
124	5
126	1
128	1
130	1
132	2
	100

The mean is 110.55 mm.
The extremes are 81 mm.
132 mm.
The standard deviation is 11.88 mm.
The coefficient of variation is 10.75

TABLE 30.

Distance from Height of Vertebra Prominens to Height of Acromion.

Standing		Sitting	
Class	*Frequency*	*Class*	*Frequency*
30 to 34.99 mm.	1	30 to 34.99 mm.	1
35	0	35	0
40	2	40	1
45	4	45	0
50	3	50	3
55	2	55	5
60	8	60	5
65	12	65	8
70	12	70	5
75	17	75	11
80	12	80	21
85	9	85	13
90	5	90	12
95	7	95	5
100	4	100	3
105	0	105	4
110	1	110	2
115	1	115	0
		120	1
	100		100
The mean is 75.94 mm. The extremes are 34 mm. 116 mm. The standard deviation is 15.65 mm. The coefficient of variation is 20.40		The mean is 81.19 mm. The extremes are 33 mm. 123 mm. The standard deviation is 15.00 mm. The coefficient of variation is 18.50	

TABLE 30a.

Comparison of the Above Measurements: Standing and Sitting.

Class	*Frequency*
−25 to −20.01 mm.	1
−20	2
−15	3
−10	11
− 5 to 0	14
0 to 4.99 mm.	25
+ 5	14
+10	12
+15	13
+20	3
+25	1
+30	1
	100

The mean is 4.69 mm. longer in sitting than in standing. In the subtraction of averages this difference is 5.25 mm.

The extremes are −23 mm.
+31 mm.

This difference is probably due to the slight shortening up of the spine and to the dropping of the shoulders when sitting.

TABLE 31.

DISTANCE FROM VERTEX TO INCISURA STERNI.

Standing		Sitting	
Class	*Frequency*	*Class*	*Frequency*
270 to 274.99 mm.	1	265 to 269.99 mm.	1
275	8	270	4
280	13	275	1
85	8	280	5
90	14	285	5
95	25	290	18
300	7	295	21
305	10	300	16
310	6	305	9
315	3	310	10
320	4	315	6
325	0	320	2
330	1	325	2
	100		100
The mean is 296.09 mm. The extremes are 273 mm. 333 mm. The standard deviation is 12.40 mm. The coefficient of variation is 4.20		The mean is 299.74 mm. The extremes are 267 mm. 325 mm. The standard deviation is 11.75 mm. The coefficient of variation is 3.95	

The distance is 3.65 mm. longer sitting than standing.

All the distances taken between vertex, tragus or vertebra prominens on the one hand and incisura or acromion on the other are actually greater when the subject is in the sitting position. As it is evident that the relative positions of the three first of these points do not change very much in different positions of the body, the difference must be due largely to a drop of the shoulders and the thoracic cage when sitting. In the data presented by Fräulein Teumin the relative position of acromion in regard to incisura is higher in the sitting subject, perhaps because in her treatment the arms may have been supported in some way. Relatively to the vertebra prominens the incisura and acromion are lower in the sitting subject.

TABLE 32.

DISTANCE FROM HEIGHT OF VERTEBRA PROMINENS TO HEIGHT OF INCISURA STERNI.

Standing		Sitting	
Class	*Frequency*	*Class*	*Frequency*
20 to 24.99 mm.	1	20 to 24.99 mm.	1
25	1	25	0
30	1	30	0
35	2	35	0
40	5	40	3
45	10	45	6
50	17	50	18
55	19	55	14
60	14	60	12
65	11	65	16
70	8	70	13
75	5	75	9
80	6	80	7
	100	85	1
			100
The mean is 57.59 mm. The extremes are 21 mm. 82 mm. The standard deviation is 12.10 mm. The coefficient of variation is 21.00		The mean is 63.19 mm. The extremes are 23 mm. 86 mm. The standard deviation is 11.35 mm. The coefficient of variation is 18.00	

TABLE 33.

Distance from Height of Tragus to Height of Incisura Sterni.

(Subject standing.)

Class	*Frequency*
130 to 134.99 mm.	1
135	0
140	1
145	2
150	7
155	9
160	20
165	13
170	20
175	9
180	9
185	5
190	2
195	2
	100

The mean is 168.89 mm.
The extremes are 130 mm.
196 mm.
The standard deviation is 12.30 mm.
The coefficient of variation is 7.30

TABLE 34.

Distance from Height of Incisura Sterni to Height of Acromion.

Standing		Sitting	
Class	*Frequency*	*Class*	*Frequency*
−10 to −5.01 mm.	1	−10 to −5.01 mm.	2
− 5 to 0 mm.	5	− 5 to 0 mm.	5
0 to 4.99 mm.	9	0 to 4.99 mm.	12
5	16	5	9
10	16	10	11
15	9	15	14
20	12	20	19
25	14	25	14
30	9	30	5
35	4	35	5
40	4	40	3
45	0	45	0
50	1	50	1
	100		100

The mean is 17.99 mm.
The extremes are −5 mm.
51 mm.
The standard deviation is 12.45 mm.
The coefficient of variation is 69.00
The incisura is higher in 92 cases
The acromium is higher in 6 cases
The height is the same in 2 cases

The mean is 18.89 mm.
The extremes are −8 mm.
53 mm.
The standard deviation is 11.90 mm.
The coefficient of variation is 63.00
The incisura is higher in 89 cases
The acromium is higher in 7 cases
The height is the same in 4 cases

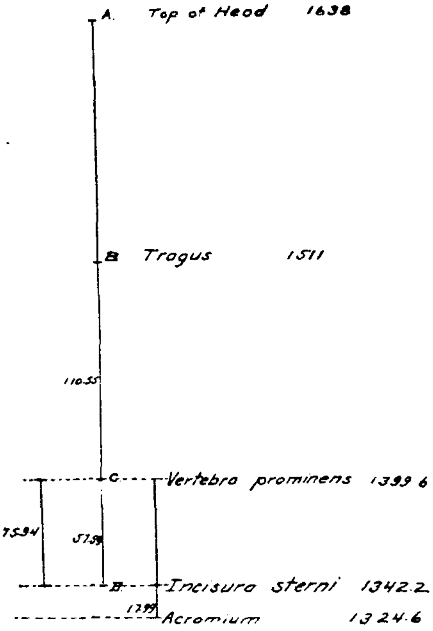


Chart showing relationships of Tragus, Vertebra prominens, Incisura sterni and Acromium

A-D = 296.09
A-C = 238.04
B-D = 168.9

FIGURE 22.

TABLE 35.

HEIGHT OF NIPPLE.

Class	*Frequency*
1060 to 1069.99 mm.	1
1070	0
1080	0
1090	0
1100	1
1110	2
1120	1
1130	2
1140	2
1150	1
1160	6
1170	3
1180	6
1190	9
1200	5
1210	5
1220	7
1230	8
1240	5
1250	1
1260	2
1270	2
1280	1
	70

The mean is 1198.79 mm.
The extremes are 1064 mm.
1280 mm.
The standard deviation is 43.20 mm.
The coefficient of variation is 3.60

TABLE 36.

Index of Height of Nipple to Total Height.

Class	*Frequency*
70 to 70.49	1
70.5	1
71.0	1
71.5	1
72.0	9
72.5	12
73.0	14
73.5	8
74.0	10
74.5	8
75.0	2
75.5	3
	70

The mean is 73.34
The extremes are 70.00
75.70
The standard deviation is 3 59
The coefficient of variation is 4.90

TABLE 37.

HEIGHT OF UMBILICUS.

Class	*Frequency*
880 to 899.99 mm.	3
900	5
920	8
940	11
960	15
980	17
1000	17
1020	15
1040	7
1060	1
1080	1
	100

The mean is 986.19 mm.
The extremes are 881 mm.
1080 mm.
The standard deviation is 42.80 mm.
The coefficient of variation is 4.34

TABLE 38.

INDEX OF HEIGHT OF UMBILICUS TO TOTAL HEIGHT.

Class	*Frequency*
55.5 to 55.99	1
56.0	0
56.5	0
57.0	0
57.5	3
58.0	5
58.5	9
59.0	12
59.5	12
60.0	16
60.5	8
61.0	11
61.5	11
62.0	4
62.5	3
63.0	1
63.5	0
64.0	1
64.5	1
65.0	1
65.5	0
66.0	1
	100

The mean is 60.41
The extremes are 55.59
66.10
The standard deviation is 6.58
The coefficient of variation is 10.80

RESULTS OF OTHER AUTHORS: HEIGHT OF UMBILICUS.

Europe

Race	*Author*	Absolute height		Index to total height ♂	♀
English students	Martin	. .	. .	57.7	. .
Parisians	"	. .	. .	58.9	. .
French	"	. .	. .	59.7	. .
Belgians	"	. .	. .	60.4	. .
French Normans	"	. .	. .	60.9	. .
Russians	Teumin	. .	. .	. .	59.0
Poles	Elkind	. .	. .	56.8	. .

Race	Author	Absolute Height		Index to total height ♂	♀
Asia					
Jews	Jakowenka	. .	. .	59.48	. .
Cossacks (Kirghiz)	Giltschenko	. .	. .	60.0	. .
" "	Charusin	. .	. .	58.68	. .
Ainu	Koganei	. .	862	. .	58.7
Japanese, thin	Bälz	. .	897	. .	59.6
" medium	"	. .	870	. .	59.1
" fat	"	. .	848	. .	58.7
Jakuts	Brodsky	. .	889	. .	59.6
Tunguses	"	. .	854	. .	58.4
Volga Kalmucks	Deniker	. .	. .	58.74	. .
" "	Metschnikoff	. .	. .	60.04	. .
Persians	Iwanowsky	. .	. .	59.25	. .
Chinese	Martin	. .	. .	59.76	. .
Torgouten	Iwanowsky	. .	. .	59.25	. .
Formosans	Martin	. .	. .	59.3	. .
Chinese	"	. .	. .	59.7	. .
Anamites	"	. .	. .	. .	57.6
Cambodians	"	. .	. .	. .	59.3
Africa					
Ba-Binga	Martin	. .	. .	58.5	59.3
Kabyles	"	. .	. .	59.4	. .
Ba-Tua	"	. .	. .	59.7	59.9
Bushmen	"	. .	. .	60.0	60.5
Dschagga	"	. .	. .	60.6	. .
Sudanese	"	. .	. .	60.9	61.2
M'Baka	"	. .	. .	60.9	61.3
Arabians	"	. .	. .	61.9	. .
Massai	"	. .	. .	61.9	. .
Oceanic Islands					
Australians	Martin	. .	. .	59.3	. .
Goliathberg Pygmies	"	. .	. .	59.6	. .
America					
Colorado Indians	Martin	. .	. .	57.9	58.4

The index of the height of the umbilicus is larger in the American women than in most of the other people measured. All the Europeans with the exception of the Belgians and the Norman French and the Cossacks have an index less than 60.0. The Asiatic peoples have an index less than 60.0, but the African tribes are over 60.0 and more like the Americans. However, we think that this measurement might be influenced largely by the physical condition of the subjects.

TABLE 39.

HEIGHT OF ANTERIOR VENTRAL SPINE OF ILIUM.

Class	*Frequency*
800 to 819.99 mm.	1
820	3
840	3
860	6
880	11
900	20
920	17
940	17
960	10
980	8
1000	4
	100

The mean is 926.79 mm.
The extremes are 811 mm.
1013 mm.
The standard deviation is 43.20 mm.
The coefficient of variation is 4.67
The index of height of spine to total height is *56.5*

RESULTS OF OTHER AUTHORS: HEIGHT OF ANTERIOR VENTRAL SPINE OF ILIUM.

Race	*Author*	Index to total height ♂	Index to total height ♀
Jakuts	Mainoff	53.34	
North Tunguses	"	55.00	55.55
Jakuts	Hecker	53.69	
Jakuts	Brodsky		55.4
Tunguses	"		55.8
Kuldscha (Kirghiz)	Iwanowsky	55.79	
Chinese	"	55.81	
Buku Cossacks	Charusin	56.04	
South Tunguses	Mainoff	56.18	56.3
Kuldscha Torgouts	Iwanowsky	56.86	
Telents	"	58.27	

TABLE 40.

HEIGHT OF SYMPHYSIS PUBIS.

Class	*Frequency*
720 to 739.99 mm.	1
740	1
760	9
780	7
800	14
820	19
840	18
860	14
880	10
900	3
920	3
940	1
	100

The mean is 839.0 mm.
The extremes are 737 mm.
942 mm.
The standard deviation is 43.20 mm.
The coefficient of variation is 5.15
The index of height of symphysis to total height is *51.2*

Results of Other Authors.

The relative symphysial height, that is, the index between the symphysial and total height, is shown by Martin in its racial variations in a table compiled from various authors. Data concerning females are rare, mainly gathered by Teumin, Koganei, and Jochelson-Brodsky. The more important are the following.

Race	♂	♀
Europe		
Englishmen	49.9	
Lapps	50.0	
Poles	50.7	
Belgians	50.7	
French	52.2	
Polish Jews		50.9
Asia		
Samoyedes	48.6	
Tungus and Yukagir		49.5
Japanese	49.0	48.4
Ainu	49.9	49.7
Kalmucks (Volga)	49.9	
Kalmucks	50.1	
Yakuts		50.3
Tatars	50.7	50.6
Anamites	51.2	
Africa		
Mawambi Pigmies	49.8	
Babinga	49.8	48.4
M'Baka	51.1	51.9
Bushmen	52.9	53.1
America		
Whites (U. S. Marines)	50.3	
U. S. Negroes	51.8	

TABLE 41.

TRUNK LENGTH.

(Tragus to symphysis pubis.)

Class	*Frequency*
590 to 599.99 mm.	1
600	1
610	2
620	7
630	4
640	13
650	9
660	12
670	12
680	8
690	13
700	7
710	3
720	6
730	1
740	1
	100

The mean is 671.29 mm.
The extremes are 597 mm.
742 mm.
The standard deviation is 31.30 mm.
The coefficient of variation is 4.55
The index of trunk length to total height is *41.0*

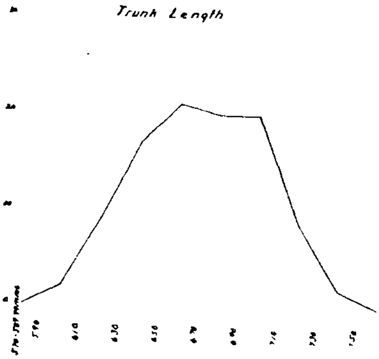


FIGURE 23.

This measurement, the distance from the tragus to the symphysis pubis, is the one used here in all tables in which a comparison is made with the trunk length. All such indices are based on this distance used as 100.

Perhaps a somewhat better trunk length is the sitting height of the suprasternale, Table 59, p. 529. The top of the table is at the plane of the sciatic tubers and furnishes the lower limit.

TABLE 42.

INDEX OF TRUNK LENGTH TO TOTAL HEIGHT.

Class	*Frequency*
36.00 to 36.99	2
37	2
38	6
39	17
40	18
41	21
42	23
43	7
44	4
	100

The mean is 41.10
The extremes are 36.60
44.50
The standard deviation is 1.70
The coefficient of variation is 4.15

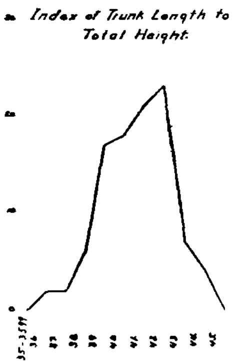


FIGURE 24.

TABLE 43.

DISTANCE FROM INCISURA STERNI TO SYMPHYSIS.

Class	*Frequency*
430 to 434.99 mm.	2
435	1
440	0
445	1
450	0
455	2
460	1
465	3
470	3
475	6
480	8
485	4
490	5
495	4
500	8
505	8
510	9
515	6
520	4
525	6
530	4
535	7
540	3
545	1
550	2
555	2
	100

The mean is 503.99 mm.
The extremes are 431 mm.
556 mm.
The standard deviation is 28.25 mm.
The coefficient of variation is 5.67
The index of this distance to total height is *30.7*

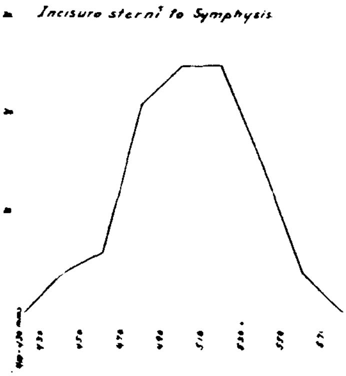


FIGURE 25.

TABLE 44.

INDEX OF THIS LATTER DISTANCE TO TOTAL HEIGHT.

Class	*Frequency*
26.0 to 26.49	1
26.5	0
27.0	1
27.5	1
28.0	4
28.5	5
29.0	8
29.5	7
30.0	13
30.5	13
31.0	15
31.5	8
32.0	16
32.5	3
33.0	2
33.5	2
34.0	1
	100

The mean is 30.78
The extremes are 26.00
34.10
The standard deviation is 1.48
The coefficient of variation is 4.80

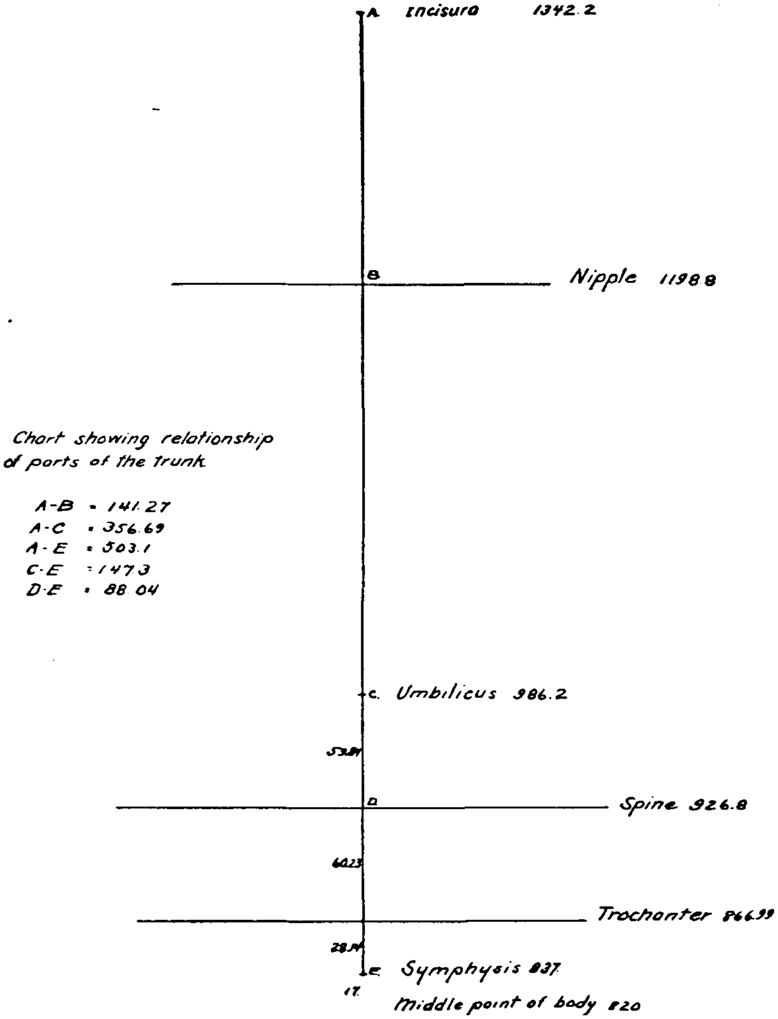

A Incisura 1342.2
B Nipple 1198.8
Chart showing relationship of parts of the trunk
A-B = 141.27
A-C = 356.69
A-E = 503.1
C-E = 147.3
D-E = 88.04
C. Umbilicus 986.2
53.84
D Spine 926.8
60.23
Trochanter 866.99
28.14
E Symphysis 837
17
Middle point of body 820

FIGURE 26.

TABLE 45.

DISTANCE FROM HEIGHT OF INCISURA STERNI TO HEIGHT OF NIPPLE.

Class	*Frequency*
100 to 104.99 mm.	0
105	1
110	2
115	4
120	4
125	6
130	6
135	12
140	11
145	9
150	4
155	3
160	2
165	3
170	2
175	0
180	0
185	0
190	0
195	0
200	1
	70

The mean is 141.27 mm.

The extremes are 105 mm.
201 mm.

The standard deviation is 16.20 mm.

The coefficient of variation is 11.50

The distance as obtained by the subtraction of averages is 143.40 mm.

The index of this distance to distance from symphysis to nipple is *39.0*

TABLE 46.

DISTANCE FROM HEIGHT OF NIPPLE TO HEIGHT OF SYMPHYSIS.

Class	*Frequency*
300 to 304.99 mm.	1
305	0
310	1
315	1
320	2
325	2
330	1
335	1
340	6
345	6
350	5
355	5
360	7
365	7
370	5
375	5
380	2
385	1
390	4
395	3
400	2
405	2
410	1
	70

The mean is 362.60 mm.
The extremes are 301 mm.
411 mm.
The standard deviation is 24.40 mm.
The coefficient of variation is 6.72
The index of this distance to trunk length is *54.2*

TABLE 47.

INDEX OF THIS LATTER DISTANCE TO DISTANCE FROM INCISURA STERNI TO SYMPHYSIS.

Class	*Frequency*
67.00 to 67.49	1
67.5	2
68.0	4
68.5	2
69.0	1
69.5	5
70.0	2
70.5	4
71.0	9
71.5	5
72.0	4
72.5	4
73.0	3
73.5	5
74.0	7
74.5	3
75.0	2
75.5	0
76.0	4
76.5	0
77.0	1
77.5	1
78.0	1
	70

The mean is 72.35
The extremes are 67.10
78.20
The standard deviation is 2.57
The coefficient of variation is 3.50

TABLE 48.

DISTANCE FROM UMBILICUS TO SYMPHYSIS.

Class	*Frequency*
90 to 94.99 mm.	1
95	0
100	1
105	2
110	0
115	6
120	3
125	8
130	4
135	9
140	13
145	5
150	12
155	6
160	10
165	8
170	4
175	3
180	5
	100

The mean is 147.30 mm.
The extremes are 90.00 mm.
180.00 mm.
The standard deviation is 21.20 mm.
The coefficient of variation is 14.46
The index of this distance to distance from symphysis to incisura (averages) is *29.4*
The index of this distance to distance from incisura to umbilicus is *41.3*

TABLE 49.

INDEX OF THIS LATTER DISTANCE TO DISTANCE FROM INCISURA STERNI TO SYMPHYSIS.

Class	*Frequency*
20 to 20.99	1
21	2
22	2
23	2
24	8
25	9
26	8
27	8
28	11
29	7
30	6
31	13
32	10
33	7
34	0
35	4
36	0
37	2
	100

The mean is 28.93
The extremes are 20.20
37.10
The standard deviation is 3.58
The coefficient of variation is 12.03

TABLE 50.

Distance from Incisura to Umbilicus.

Class	*Frequency*
295 to 299.99 mm.	1
300	0
305	0
310	1
315	1
320	1
325	7
330	9
335	5
340	10
345	5
350	7
355	8
360	9
365	6
370	9
375	5
380	5
385	2
390	4
395	1
400	3
405	1
	100

The mean is 356.69 mm.
The extremes are 295 mm.
405 mm.
The standard deviation is 22.00 mm.
The coefficient of variation is 6.20
The index of this distance to distance from the symphysis to incisura is *71.6*

TABLE 51.

HEIGHT OF MID-POINT OF BODY.

(When standing erect.)

Class	*Frequency*
750 to 759.99 mm.	1
760	1
770	7
780	8
790	5
800	10
810	15
820	13
830	13
840	17
850	2
860	4
870	3
880	1
	100

The mean is 820 mm.
The extremes are 750 mm.
880 mm.
The standard deviation is 27.40 mm.
The coefficient of variation is 3.34

The average height of the symphysis is 839 mm. This average is 19 mm. higher than that of the middle point of the body.

The symphysis is higher in	72%
The middle point is higher in	23%
They are equal in	5%
Total	100%

TABLE 52.

DIFFERENCE BETWEEN HEIGHT OF UMBILICUS AND MID-POINT OF BODY.
(The umbilicus is always higher than this middle point.)

Class	*Frequency*
120 to 124.99 mm.	1
125	4
130	6
135	6
140	7
145	4
150	10
155	5
160	7
165	12
170	6
175	8
180	6
185	5
190	2
195	3
200	6
205	1
210	1
	100

The mean is 163.99 mm.
The extremes are 122 mm.
211 mm.
The standard deviation is 21.80 mm.
The coefficient of variation is 13.30

TABLE 53.

Distance from Height of Umbilicus to Height of Anterior Ventral Spine of Ilium.

Class	*Frequency*
15 to 19.99 mm.	1
20	0
25	2
30	8
35	7
40	5
45	6
50	9
55	10
60	13
65	10
70	8
75	13
80	2
85	3
90	1
95	1
100	1
	100

The mean is 59.84 mm.
The extremes are 16 mm.
100 mm.
The standard deviation is 17.90 mm.
The coefficient of variation is 30.00.
The distance obtained by the subtraction of averages is 59.40 mm.

TABLE 54.

DISTANCE FROM HEIGHT OF ANTERIOR VENTRAL SPINE OF ILIUM TO HEIGHT OF SYMPHYSIS.

Class	*Frequency*
35 to 39.99 mm.	1
40	1
45	0
50	1
55	6
60	5
65	3
70	9
75	8
80	7
85	9
90	12
95	12
100	8
105	6
110	5
115	3
120	2
125	0
130	1
135	1
	100

The mean is 88.04 mm.
The extremes are 35 mm.
137 mm.
The standard deviation is 20.30 mm.
The coefficient of variation is 23.05
The distance obtained by the subtraction of averages is 87.79 mm.

TABLE 55.

Distance from Height of Trochanter to Height of Symphysis.

Class	*Frequency*
− 5 to 0.0 mm.	1
0.0 to 5.0	2
5	8
10	8
15	8
20	15
25	13
30	14
35	7
40	7
45	4
50	6
55	2
	95

The mean is 28.14 mm.
The extremes are − 5 mm.
56 mm.
The standard deviation is 15.00 mm.
The coefficient of variation is 53.30
The distance obtained by the subtraction of average heights is 27.99 mm.

The trochanter is higher than the symphysis in 92 cases.
The symphysis is higher than the trochanter in 1 case.
The height is equal in 2 cases.
In all, 95 cases.

TABLE 56.

Distance from Height of Anterior Ventral Spine of Ilium to Height of Trochanter.

Class	*Frequency*
10 to 19.99 mm.	1
20	2
30	5
40	13
50	23
60	30
70	4
80	7
90	5
100	3
110	1
	94

The mean is 60.23 mm.
The extremes are 11 mm.
110 mm.
The standard deviation is 17.50 mm.
The coefficient of variation is 29.05

Checks to Certain of the Preceding Tables.

The average measurements of distances between two levels (or "Heights") may be found, as in many of the preceding tables, by obtaining the individual distances by subtraction and then averaging these, or by obtaining the averages of certain heights, and subtracting these. Thus, in the first instance below, the distance from umbilicus to symphysis was first found in each of the 95 subjects from whom the data were obtained, and these gave a *mean distance* of 147.30 mm. (p. 516). This distance was again obtained by subtracting the average height of the symphysis, taken from the 95 individual heights, from a similar average for the height of the umbilicus, and this method gave 147.19 mm. The disparity, .11 mm., is negligible and proves the correctness of the general method. In the other cases given, the results are also nearly coincident, and furnish a satisfactory proof of the work. The results, as shown in a few of the preceding tables, are as follows:

Measurement	(1) By averaging individual distances obtained by subtracting heights	(2) By subtracting average heights
Umbilicus to symphysis	147.30	986.19 839.00 147.19
Umbilicus to anterior spine of ilium	59.84	986.19 926.79 59.40
Anterior spine to trochanter	60.23	926.79 866.99 59.80
Trochanter to symphysis	28.14	866.99 839.00 27.99

Again, certain of the results given here, representing the components of a larger distance, may be added together and the result compared with those for the distance of which they are the components. Thus, the distance umbilicus–symphysis is made up of the three distances umbilicus–ant. spine; ant. spine–trochanter, and trochanter–symphysis, and the sum of the results of the last three practically equals the result from the entire distance.

Umbilicus–ant. spine	59.84	59.40
Ant. spine–trochanter	60.23	59.80
Trochanter–symphysis	28.14	27.99
Umbilicus–symphysis	148.21	147.19

By using the distance ant. spine–symphysis, instead of its two components, we add to the 59.84 (umbilicus–ant. spine) the mean, 88.04, which gives 147.88. In the same way the distance ant. spine–symphysis may be broken up into ant. spine–trochanter, and trochanter–symphysis, and using these independent results we get 60.23 + 28.14 = 88.37.

Other similar results that concern distances between levels, such as those that involve vertex, tragus, incisura, acromion, and nipple, may be tested in the same way.

TABLE 57.

TOTAL SITTING HEIGHT.

Class	*Frequency*
810 to 819.99 mm.	1
820	8
830	12
840	16
850	11
860	13
870	8
880	12
890	9
900	5
910	2
920	1
930	1
940	0
950	1
	100

The mean is 864.49 mm.
The extremes are 811 mm.
952 mm.
The standard deviation is 27.90 mm.
The coefficient of variation is 3.23
The index of the total sitting height to the total height is *52.7*

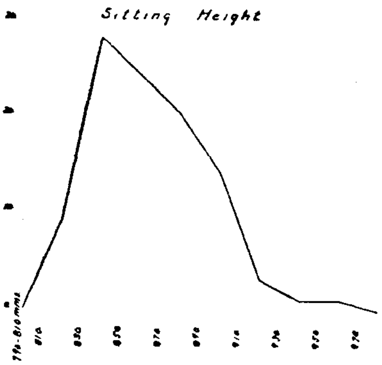


FIGURE 27.

RESULTS OF OTHER AUTHORS: INDEX OF SITTING HEIGHT TO TOTAL HEIGHT.

From Martin.

Race	*Author*	♂	♀
	Europe		
Ukrainian Jews		51.4	
Russian Jews	Blechmann	51.6	
Letts		51.9	
French		51.9	53.6
Lithuanians		52.1	
Belgians		52.2	53.4
Norman French		52.3	
English		52.4	
Livonians and Ests		52.5	
Albanians		52.6	
Balkan Tatars		52.6	53.5
Swedes		52.9	
Norwegians		53.0	
Russian Jews	Weissenberg	53.0	
Great Russians		53.0	53.2
	Asia		
Malser		50.8	
Cambodians			51.7
Anamites			51.9
Toda		51.5	
Kurumbar		51.6	
Brahmans		51.8	
Sudra		52.1	
Lolo		52.7	
Kalmucks		52.7	52.7
Jakuts		53.0	
Thibetans		53.2	54.3
South Chinese		53.2	
Setchuan Chinese		53.6	
Armenians		53.6	
North Chinese		53.7	
Kubu		53.9	54.0
Formosans		54.0	
Ainu		54.8	54.6
	Africa		
Massai		48.9	
Bushmen		49.5	50.5

Race	*Author*	♂	♀
Bugu		49.8	
M'Baka		50.4	50.6
Dschagga		50.5	
Somali		51.0	
Suaheli		51.0	
Jakoma		51.2	
Fans		51.3	50.4
Egyptians		51.3	
Kabyles		51.4	
Fiot		51.5	
Ba-Tua		51.8	52.5
Togo		51.9	
Duala		52.2	
Mawambi Pygmies		52.3	50.7
Kagoro		53.8	
Ba-Binga		54.0	53.2
	Oceanic Islands		
Australia		46.5	48.4
	America		
Trumai		50.6	51.1
Auetö		51.4	51.3
Eskimos		51.4	53.2
Nahuqua		51.8	52.2
Shoshoni		52.2	52.7
Polar Eskimos		52.5	53.7
Pima		52.9	
Apaches		53.2	
Koukpagmiut Eskimos		53.5	

In general this index is larger in the female than in the male, the legs of the male being longer and increasing the total height without a correspondingly large increase in the length of the body. In general too this index is larger in the shorter peoples as their bodies are longer in proportion. Curiously, though, the Mawambi pygmies have not a very large index and it is smaller in the female. The index of the women of the United States (52.7) is less than that of any of the Europeans measured and less than the Asiatic races. It is closer to the index of some of the African tribes and North American Indians.

TABLE 58.

HEIGHT OF VERTEBRA PROMINENS.

(Subject sitting.)

Class	*Frequency*
580 to 589.99 mm.	5
590	6
600	13
610	11
620	26
630	15
640	11
650	3
660	5
670	2
680	1
690	0
700	1
710	1
	100

The mean is 626.89 mm.
The extremes are 580 mm.
711 mm.
The standard deviation is 24.40 mm.
The coefficient of variation is 3.89
The index of height of vertebra prominens to total sitting height is *72.5*

TABLE 59.

HEIGHT OF INCISURA STERNI.

(Subject sitting.)

Class	*Frequency*
510 to 519.99 mm.	0
520	5
530	9
540	14
550	20
560	16
570	10
580	13
590	6
600	4
610	1
620	2
	100

The mean is 564.19 mm.
The extremes are 521 mm.
626 mm.
The standard deviation is 21.60 mm.
The coefficient of variation is 3.83
The index of height of incisura to total height is *65.2*

This measurement, the sitting height of the suprasternale, is often used as the trunk height. The principal objection to this is that it does not include the neck, as is done if the tragus be taken as the upper limit. Cf. p. 508, Table 41.

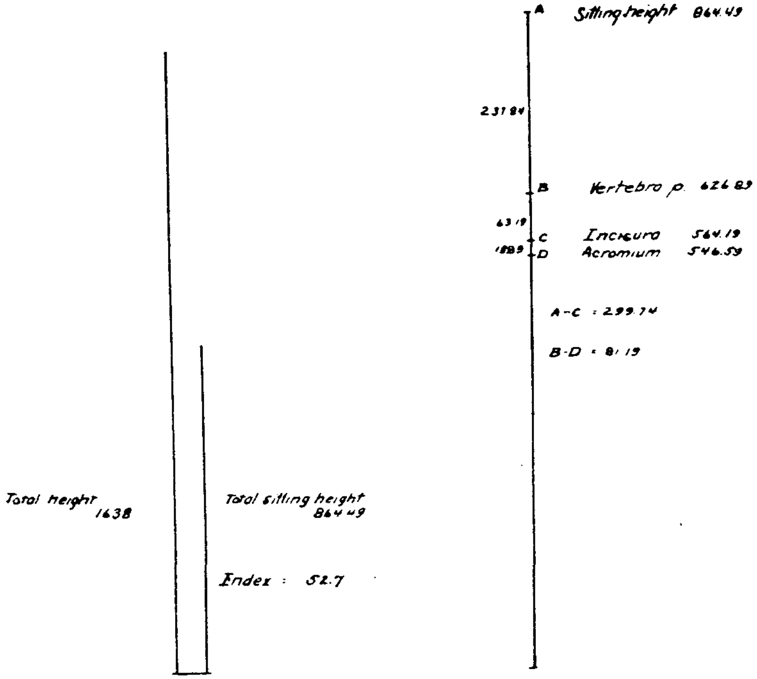
TABLE 60.

HEIGHT OF ACROMION.

(Subject sitting.)

Class	*Frequency*
500 to 509.99 mm.	3
510	9
520	17
530	15
540	14
550	15
560	12
570	6
580	5
590	3
600	0
610	0
620	1
	100

The mean is 546.59 mm.
The extremes are 500 mm.
629 mm.
The standard deviation is 23.10 mm.
The coefficient of variation is 4.23
The index of height of acromium to total height is *33.3*


A
Sitting height 864.49
231.84
B
Vertebra p. 626.83
63.19
C
Incisura 564.19
18.89
D
Acromium 546.59
A-C = 299.74
B-D = 81.19
Total height 1638
Total sitting height 864.49
Index : 52.7

Sitting Positions.

FIGURE 28.*

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* The scale of the right-hand diagram of Figure 28 is twice that of the left-hand half.

III. ARM MEASUREMENTS.

TABLE 61.

HEIGHT OF ACROMION.

Class	*Frequency*
1200 to 1219.99 mm.	1
1220	4
1240	6
1260	5
1280	11
1300	16
1320	18
1340	17
1360	14
1380	3
1400	3
1420	2
	100

The mean is 1324.59 mm.
The extremes are 1207 mm.
1425 mm.
The standard deviation is 50.60 mm.
The coefficient of variation is 3.74
The index of height of acromion to total height is *81.0*

TABLE 62.

HEIGHT OF HEAD OF RADIUS.

Class	*Frequency*
920 to 939.99 mm.	2
940	3
960	5
980	6
1000	19
1020	19
1040	19
1060	12
1080	10
1100	4
1120	1
	100

The mean is 1035.2 mm.
The extremes are 924 mm.
1124 mm.
The standard deviation is 41.00 mm.
The coefficient of variation is 3.96
The index of height of olecranon to total height is *63.2*

TABLE 63.

HEIGHT OF STYLOID PROCESS OF RADIUS.

Class	*Frequency*
690 to 699.99 mm.	1
700	0
710	1
720	2
730	3
740	5
750	7
760	6
770	7
780	12
790	15
800	12
810	5
820	11
830	9
840	3
850	1
	100

The mean is 790.49 mm.
The extremes are 693 mm.
857 mm.
The standard deviation is 33.00 mm.
The coefficient of variation is 4.17
The index of height of styloid process to total height is *48.3*

TABLE 64.

HEIGHT OF TIP OF MIDDLE FINGER.

Class	*Frequency*
540 to 549.99 mm.	1
550	0
560	9
570	4
580	6
590	9
600	14
610	14
620	16
630	11
640	5
650	4
660	6
670	0
680	1
	100

The mean is 613.79 mm.
The extremes are 547 mm.
681 mm.
The standard deviation is 28.40 mm.
The coefficient of variation is 4.65
The index of height of middle finger to total height is 37.5

TABLE 65.

Length of Arm.

(Acromion to tip of middle finger.)

Class	*Frequency*
640 to 649.99 mm.	4
650	1
660	9
670	8
680	7
690	6
700	14
710	18
720	12
730	9
740	3
750	4
760	3
770	1
780	0
790	1
	100

The mean is 707.49 mm.
The extremes are 640 mm.
792 mm.
The standard deviation is 33.60 mm.
The coefficient of variation is 4.75
The index of length of arm to length of the leg is *81.5*

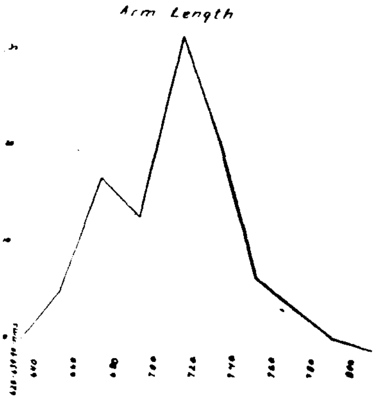

Figure 29.

TABLE 66.

INDEX OF ARM LENGTH TO TOTAL HEIGHT.

Class	*Frequency*
39.5 to 39.99	1
40.0	1
40.5	1
41.0	5
41.5	8
42.0	11
42.5	14
43.0	14
43.5	14
44.0	12
44.5	12
45.0	5
45.5	1
46.0	0
46.5	1
	100

The mean is 43.29
The extremes are 39.5
46.6
The standard deviation is 1.14
The coefficient of variation is 2.64

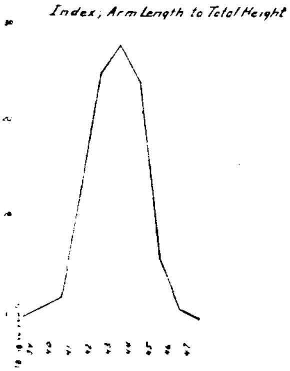

FIGURE 30.

TABLE 67.

Index of Arm Length to Trunk Length.

Class	*Frequency*
92 to 92.99	1
94	1
96	5
98	7
100	16
102	9
104	14
106	10
108	12
110	8
112	8
114	3
116	3
118	1
120	2
	100

The mean is 106.03
The extremes are 92.1
121
The standard deviation is 6.62
The coefficient of variation is 6.25

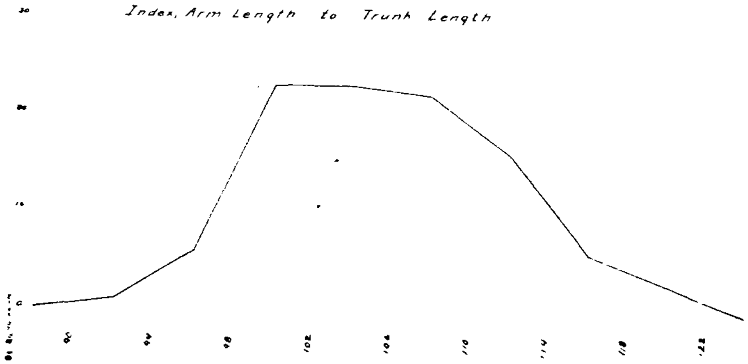

Figure 31.

RESULTS OF OTHER AUTHORS: INDEX OF ARM LENGTH TO TOTAL HEIGHT.

Compiled from data of Brodsky, Martin and Teumin.

Europe

Race	*Author*	♂	♀
Europeans	Quetelet	44.2	
Herzogovinians	Martin	44.1	
French Normans	"	44.7	
French	"	44.8	44.5
Czechs	"	45.0	
People of Baden	"	45.1	
Jews	Blechmann	45.2	
"	Weissenberg	45.2	
"	Jakowenko	44.7	
Russian Jews	Martin	45.1	44.1
Letts	"	44.9	44.4
Livonians	"	45.4	
Polish Jews	"	45.5	43.5
Belgians	"	45.5	44.2
Poles	Elkind	45.6	
Lapps	Iwanowsky	45.95	
Armenians	Martin	45.8	
Galicians	"	46.0	
Gypsies	"	46.0	
Jews	Weisbach	46.0	
Great Russians	Martin	46.4	43.7
Little Russians	Diebold	46.6	
Magyars	Martin	46.7	
Alsatians	"	46.8	
Lithuanians	Brenson		46.7
Russians	Teumin		42.0
Little Russian Jews	"		43.0
Polish Jews	"		42.0

Asia

Anamites	Martin		43.2
South Chinese	"		44.6
Cambodians	"		44.7
Kuldscha (Chinese)	Iwanowsky	44.4	
Zirjans	Martin	43.1	
Toala	"	43.2	
Japanese	"	43.2	
Senoi	"	43.6	42.9
Lolo	"	43.7	

Race	*Author*	♂	♀
Igorotes	Martin	43.8	
Cochin Chinese	"	43.9	41.2
Badaga	"	44.2	
Kubu	"	44.4	43.4
South Tunguses	Mainoff	44.4	44.1
Kalmucks	Martin	44.4	43.6
Menangkabau Malays	"	44.5	
Tonkinese	"	44.5	
Buriats	Korotoff	44.54	44.19
Cossacks (Kirghiz)	Iwanowsky	44.52	45.12
Orotsches	Margaritoff	44.84	
Ossets	Giltschenko	44.19	
"	Martin	44.9	44.3
Kayan	"	45.0	
Kitai	"	45.0	
Baschkirs	"	45.0	
Moi	"	45.1	
North Chinese	"	45.2	
Tatars	Iwanowsky	44.88	
Ostjaks	"	44.98	
Samoyedes	"	45.29	44.95
Kara Kirghiz	"	45.31	44.62
Bontoc Igorotes	Martin	45.2	42.7
Kurds	"	45.3	45.1
Javanese	"	45.4	
Torgouts	Iwanowsky	45.8	
Battaks	Martin	45.8	44.2
Telengets	"	46.0	
North Tunguses	Mainoff	45.75	44.97
Transbaikal Tunguses	Iwanowsky	45.81	
Astrakhan Kalmucks	"	45.96	
Ainu	Koganei	45.98	45.1
Jakuts	Mainoff	46.1	45.70
Tamils	Martin	46.3	
Chinese Setchuan	"	46.4	
Sikhs	"	46.6	
Bashkirs	Iwanowsky	46.09	
Tatars	Martin	46.9	44.6
Veddahs	"	47.0	
Persians	"	48.3	
Japanese, medium	Bälz		42.9
" delicate	"		43.9
Tunguses	Brodsky		44.5
Jakuts	"		44.6

Race	*Author*	♂	♀
Sojots	Gorostschenko		45.06
	Africa		
Amhara Abyssinians	Martin	44.5	
Mawambi Pygmies	"	44.8	46.6
Somali	"	45.0	
Kongo Negroes	"	45.0	
Baamba	"	45.2	
Bangambo	"	45.2	
Ba-Tua	"	45.3	45.8
M'Baka	"	45.6	44.9
Dahome Negroes	"	45.6	
Suaheli	"	45.6	
Togo	"	45.9	
Bushmen	"	46.0	45.5
Fiot	"	46.0	
Fans	"	46.2	
Jakoma	"	46.6	
Duala	"	46.7	
Ba-Binga	"	47.2	46.3
Lobi	"	47.4	46.9
Bugu	"	47.7	
	Oceanic Islands		
Micronesians	Martin	45.1	
New Mecklenbergers	"	45.1	
Solomon Islanders	"	45.5	
Samoans	"	45.7	
Buka	"	46.3	
Papuans	"	46.5	
Australians	"	47.0	
New Caledonians	"	47.2	
Goliathberg Pygmies	"	48.5	
	America		
Toba	Martin	43.4	
Koukpagmiut Eskimos	"	44.0	42.7
Nunatagmiut Eskimos	"	44.1	42.6
Matakis	"	44.1	
Athabascan Indians	Loucheux	44.4	42.7
Indians (Harrison Lake)	Boas	44.45	44.43
Vancouver Islanders	Martin	44.87	
Athabascan Indians	Boas	44.9	43.0
Shoshoni Indians	Martin	44.6	44.1

Race	*Author*	♂	♀
Bella Coola Indians	Boas	45.04	44.4
Iroquois	Martin	45.1	
Terra del Fuegians	"	45.6	
Karaja	"	45.8	
Botokudes	"	46.3	44.3

In general the Asiatic and African women have a larger index than the European. The index of the women of the United States is approximately that of the Europeans. The women of the American Indian races also have a larger index than these. In practically every case this index is larger in the male than in the female.

RESULTS OF OTHER AUTHORS: INTERMEMBRAL INDEX.

Race	*Author*	♂	♀
People of Baden		73.2	67.9
Veddahs		67.1	
Malayans (Asia)		69.1	64.4
Tamils		70.1	
Igorotes		70.0	
Menangkabau Malays		70.2	
Sikhs		70.8	
South Chinese		71.6	
Javanese		72.2	
Battaks		73.3	
Meraukė		70.9	71.7
Jabim Islanders		71.0	
New Mecklenbergers		71.0	69.5
Buka		72.2	
Brazilian Indians		74.4	74.9

This index is the relation of the humerus + the radius to the femur + the tibia. The figures are from Martin.

This is larger than the Asiatics and very much smaller than the Oceanic Islanders, or the Brazilian Indians. Except in the latter this index is smaller in females than in males.

Our determination of this index for the women of the United States is 67.6, approximately that of the women of Baden.

TABLE 68.

ARM STRETCH.

Class	*Frequency*
1500 to 1519.99 mm.	2
1520	0
1540	6
1560	9
1580	6
1600	10
1620	11
1640	16
1660	13
1680	7
1700	6
1720	5
1740	4
1760	4
1780	1
	100

The mean is 1647.4 mm.
The extremes are 1509 mm.
1792 mm.
The standard deviation is 59.00 mm.
The coefficient of variation is 3.58

TABLE 69.

INDEX OF ARM STRETCH TO TOTAL HEIGHT.

Class	*Frequency*	
95.5 to 95.99	1	
96.0	1	
96.5	1	
97.0	4	
97.5	9	
98.0	4	
98.5	7	
99.0	6	
99.5	9	The mean is 101.00
100.0	6	The extremes are 95.51
100.5	3	105.00
101.0	12	The standard deviation is 2.22
101.5	7	The coefficient of variation is 2.20
102.0	8	
102.5	3	
103.0	8	
103.5	4	
104.0	4	
104.5	1	
105.0	2	
	100	

Results of Other Authors: Index of Arm Stretch to Total Height.

Europe

Race	*Author*	♂	♀
Jews	Blechmann	103.3	
Albanians	Martin	101.6	
Russians	Teumin		100
Jews (Polish, Russian)	"		100
Great Russians	Martin	103.6	103.4
People of Baden	"		103.4
Jews	Weissenberg	103	
"	Giltschenko	103.4	
Norwegians	Martin	103.7	
South Russian Jews	"	103.8	102.9
Jews	Jakowenko	104.0	
Swedes	Martin	104.0	
French	Collignon	104.4	
Livonians	Martin	104.5	
Gypsies	"	104.6	
Lithuanians	Baronas	104.7	104.6
Belgians	Martin	104.8	101.6
French	Godin	106.1	
Lithuanians	Waeber	106.6	
Ests	Martin	108.0	
European women	Ricardo		101.5
	Asia		
Lolo	Martin	99.1	
North Chinese	"	102.1	
Chinese	"	102.1	101.5
Japanese	"	102.6	100.5
Perak Malays	"	102.8	
South Chinese	Girard	102.9	
Senoi	Martin	103.0	102.6
Formosans	"	103.7	
Koreans	"	104.0	
Toda	"	104.1	
Brahmans	"	104.4	
Payaks	"	104.5	
Thibetans	"	104.7	
Buriats	"	104.8	
Jakuts	Brodsky		104.0
Tunguses	"		103.5
Menangkabau Malays	Martin	104.9	

Race	*Author*	♂	♀
Tatars	Martin	105.0	103.0
Ainu	Koganei	105.9	104.9
Kurumbar	Martin	106.3	
Irular	“	106.3	
Kadir	“	107.0	
Anamites	“		101.9
Cambodians	“		104.5
Africa			
Kabyles	Martin	101.5	
Bushmen	“	102.1	100.8
Somali	“	103.2	
Fiot	“	104.0	
Ba-Binga	“	104.0	107.2
Ba-Tua	“	104.0	102.3
M'Baka	“	105.0	104.1
Massai	“	105.0	
Batanga	“	105.0	
Fans	“	105.0	105.0
Mawambi Pygmies	“	105.2	107.3
Jakoma	“	105.6	
Togo	“	105.9	
Lobi	“	105.9	104.5
Bugu	“	106.2	
Duala	“	108.0	
Dschagga	“	108.6	106.4
Oceanic Islands			
Goliathberg Pygmies	Martin	104.4	
Australians	“	104.9	
Meraukė	“	106.5	104.8
Jakumul (New Guinea)	“	106.9	
Arup “ “	“	108.8	
America			
Polar Eskimos	Martin	99.3	97.9
Colorado Indians	“	100.7	
Terra del Fuegians	“	101.4	
Koukpagmiut Eskimos	Boas	102.5	100.3
Nunatagmiut Eskimos	“	103.1	102.4
Thaltan Indians	“	103.5	101.0
Indians (Harrison Lake)	“	104.6	104.1
Shoshoni Indians	Martin	104.3	

Race	*Author*	♂	♀
Brazilian Indians	Martin	104.5	103.2
Bororó	“	104.7	
Nahuqua	“	105.0	104.7
Trumai	“	105.3	104.6
Auetö	“	106.1	105.1
Iroquois Indians	“	108.9	

This index is smaller in the female than in the male. It is smaller in women of the United States than in the Asiatic or African women; and is almost exactly equal to the general average for European women given by Ricardo.

TABLE 70.

Length of Humerus.

Class	*Frequency*
250 to 254.99 mm.	4
255	3
260	8
265	8
270	7
275	7
280	11
285	10
290	11
295	12
300	5
305	3
310	6
315	1
320	2
325	1
330	1
	100

The mean is 286.74 mm.
The extremes are 252 mm.
332 mm.
The standard deviation is 18.20 mm.
The coefficient of variation is 6.35
The index of humerus to total height is *17.5*
The index of humerus to arm length is *40.53*
The index of length of humerus to length of the femur is *63.3*

TABLE 71.

INDEX OF HUMERUS LENGTH TO TRUNK LENGTH.

Class	*Frequency*
35 to 35.99	1
36	1
37	2
38	3
39	12
40	13
41	11
42	13
43	11
44	7
45	11
46	9
47	2
48	1
49	1
50	1
51	0
52	1
	100

The mean is 42.76
The extremes are 35.6
52.1
The standard deviation is 3.05
The coefficient of variation is 7.13
The index of length of humerus to length of femur is *63.3*

RESULTS OF OTHER AUTHORS: INDEX OF HUMERUS LENGTH TO TOTAL HEIGHT.

Compiled from data of Brodsky, Martin and Teumin.

Europe

Race	*Author*	♂	♀
French	Martin	18.9	
Austrian Jews	Weisbach	17.90	
Germans	Martin	19.0	18.8
Norman French	"	19.4	
People of Baden	"	19.8	19.1
Jews	Jakowenko	19.04	19.23
Lithuanians	Brenson		19.4
Polish Jews	Teumin		19
Little Russian Jews	"		19.3
Russians	"		19
Jews	Blechmann	19.8	

Race	*Author*	♂	♀
White Russians	Eichholz	19.39	
Little Russians	Diebold	20.4	
	Asia		
Tonkinese	Martin	17.5	
Lolo	"	17.7	
Buriats	"	17.8	
Senoi	"	18.1	18.0
Japanese, heavy	Bälz		18.3
" light	"		18.4
Menangkabau Malays	Martin	18.3	
Javanese	"	18.3	
Igorotes	"	18.7	
Tengereses	"	19.2	
Jakuts	Brodsky		19
Tunguses	"		19
Kubu	Martin	18.8	18.0
South Chinese	"	19.2	
Bontoc Igorotes	"	19.2	18.5
Ainu	Koganei	19.3	17.7
Veddahs	Martin	19.4	
Tamils	"	19.5	
Malays	"	19.5	
Kalmucks	"	19.9	
Sikhs	"	20.1	
	Africa		
Kongo Negroes	Martin	18.0	19.2
Lobi	"	18.4	18.4
Fiot	"	18.5	
Jakoma	"	18.5	
Fans	"	18.6	18.9
Mawambi Pygmies	"	19.1	19.2
M'Baka	"	19.5	19.1
Ba-Tua	"	19.5	19.7
Bushmen	"	19.6	18.9
Ba-Binga	"	20.2	20.0
Hottentots	"	21.9(?)	
	Oceanic Islands		
New Mecklenbergers	Martin	19.2	
	America		
Colorado Indians	Martin	18.5	
Terra del Fuegians	"	19.2	19.7

Race	*Author*	♂	♀
Brazilian Indians	Martin	19.6	19.2
Patagonians	"	20.9	

This index is shorter in the women of the United States than in any others except the Japanese. It is about the same in the two sexes.

RESULTS OF OTHER AUTHORS: FEMORO-HUMERAL INDEX.

From Martin.

Race	*Author*	♂	♀
	Europe		
Bajuvars	Martin	71.0	73.1
German Swiss	"	71.1	70.9
Tyroleans	"	71.6	70.4
Europeans	Broca	72.5	71.9
Italians	Martin	72.9	72.1
Russians	Teumin		67.5
Polish Jews	"		67.5
Little Russian Jews	"		69
Little Russians	Diebold	68.7	68.7
Torgouts	Iwanowsky	75.6	72.6
	America		
Terra del Fuegians	Martin	69.8	
Lower Californians	"	70.6	71.1
Paltacalo Indians	"	72.0	73.2
Salado Indians	"	72.6	72.2
	Asia		
Andaman Islanders	Flower	68.8	
Senoi	Martin	69.9	
Hindus	Turner	70.5	
Veddahs	Martin	71.2	68.1
Ainu	"	72.4	71.7
	Africa		
Negroes	Broca	69.8	68.8
Bushmen	Schrubsell	70.0	
Massai	Martin	71.0	
Egyptians	Naqada	71.0	70.1
Negroes	Humphrey	71.2	
"	Turner	71.6	
Fiot	Martin	72.6	
	Oceanic Islands		
Australians	Turner	71.4	

TABLE 72.

LENGTH OF RADIUS.

Class	*Frequency*
210 to 214.99 mm.	1
215	1
220	5
225	11
230	12
235	12
240	16
245	8
250	9
255	7
260	5
265	4
270	1
275	2
280	4
285	2
	100

The mean is 245.24 mm.
The extremes are 211 mm.
289 mm.
The standard deviation is 16.60 mm.
The coefficient of variation is 6.77
The index of length of radius to total height is *14.9*
The index of length of radius to arm length is *34.8*
The index of length of radius to length of humerus is *85.5*
The index of length of radius to length of tibia is *73.4*

TABLE 73.

INDEX OF RADIUS LENGTH TO TRUNK LENGTH.

Class	*Frequency*
31 to 31.99	4
32	4
33	7
34	20
35	12
36	11
37	11
38	6
39	12
40	5
41	2
42	5
43	1
	100

The mean is 36.66
The extremes are 31.3
43.0
The standard deviation is 2.89
The coefficient of variation is 7.90

RESULTS OF OTHER AUTHORS: INDEX OF RADIUS LENGTH TO TOTAL HEIGHT.

Race	*Author*	♂	♀
	Europe		
Jews	Weisbach	16	
Parisians	Martin	14.3	
Belgians	"	14.4	13.9
Polish Jews	"	14.9	14.4
People of Baden	"	15.5	14.4
Germans	"	15.9	
Little Russian Jews	Teumin		14.0
Polish Jews	"		14.0
Russians	"		13.0
Jews	Martin	15.9	14.0
"	Jakowenko	16.07	
White Russians	Eichholz	16.59	
Little Russians	Diebold	19.91	
	Asia		
Burjats	Martin	13.7	
Senoi	"	13.8	13.9

Race	*Author*	♂	♀
Mongolian Torgouts	Iwanowsky	14.3	
Igorotes	Martin	14.5	
Kubu	"	14.6	14.4
South Chinese	"	14.8	
Tengereses	"	15.1	
Lolo	"	15.2	
Tamils	"	15.2	
Ainu	Koganei	15.3	15.0
Menangkabau Malays	Martin	15.4	
Kalmucks	"	16.2	16.5
Tonkinese	"	16.3	
Ossets	Giltschenko	16.52	
Kirghiz	"	16.6	
Japanese, heavy	Bälz		15.1
" medium	"		14.9
" light	"		14.8
Jakuts	Brodsky		14.2
Tunguses	"		14.2
	Africa		
Mawambi Pygmies	Martin	14.2	13.8
Ba-Tua	"	15.2	15.4
M'Baka	"	15.4	15.3
Bushmen	"	15.4	15.4
Ba-Binga	"	15.9	15.8
Fiot	"	16.2	
Fans	"	16.3	15.8
Jakoma	"	16.4	
Bugu	"	16.7	
Lobi	"	17.7	17.1
	America		
Colorado Indians	Martin	14.1	14.0
Botokudo	"	15.3	14.3
Terra del Fuegians	"	15.6	

This index in the women of the United States is slightly larger than in the Europeans, while the index of the humeral length is smaller. The lower arm is therefore longer in proportion. The radial index of the African tribes is longer than in the other races. There is no difference between the sexes in this measurement.

Results of Other Authors: Index of Arm Length without Hand to Total Height.

From Martin.

Race	♂	♀
People of Baden	35.4	33.6
French	35.0	34.1
Parisians	33.2	
Igorotes	33.3	
Menangkabau Malays	33.7	
Bontoc Igorotes	33.9	32.4
African Negroes	35.5	35.3
Lolo	32.5	
Fans	35.4	
Fiot	35.1	

This index in the women of the United States we have found to be 32.4

Results of Other Authors: Tibio-radial Index.

From Martin.

Race		♂	♀
Parisians (St. Germains)	Martin	63.5	
“ (St. Marcel)	“	64.6	66.2
Tyroleans	“	64.9	68.1
Australians	“	64.9	65.2
Negroes	“	65.4	65.7
Germans	“	66.0	68.7
Massai	“	66.1	62.2
Paltacalo Indians	“	66.1	66.8
Lower Californians	“	66.4	64.3
Veddahs	“	66.5	65.1
Ainu	“	67.7	66.2
Andamen Islanders	“	67.8	65.4
Bajuvars	“	68.1	68.4
Salado Indians	“	69.9	68.5
Terra del Fuegians	“	70.9	
Jews	Jakowenko	47.6	49
Little Russian Jews	Teumin		50
Polish Jews	“		48.5
Russians	“		50

The tibio-radial index for the women of the United States we have found to be 73.4.

TABLE 74.

LENGTH OF HAND.

Class	*Frequency*
130 to 134.99 mm.	1
135	0
140	1
145	0
150	4
155	5
160	7
165	15
170	18
175	8
180	11
185	8
190	6
195	7
200	5
205	1
210	1
215	1
220	1
	100

The mean is 177.64 mm.
The extremes are 130 mm.
220 mm.
The standard deviation is 15.90 mm.
The coefficient of variation is 9.00
The index of length of hand to total height is *10.85*
The index of length of hand to arm is *25.1*
The index of length of hand to length of foot is *73.6*

TABLE 75.

INDEX OF HAND LENGTH TO TRUNK LENGTH.

Class	*Frequency*
20 to 20.99	1
21	0
22	2
23	7
24	20
25	13
26	20
27	11
28	6
29	10
30	2
31	4
32	3
33	0
34	0
35	1
	100

The mean is 26.69
The extremes are 20.0
35.1
The standard deviation is 2.59
The coefficient of variation is 9.70

RESULTS OF OTHER AUTHORS: INDEX OF HAND LENGTH TO TOTAL HEIGHT.

From Martin

Europe

Race	*Author*	♂	♀
Little Russian Jews	Teumin		10
Polish Jews	"		10
Russians	"		10
Little Russians	Martin	10.9	
Polish Jews	"	10.9	10.9
White Russians	Eichholz	10.9	
Little Russians	Diedold	11.0	
Parisians	Martin	11.1	
Jews	"	11.1	11.2
French	Godin	11.2	
Jews	Weissenberg	11.2	
People of Baden	Martin	11.2	11.0
Belgians	"	11.3	11.3
Kirghiz	Giltschenko	11.3	
Norman French	Martin	11.4	
Jews	Blechmann	11.5	
"	Weisbach	11.8	
"	Jakowenko	11.13	11.24
Lithuanians	Martin	11.9	12.2
Lithuanians	Brenson		11.11

Asia

Lolo	Martin	9.8	
Kubu	"	9.8	
South Chinese	"	10.1	
Javanese	"	10.2	
Tamils	"	10.3	
Battaks	"	10.6	
Sikhs	"	10.7	
Tonkinese	"	10.7	
Menangkabau Malays	"	10.7	
Senoi	"	10.9	11.3
Jakuts	Brodsky		11.0
Kalmuks	Martin	11.1	11.0
Tunguses	Brodsky		11.2
Bontoc Igorotes	Martin	11.3	10.3
Veddahs	"	11.5	
Chinese	"	11.6	11.7
Ainu	Koganei	11.7	11.5

Race	*Author*	♂	♀
Japanese, coarse	Bälz		11.5
" medium	"		11.5
" delicate	"		11.6
Anamites	Martin	11.8	10.2
Cambodians	"	11.9	
	Africa		
Ba-Tua	Martin	10.5	10.8
M'Baka	"	10.8	10.5
Fiot	"	10.8	
Mawambi Pygmies	"	10.8	11.2
Bushmen	"	10.9	11.1
Massai	"	11.1	
Fans	"	11.2	11.2
Ba-Binga	"	11.4	11.0
Lobi	"	11.4	
Dschagga	"	11.4	11.0
Egyptians	"	11.6	
Jakom	"	11.7	
Bugu	"	11.8	
Kabyles	"	11.9	
	Oceanic Islands		
New Mecklenbergers	Martin	10.6	
Merauké	"	11.0	
	America		
Brazilian Indians	Martin	10.4	10.3
Galibi	"	10.5	
Botokudes	"	10.6	
North American Indians	"	11.0	
Terra del Fuegians	"	11.2	11.1

IV. LEG MEASUREMENTS.

TABLE 76.

LENGTH OF LEG.

(Height of trochanter.)

Class	*Frequency*
760 to 779.99 mm.	2
780	1
800	9
820	11
840	20
860	18
880	21
900	12
920	3
940	2
960	0
980	1
	100

The mean is 866.99 mm.
The extremes are 763 mm.
980 mm.
The standard deviation is 38.6 mm.
The coefficient of variation is 4.42
The index of length of leg to total height is *53.0*

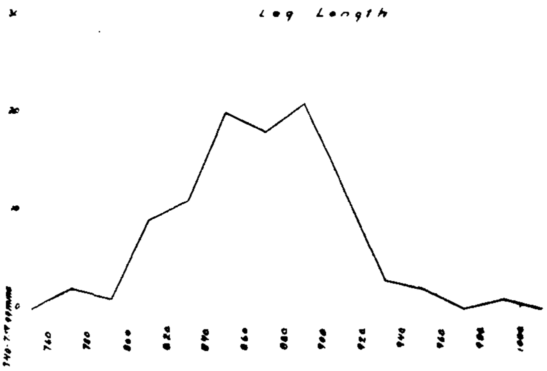

FIGURE 32.

TABLE 77.

INDEX OF LEG LENGTH TO TOTAL HEIGHT.

Class	*Frequency*
49.5 to 49.99	1
50.0	3
50.5	4
51.0	8
51.5	10
52.0	8
52.5	14
53.0	12
53.5	17
54.0	9
54.5	8
55.0	3
55.5	1
56.0	1
56.5	0
57.0	1
	100

The mean is 53.01
The extremes are 49.6
57.0
The standard deviation is 1.45
The coefficient of variation is 2.74

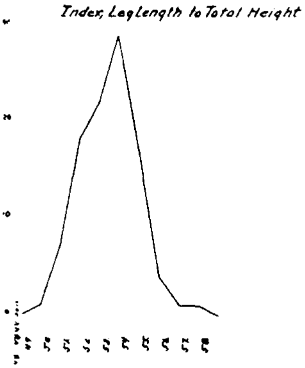

FIGURE 33.

TABLE 78.

Index of Leg Length to Trunk Length.

Class	*Frequency*	
110 to 111.99	1	
112	0	
114	0	
116	5	
118	7	
120	8	
122	6	
124	8	
126	7	
128	11	The mean is 128.60
130	10	The extremes are 111.0
132	9	154.1
134	8	The standard deviation is 8.08
136	7	The coefficient of variation is 6.30
138	4	
140	2	
142	3	
144	1	
146	1	
148	0	
150	1	
152	0	
154	1	
	100	

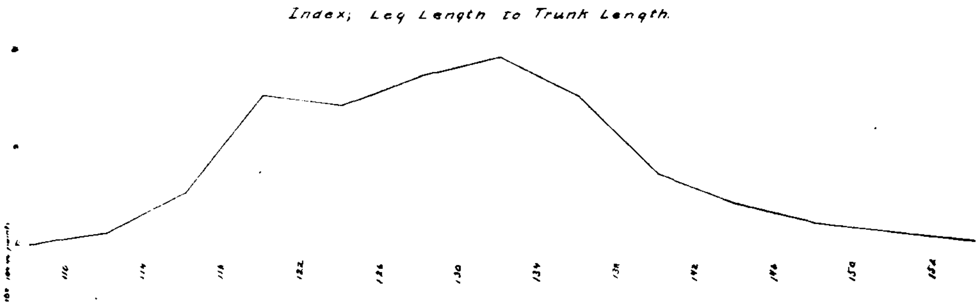

FIGURE 34.

It is interesting to note how much more variation there is in the index of length of the leg to the trunk length than to the total height. This corresponds almost exactly to the difference between the corresponding coefficients of variation of the indices of the arm lengths. The coefficient of variation of index of the arm length to the total height is 2.64; that of the leg 2.74.

The coefficient of variation of the index of the arm length to the trunk length is 6.25; that of the leg 6.30.

The correlation between the lengths of the arm and leg, and the total height is evidently much larger but there is a large amount of spuriousness in a correlation between the length of leg and total height.

RESULTS OF OTHER AUTHORS: INDEX OF LEG LENGTH TO TOTAL HEIGHT.

I. *Taking the symphysis as the upper limit of the leg.* (See index of the symphysis height to the total height.)

II. *Taking the height of the anterior ventral spine less 40 mm. as the upper limit of the leg.*

Race	*Author*	♂	♀
	Europe		
Lithuanians	Martin	50.7	50.2
Great Russians	"	51.1	50.6
Polish Jews	"	52.2	52.0
Swiss	Schaffhausen	53.3	
	Asia		
Japanese	Martin	48.5	49.0
Kalmucks	"	50.8	
Igorotes	"	51.6	50.5

Race	*Author*	♂	♀
Senoi	Martin	52.1	52.6
Anamites	“	52.2	
Tatars	“	52.3	51.3
Tengereses	“	52.8	
Moi	“	53.1	
Menangkabau Malays	“	53.6	
Jakuts	“	53.7	54.1
Sikhs	“	54.3	
Veddahs	“	54.8	
Jakuts	Brodsky		52.8
Tunguses	“		52.8

Africa

Jakoma	Martin	48.7	
Bugu	“	50.1	
Mawambi Pygmies	“	50.8	52.2
Kabyles	“	52.4	
Arabs	“	53.7	

Oceanic Islands

Meraukė	Martin	55.7	

America

Eskimos	Martin	54.0	

III. *Taking the trochanter as the upper limit of the leg.*

Europe

White Russians	Martin	50.6	
Russian Jews	“	50.7	
Norman French	“	50.7	
French	Collignon	51.7	
Belgians	Martin	52.0	50.8
South Russian Jews	“	52.1	52.2

Asia

Japanese, medium	Bälz	48.5	49.0
“ light	“	49.6	50.9
“ heavy	“	50.0	49.2
Cochin Chinese	Martin	49.7	
Chinese	“	50.0	52.9
Anamites	“	50.2	51.3
Kubu	“	50.9	51.5
Zirjans	“	51.0	

Race	*Author*	♂	♀
Tatars	Martin	51.3	
Armenians	"	51.5	
Ainu	Koganei	51.5	50.6
"	Iwanowsky	50.80	50.59
Kirghiz	"	50.09	49.04
Kara Kirghiz	"	49.52	
Jakuts	Brodsky		49.7
Kuldscha Chinese	Iwanowsky	49.67	
Lapps	"	49.7	
Tunguses	Brodsky		50.0
Jukagirs	"		50.0
Torgouts	Iwanowsky	50.55	
Astrakhan Kalmucks	"	50.59	
Buriats	"	50.82	50.28
Battaks	Martin	51.7	
South Chinese	"	51.8	
Kasan Tatars	Iwanowsky	51.29	
Japanese	Soller	51.7	
Javanese	Martin	52.2	
Arabians	"	52.6	
Tamils	"	53.8	
Sikhs	"	54.3	
Cambodians	"		53.3
	Africa		
Hottentots	Martin	50.5	
Ba-Binga	"	52.3	52.0
Congo Negroes	"	52.6	
Egyptians	"	52.9	
Ba-Tua	"	53.0	53.0
Dahome	"	53.1	
Berbers	"	53.4	
Fiot	"	54.0	
M'Baka	"	54.1	52.8
Fans	"	54.5	55.1
Aschanti	"	54.8	
Kru	"	56.0	52.2
Lobi	"	56.5	58.5
	Oceanic Islands		
New Mecklenbergers	Martin	51.6	
Buka	"	51.7	
Merauké	"	52.1	

Race	*Author*	♂	♀
Papuans	Martin	52.2	
Australians	"	52.7	
	America		
Botokudo	Martin	49.8	50.0
Eskimos	"	50.7	
Terra del Fuegians	"	50.9	50.4
Colorado Indians	"	51.0	52.3
Bororó	"	51.2	
Arowaks	"	51.2	
Tupi	"	51.3	
Bella Coola	"	51.8	
Bakairi	"	52.0	
Omaha Indians	"	52.4	

In Group I, where the symphysis is taken as the upper limit, the legs of the United States women are a little longer than those of the Asiatic and European women. In the second group the United States women have a larger index than any of the Europeans or Asiatics with the exception of the Jakuts measured by Brodsky. The index of United States women, as measured here, is about equal to that of the African tribes. In the third group the index of the United States women is larger than that of the European and Asiatic women and more nearly equal to that of the African tribes.

As the index of the height of the symphysis is about equal in all races, the difference seems to lie in the relative heights of the trochanter and spine. In the Russian and Jewish women measured by Dr. Teumin the pelvis, from the data, appears to be flatter and more flaring than in the women from the United States. The index of the cristal breadth in the Russians is 17.8, while in the Americans it is 16.63. The proportion is more clearly shown by the biacromial-cristal index which is 83.0 in the Russians and only 77.3 in the Americans. The breadth between the spines of the ilia is about the same in relation to the total height, but is much larger in comparison with the cristal breadth, in the Americans. The cristal-spinal index of the Russians is 88 and that of the Americans 94.1. If this is true it appears that the shape of the American pelvis may be more like that of the Africans, deeper and not so broad, with the trochanter above the symphysis.

TABLE 79.

HEIGHT OF KNEE.

(Articulation of femur and tibia on inner side of leg.)

Class	*Frequency*	
350 to 359.99 mm.	1	The mean is 414.59 mm. The extremes are 353 mm. 470 mm. The standard deviation is 23.30 mm. The coefficient of variation is 5.61 The index of height of knee to total height is *25.3*
360	1	
370	4	
380	8	
390	12	
400	18	
410	18	
420	13	
430	12	
440	5	
450	4	
460	3	
470	1	
	100	

TABLE 80.

HEIGHT OF INNER MALLEOLUS.

Class	*Frequency*	
55 to 59.99 mm.	1	The mean is 78.69 mm. The extremes are 57 mm. 95 mm. The standard deviation is 8.25 mm. The coefficient of variation is 10.50 The index of height of inner malleolus to total height is *4.8* The index of height of inner malleolus to length of leg is *9.1* The index of height of inner malleolus to length of trunk is *11.7*
60	1	
65	13	
70	25	
75	11	
80	19	
85	24	
90	5	
95	1	
	100	

TABLE 81.

LENGTH OF FEMUR.

Class	*Frequency*
390 to 399.99 mm.	1
400	6
410	9
420	6
430	8
440	15
450	14
460	11
470	19
480	5
490	3
500	2
510	0
520	1
	100

The mean is 452.99 mm.
The extremes are 390 mm.
523 mm.
The standard deviation is 27.60 mm.
The coefficient of variation is 6.07
The index of length of femur to total height is *27.6*
The index of length of femur to length of leg is *52.3*

TABLE 82.

INDEX OF FEMUR LENGTH TO TRUNK LENGTH.

Class	*Frequency*
52 to 53.99	1
54	0
56	4
58	1
60	5
62	14
64	12
66	15
68	11
70	21
72	5
74	8
76	1
78	0
80	1
82	1
	100

The mean is 67.67
The extremes are 51.9
82.0
The standard deviation is 5.18
The coefficient of variation is 7.76

Results of Other Authors: Index of Femur Length to Total Height.

I. *Taking the symphysis as the upper limit of the leg.*

Race	*Author*	♂	♀
Jews	Weisbach		23.7
Lithuanians	Brenson		29.43
Little Russian Jews	Teumin		27
Polish Jews	"		27
Russians	"		27
White Russians	Eichholz	25.08	
Ossets	Giltschenko	22.9	
Little Russians	Talko Grinzewitch	29.2	
" "	Diebold	30.30	
Ainu	Koganei		21.7

II. *Taking the spinal height less 40 mm. as the upper limit of the leg.*

Europe

Race	*Author*	♂	♀
People of Baden	Martin	27.1	28.4

Asia

Race	*Author*	♂	♀
Kalmucks	Martin	23.3	
Sikhs	"	26.0	
Senoi	"	26.3	25.6
Menangkabau Malays	"	26.5	
Veddahs	"	27.8	
Jakuts	Brodsky		26.6
Tunguses	"		27.1

Africa

Race	*Author*	♂	♀
Fiots	Martin	25.6	
Mawambi Pygmies	"	26.7	28.6
Fans	"	26.8	27.5
Bushmen	"	27.2	27.4
Lobi	"	29.9	30.8

III. *Taking the trochanter as the upper limit of the leg.*

Europe

Race	*Author*	♂	♀
French	Godin	25.4	
People of Baden	Martin	26.0	

	Asia		
Race	*Author*	♂	♀
Japanese, medium	Bälz	22.5	23.2
" light	"	23.3	24.2
" heavy	"	23.7	23.2
Kubu	Martin	24.2	24.5
Mongolian Torgouts	Iwanowsky	24.24	
Menangkabau Malays	"	24.5	
Battaks	"	24.6	
Javanese	"	24.6	
Igorotes	"	24.7	23.1
South Chinese	"	24.9	
Kling	"	25.8	
	Africa		
Jakoma	Iwanowsky	22.9	
Bugu	"	23.1	
Ba-Tua	"	25.8	25.9
Ba-Binga	"	25.9	24.7
M'Baka	"	26.2	25.4
	Oceanic Islands		
New Mecklenbergers	Iwanowsky	24.1	
Buka	"	24.2	
Jabim	"	25.0	
	America		
Colorado Indians	Martin	24.3	25.3

Compared with this last set of indices that of the United States women (27.6) is larger as in most of our subjects we found the trochanter to be above the symphysis. It was 60 mm. below the spine of the ilium so that there is only 20 mm. difference between the height of the trochanter and the height of the point defined by Martin as the upper limit of the leg, 40 mm. below the spine of the ilium. This index is more nearly like those in the first group where this point has been taken as the upper limit of the leg. In the first group there is a great deal of variation, the index of the United States women (25.7) being a little smaller than the average. In general this index seems to be a little larger in the female than in the male.

TABLE 83.

LENGTH OF TIBIA.

Class	*Frequency*
270 to 279.99 mm.	1
280	1
290	3
300	7
310	14
320	19
330	15
340	19
350	9
360	6
370	4
380	1
390	1
	100

The mean is 334.19 mm.
The extremes are 274 mm.
390 mm.
The standard deviation is 21.40 mm.
The coefficient of variation is 6.10
The index of length of tibia to total height is *20.4*
The index of length of tibia to length of the leg is *38.7*
The index of femur plus tibia to length of leg is *91.0*
The index of length of tibia to length of femur is *73.8*

TABLE 84.

Index of Tibia Length to Trunk Length.

Class	*Frequency*
42 to 43.99	3
44	12
46	12
48	22
50	22
52	12
54	10
56	6
58	1
	100

The mean is 50.19
The extremes are 42.1
59.0
The standard deviation is 3.64
The coefficient of variation is 7.25

Results of Other Authors: Index of Tibia Length to Total Height.

Race	*Author*	♂	♀
	Europe		
Jews	Jakowenko (to outer malleolus)	23.45	22.98
Little Russian Jews	Teumin		20
Polish Jews	"		20
Russian	"		20
White Russians	Eichholz	23.15	
People of Baden	Martin	22.4	21.2
French	Godin	22.6	
	Asia		
Mongolian Torgouts	Iwanowsky	20.39	
Japanese, medium	Bälz	21.2	23.8
" light	"	21.9	23.4
" heavy	"	22.0	22.5

Race	*Author*	♂	♀
Japanese	Martin	21.9	22.5
Javanese	"	22.8	
Battaks	"	23.0	
Toda	"	22.1	
Irular	"	22.2	
Brahmans	"	22.3	
Senoi	"	22.4	22.7
Menangkabau Malays	"	22.7	
Kalmucks	"	23.1	
Veddahs	"	23.3	
South Chinese	"	23.1	
Igorotes	"	23.4	
Sikhs	"	24.1	
Tamils	"	24.3	
Tunguses	Brodsky		20.3
Jakuts	"		20.9
	Africa		
Jakoma	Martin	21.8	
Ba-Binga	"	22.4	23.4
Mawambi Pygmies	"	22.7	23.4
Bugu	"	23.1	
Ba-Tua	"	23.4	
Bushmen	"	23.8	23.9
M'Baka	"	23 9	23.8
Lobi	"	25.7	25.2
Fans	"	27.4	27.2
Fiot	"	28.0	
	Oceanic Islands		
Buka	Martin	23.4	
New Mecklenbergers	"	23.5	
Jabim	"	24.0	
	America		
Colorado Indians	Martin	22.2	22.6

The index of the American women (20.4) is about the same as that of the Russian and Jewish women measured by Teumin and as that of the Jakut and Tungus women measured by Brodsky. It is smaller than the index of the rest of the Asiatic women. There seems to be no sexual difference.

RESULTS OF OTHER AUTHORS: INDEX OF TIBIA LENGTH TO FEMUR LENGTH

(Crural or Tibio-femoral Index.)

From Martin.

Race	*Author*	♂	♀
Tunguses	Brodsky		74.7
Jukagirs	"		74.7
Jakuts	"		77.6
Swiss	Aargau	84.2	
Bologna Italians	Peli	85.0	85.0
Menangkabau Malays	Martin	85.8	
People of Baden	"	86.5	75.0
Senoi	"	86.6	89.3
Malays (Sumatra)	"	87.0	
French	Papillaud	87.7	89.4
Setchuan Chinese	Martin	88.6	
Jews	Jakowenko	88.7	86.9
Little Russian Jews	Teumin		76.1
Polish Jews	"		72.2
Russians	"		76.1

In this set of data there are few cases in which both sexes of the same race were measured. In general this index seems much smaller in the female than in the male. It is hard to reach any definite conclusion as in four cases where both sexes are measured, the index is about equal, while in only one is it much smaller in the female.

TABLE 85.

LENGTH OF FOOT.

Class	*Frequency*
205 to 209.99 mm.	1
210	0
215	2
220	6
225	9
230	10
235	21
240	18
245	10
250	14
255	4
260	3
265	1
270	1
	100

The mean is 240.69 mm.
The extremes are 207 mm.
273 mm.
The standard deviation is 11.80 mm.
The coefficient of variation is 4.66
The index of foot length to total height is *15.1*
The index of foot length to length of leg is *24.1*
The index of hand length to foot length is *73.6*

TABLE 86.

INDEX OF FOOT LENGTH TO TRUNK LENGTH.

Class	*Frequency*
31 to 31.99	2
32	5
33	14
34	10
35	14
36	19
37	20
38	8
39	6
40	2
	100

The mean is 36.05
The extremes are 31.1
40.0
The standard deviation is 2.16
The coefficient of variation is 6.00

Results of Other Authors: Index of Foot Length to Total Height

Mainly from Martin.

Europe

Race	*Author*	♂	♀
Jews	Jakowenko	15.43	14.04
Lithuanians	Brenson		15.5
“	Martin	14.6	14.4
Little Russian Jews	Teumin		14
Polish Jews	“		14
Russians	“		15
Jews	Weisbach	15.06	
Little Russian Jews	Weissenberg	15.6	
Jews	Blechmann	15.5	
White Russians	Eichholz	15.5	
Letts	Martin	14.6	14.8
Parisians	“	14.8	
French Normans	“	15.6	
South Russian Jews	“	15.7	
Belgians	“	15.7	14.9
Jews	“	15.9	
People of Baden	“	16.0	15.5
	Asia		
Japanese, medium	Bälz	13.8	14.4
“ thin	“	14.5	14.6
“ heavy	“	14.6	15.6
Malser	Martin	15.0	
Setchuan Chinese	“	15.0	
Chinese	“	15.1	
Toda	“	15.2	
Irular	“	15.3	
Brahmans	“	15.4	
Javanese	“	15.4	
Siamese	“	15.5	
Battaks	“	15.5	
Sikhs	“	15.5	
Ainu	Koganei	15.5	14.8
Ajini	Anutschin	15.3	
Mongolian Torgouts	Iwanowsky	15.3	
Cambodians	Martin	15.5	
Senoi	“	15.6	
Tamils	Martin	15.8	

Race	*Author*	♂	♀
Anamites	Martin	15.0	
Tunguses	Brodsky		14.6
Jakuts	"		14.9
	Africa		
Bushmen	Martin	14.6	14.4
Suaheli	"	14.8	
Fiot	"	14.8	
Duala	"	15.0	
Fans	"	15.4	14.7
Bugu	"	15.4	
Togo	"	15.4	
Lobi	"	15.4	15.0
Massai	"	15.5	
Egyptians	"	15.5	
Ba-Tua	"	15.5	14.8
Ba-Binga	"	15.6	15.2
M'Baka	"	15.6	15.0
Dschagga	"	15.6	15.0
Mawambi Pygmies	"	15.6	15.5
Jakoma	"	15.6	
	Oceanic Islands		
Buka	Martin	15.9	
Merauké	"	15.9	
New Mecklenbergers	"	16.0	
	America		
Galibi	Martin	13.7	
Colorado Indians	"	15.0	14.5
Schingu Indians	"	15.1	
Brazilian Indians	"	15.2	14.8

The index of the United States women (15.1) is slightly larger than the majority of both European and Asiatic races. It is in general subject to but little variation.

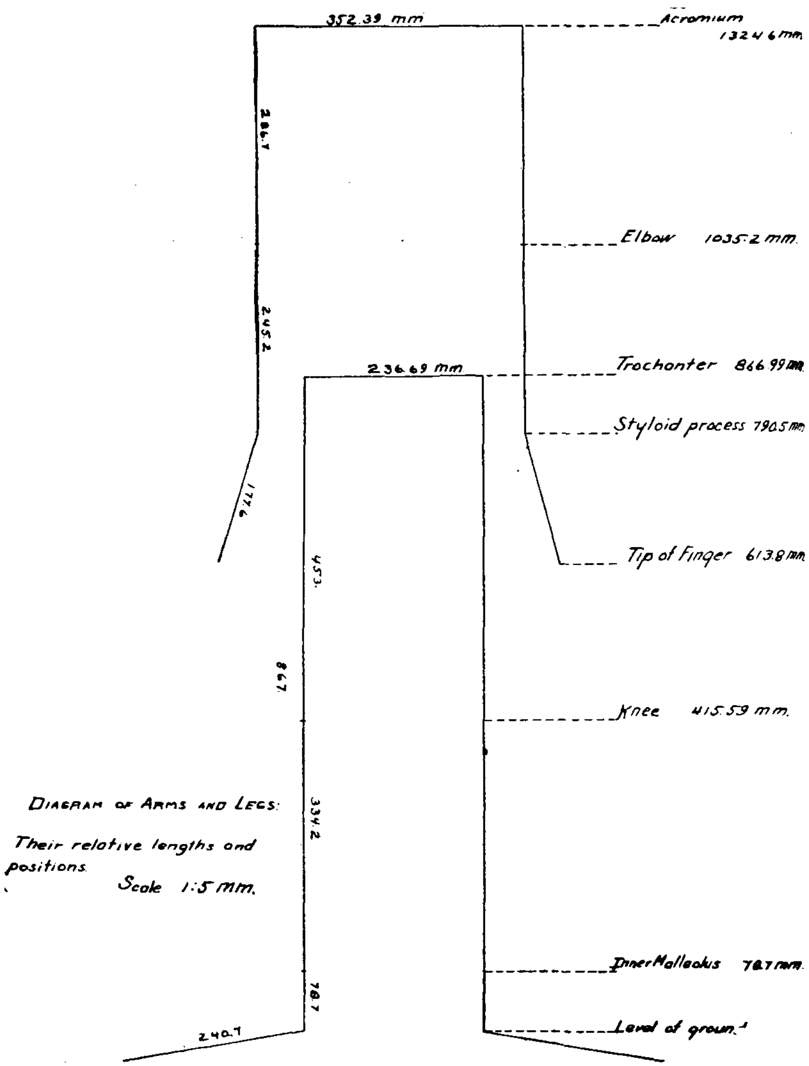
352.39 mm
Acromium
1324.6 mm
286.7
Elbow 1035.2 mm.
245.2
236.69 mm
Trochonter 866.99 mm
Styloid process 790.5 mm
177.6
Tip of Finger 613.8 mm
453.
867.
Knee 415.59 mm.
Diagram of Arms and Legs:
Their relative lengths and positions.
Scale 1:5 mm.
334.2
Inner Malleolus 78.7 mm.
78.7
240.7
Level of groun.

FIGURE 35.

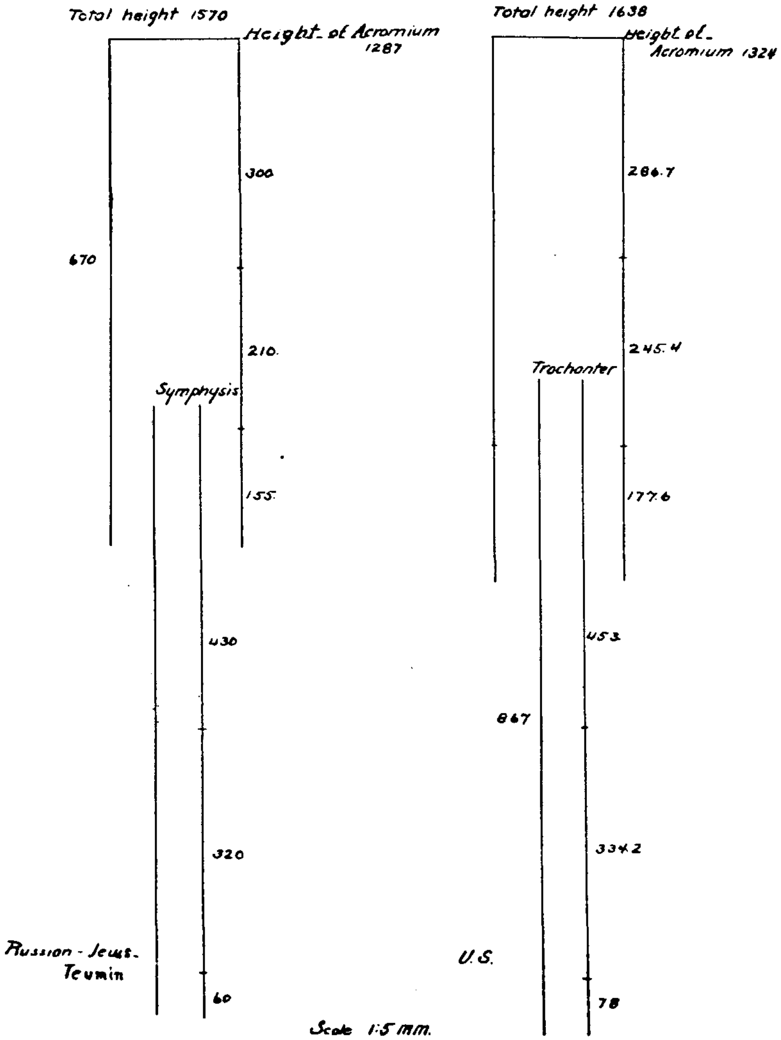
Total height 1570
Height of Acromium 1287
300
670
210.
Symphysis
155.
430
320
60
Russion - Jews-
Teumin
Total height 1638
Height of
Acromium 1324
286.7
245.4
Trochonter
177.6
453
867
334.2
78
U.S.
Scale 1:5 mm.

FIGURE 36.

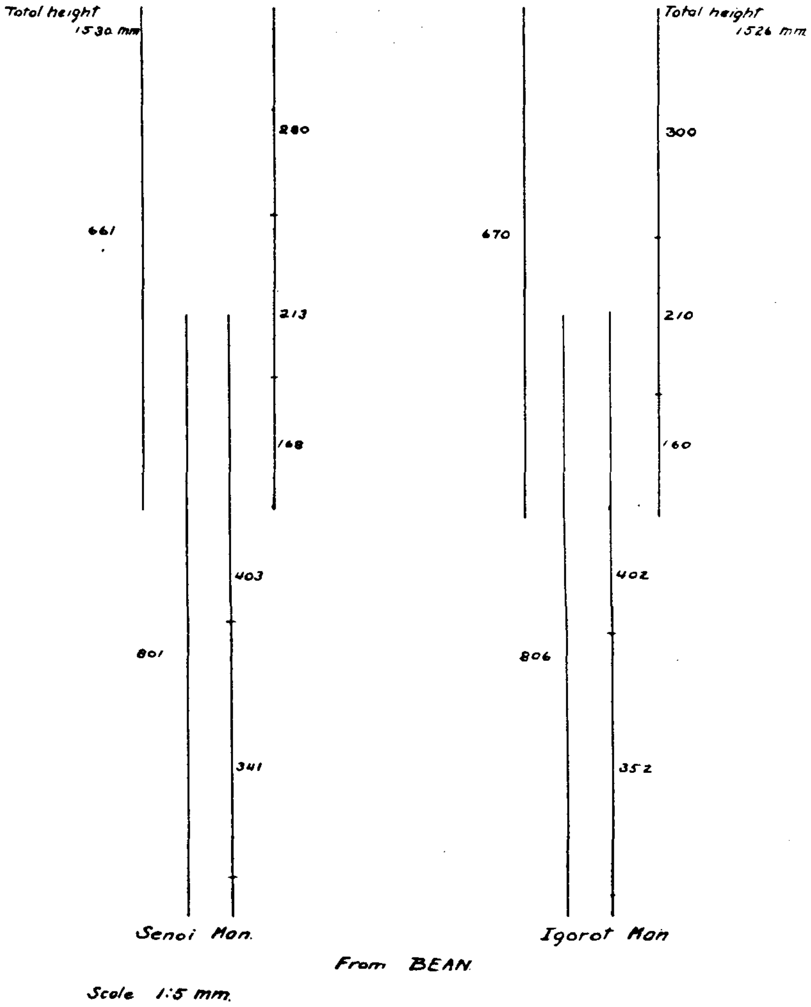
Total height
1530 mm
Senoi Man.
Total height
1526 mm
Igorot Man
From BEAN
Scale 1:5 mm.

FIGURE 37.

V. Intermembral Correlations.

The amounts of correlation in such measurements as the length of the arm as compared with that of the leg, or between the lengths of their component parts, seem to us of great value and interest. Correlations between the actual lengths of different parts of the body, as well as correlations between indices, offer an almost untouched field to the anthropometrist. In this place we present such correlations only between absolute lengths of arm and leg.

I The coefficient of correlation between the length of the arm and the length of the leg is .63 .63
This is a large coefficient and shows a large amount of correlation.

II The coefficient of correlation between the length of the humerus and the length of the femur is .17. This shows only a small amount of correlation. .17

III The coefficient of correlation between the length of the radius and the length of the tibia is .40, showing a moderate degree of correlation. .40

IV The coefficient of correlation between the length of the hand and the length of the foot is .43, showing a moderate degree of correlation. .43

V The coefficient of correlation between the length of the arm and the total height is .63, showing a large amount of correlation. .63

VI. Breadths.

TABLE 87.

Biacromial Breadth.

Class	*Frequency*
300 to 304.99 mm.	1
305	0
310	1
315	0
320	4
325	5
330	1
335	9
340	17
345	12
350	7
355	9
360	8
365	9
370	7
375	3
380	3
385	3
390	1
	100

The mean is 352.39 mm.
The extremes are 302 mm.
391 mm.
The standard deviation is 17.25 mm.
The coefficient of variation is 4.90
The index of biacromial breadth to total height is *21.5*
The index of biacromial breadth to trunk length is *52.5*

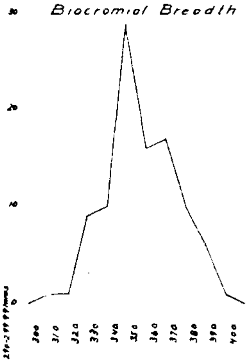

Figure 38.

RESULTS OF OTHER AUTHORS: INDEX OF BIACROMIAL BREADTH TO TOTAL HEIGHT.

Mainly from Martin.

Europe

Race	*Author*	♂	♀
Lithuanians	Martin	18.1	18.0
French	Topinard	18.9	16.3
"	Collignon	20.3	
"	Lyon	21.2	19.9
"	Godin	22.8	
Russians	Teumin		21.0
Jews	"		21.0
"	Weissenberg	22.0	21.9
Polish Jews	Martin	22.1	20.6
Letts	"	23.0	
People of Baden	"	23.0	22.4
Belgians	"	23.4	22.0

Asia

Sojots	Gorostschenko		18.5
Tatars	Martin	20.3	19.8
Jakuts	Mainoff	21.2	20.9
North Tunguses	"		21.76
North Chinese	Martin	22.0	
Igorotes	"	22.6	21.8
Setchuan Chinese	"	23.1	
Cossacks	"	23.2	
Tunguses	Brodsky		22.2
Jukagirs	"		22.2
Jakuts	"		21.6
Buriats	Martin	23.5	
Ainu	Koganei	23.6	23.2
Anamites	Martin		21.4
Cambodians	"		21.9
Japanese, medium	Bälz	23.5	23.2
" heavy	"	25.0	25.3
Kubu	Martin	24.1	22.7
Kalmucks	"	24.5	23.4
Formosans	"	25.3	

Africa

Lobi	Martin	18.6	17.7
Dschagga	"	19.8	

Race	*Author*	♂	♀
Somali	Martin	20.0	
Ba-Binga	“	20.0	20.1
Bushmen	“	20.0	20.0
Bugu	“	20.5	
Jakoma	“	20.6	
Massai	“	20.7	
Tunisians	“	21.3	
Fiot	“	21.7	
Sudanese	“	21.8	
Ba-Tua	“	22.0	20.6
M'Baka	“	22.0	21.0
Fans	“	22.4	21.4
Mawambi Pygmies	“	22.7	20.4
Suaheli	“	24.4	
Togo	“	25.0	
Duala	“	25.3	

Oceanic Islands

Goliathberg Pygmies	Martin	21.9	
Merauké	“	23.1	22.1

America

Iroquois	Gould	18.8	
Trumai	Martin	21.0	21.5
Athabascan Indians	“	22.1	22.0
“ “	Boas		22.24
Colorado Indians	Martin	22.5	22.7
Eskimos	Boas		21.83
Nunatagmiut Eskimos	“		21.2
Koukapamiut “	“	22.8	22.0
Nahuqua	Martin	22.9	21.6
Shoshoni Indians	“	23.2	22.6
Bororó	“	23.6	21.5
Bakairi	“	24.7	23.9
Eskimos	“	24.3	22.7

The biacromial index is smaller in the female than in the male. This index is larger in the women of the United States (21.5) than in the European and is smaller than in the Asiatic women.

TABLE 88.

THORACIC BREADTH.

Class	*Frequency*
220 to 224.99 mm.	8
225	6
230	10
235	10
240	10
245	4
250	7
255	5
260	3
265	1
270	2
275	0
280	1
	67

The mean is 239.14 mm.
The extremes are 222 mm.
281 mm.
The standard deviation is 13.35 mm.
The coefficient of variation is 5.60
The index of thoracic breadth to total height is *14.6*
The index of thoracic breadth to trunk length is *35.6*
The index of thoracic breadth to biacromial breadth is *68.1*
The index of thoracic breadth to cristal breadth is *87.8*
The index of thoracic breadth to spinal breadth is *93.5*

TABLE 89.

THORACIC DEPTH.

Class	*Frequency*
135 to 139.99 mm.	2
140	1
145	2
150	2
155	1
160	6
165	16
170	10
175	10
180	7
185	4
190	2
195	1
200	0
205	1
210	0
215	1
220	0
225	0
230	1
	67

The mean is 173.69 mm.
The extremes are 139 mm.
230 mm.
The standard deviation is 15.75 mm.
The coefficient of variation is 9.10
The index of thoracic depth to total height is *10.6*
The index of thoracic depth to trunk length is *25.9*
The index of the mean thoracic depth to mean thoracic breadth is *72.6*

TABLE 90.

THORACIC INDEX.

(Depth to breadth.)

Class	*Frequency*
60 to 61.99	3
62	4
64	1
66	8
68	6
70	10
72	13
74	10
76	7
78	2
80	1
82	1
84	1
	67

The mean is 71.71
The extremes are 60.00
85.00
The standard deviation is 5.08
The coefficient of variation is 7.08
There is a difference of .92 between the mean thoracic index, and the index of the mean thoracic depth to the mean thoracic breadth.

RESULTS OF OTHER AUTHORS: THORACIC INDEX.

	Depth	Breadth	Index
African Negroes	195	269	138.0
Hova	179	260	143.5
Navajo	216	297	137.5
French	194	269	138.6
Jakoma	..	..	133.0
Bugu	..	..	124.0

These figures are from Martin and give the more usual relation of the *breadth* to the *depth*. The corresponding figures for the women of the United States we find to be 137.9.

TABLE 91.

BIMAMMILLARY BREADTH.

Class	*Frequency*
150 to 154.99 mm.	1
155	0
160	1
165	2
170	2
175	2
180	7
185	11
190	9
195	10
200	8
205	3
210	4
215	6
220	4
	70

The mean is 195.49 mm.
The extremes are 150 mm.
223 mm.
The standard deviation is 15.10 mm.
The coefficient of variation is 7.72
The index of bimammillary breadth to total height is *11.9*
The index of bimammillary breadth to trunk length is *29.1*
The index of bimammillary breadth to biacromial breadth is *55.5*

RESULTS OF OTHER AUTHORS: BIMAMMILLARY BREADTH.

From Martin.

	♂	♀
Massai	113 mm.	. .
Dschagga	113 mm.	. .
Ba-Tua	113 mm.	. .
M'Baka	120 mm.	126 mm.
People of Baden	122 mm.	. .
Ba-Binga	126 mm.	. .
Tatars	127 mm.	132 mm.
Russians and Jews (Teumin)	. .	200 mm.

The above data are not of much value for comparison as they are mostly measurements of males. The bimammillary breadth of Russian women is larger than that of Americans, and the index of this breadth to the total height is 60.0 in the case of the Russian women, who are short, and only 55.0 in the case of the Americans. This difference may be due entirely to the amount of development of the glands. We have endeavored to eliminate as much of this error as possible by omitting all subjects with pendulous breasts.

TABLE 92.

SPINAL BREADTH.

Class	*Frequency*
210 to 214.99 mm.	1
215	1
220	0
225	3
230	2
235	3
240	4
245	6
250	9
255	13
260	7
265	4
270	5
275	3
280	5
285	1
	67

The mean is 256.22 mm.
The extremes are 211 mm.
285 mm.
The standard deviation is 14.85 mm.
The coefficient of variation is 5.80
The index of spinal breadth to total height is *15.6*
The index of spinal breadth to trunk length is *38.4*
The index of spinal breadth to biacromial breadth is *72.9*
The index of spinal breadth to cristal breadth is *94.1*

TABLE 93.

CRISTAL BREADTH.

Class	*Frequency*
210 to 214.99 mm.	2
215	0
220	0
225	1
230	0
235	1
240	2
245	3
250	5
255	7
260	7
265	14
270	12
275	15
280	8
285	6
290	7
295	6
300	2
305	1
310	1
	100

The mean is 272.34 mm.
The extremes are 210 mm.
313 mm.
The standard deviation is 18.80 mm.
The coefficient of variation is 6.90
The index of cristal breadth to total height is *16.6*
The index of cristal breadth to trunk length is *40.6*
The index of cristal breadth to biacromial breadth is *77.3*

TABLE 94.

TROCHANTERIC BREADTH.

Class	*Frequency*
205 to 209.99 mm.	6
210	4
215	10
220	11
225	12
230	8
235	11
240	7
245	11
250	5
255	4
260	2
265	2
270	1
275	2
280	3
285	0
290	0
295	0
300	1
	100

The mean is 236.69 mm.
The extremes are 205 mm.
304 mm.
The standard deviation is 19.40 mm.
The coefficient of variation is 8.19
The index of trochanteric breadth to total height is *14.4*
The index trochanteric breadth to trunk length is *35.3*
The index of trochanteric breadth to biacromial breadth is *67.2*
The index of trochanteric breadth to cristal breadth is *86.9*

RESULTS OF OTHER AUTHORS: SPINAL, CRISTAL AND TROCHANTERIC BREADTHS.
(Females only.)

From Brodsky.

Index to total height.

		Spinal	Cristal	Trochanteric
Race	*Author*		♂	♀
Russians and Jews	Teumin	16	18	..
North Tunguses	Mainoff	..	18.6	..
South Tunguses	"	..	19.22	..
Sojots	Gorotschenko	..	..	..
Ainu	Koganei	16.3	18	20.4
Japanese	Osawa	16.1	17.9	19.8

		Spinal	Cristal	Trochanteric
Race	*Author*		♂	♀
Jakuts	Brodsky	16.6	18.6	20.2
Tunguese	"	16.6	18.4	19.9
	Absolute lengths.			
Russians and Jews	Teumin	250 mm.	280 mm.	. .
North Tunguses	Mainoff	. .	268	. .
South Tunguses	"	293.6	293.6	. .
Sojots	Gorostschenko	. .	267	. .
Javans	Strasser	230	250	280
Ainos	Koganei	239	264	300
Japanese	Osawa	235	262	284
Jakuts	Brodsky	254	277	301
Tunguses	"	243	268	293

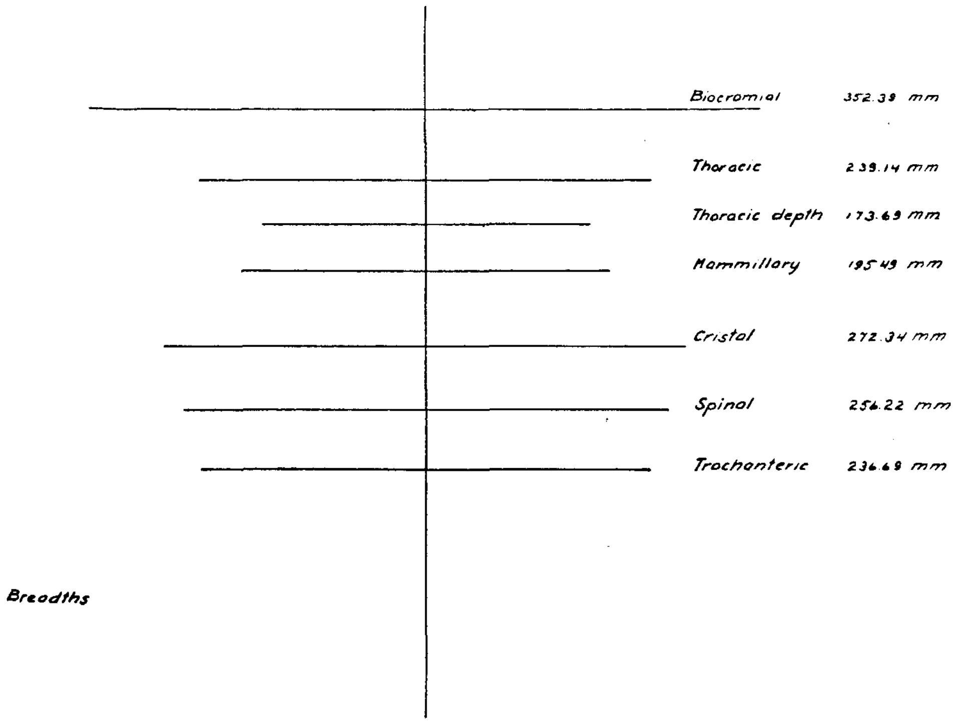

FIGURE 39.

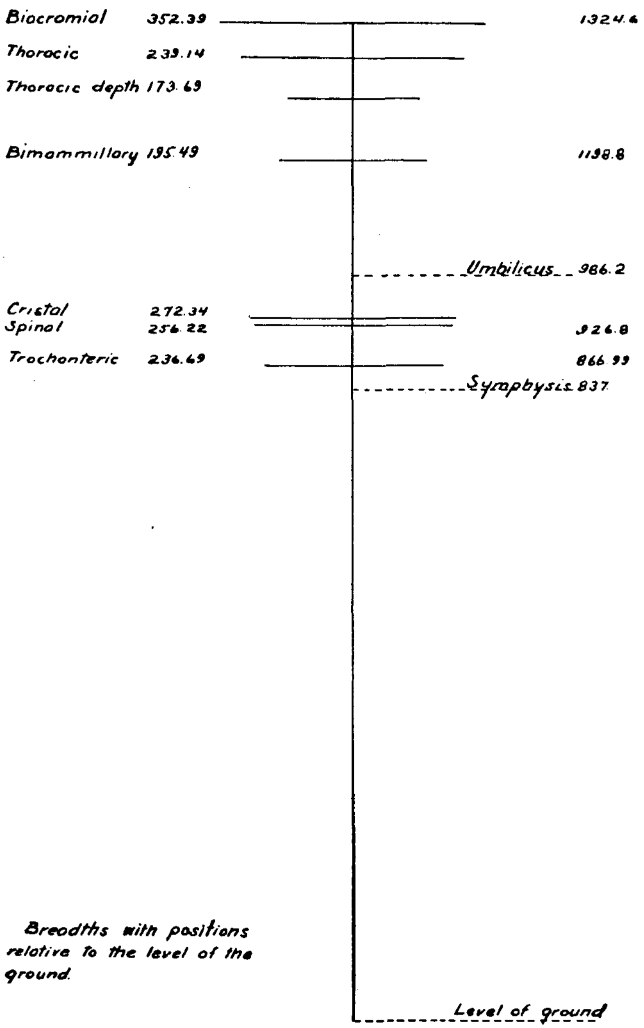
Biacromial 352.39
1324.6
Thoracic 233.14
Thoracic depth 173.69
Bimammillary 195.49
1198.8
Umbilicus 986.2
Cristal 272.34
Spinal 256.22
926.8
Trochanteric 236.69
866.99
Symphysis 837
Breadths with positions relative to the level of the ground
Level of ground

FIGURE 40.

VII. INDIVIDUAL MEASUREMENTS OF THE ONE HUNDRED SUBJECTS USED IN THIS PAPER.

The individual measurements of the 100 subjects used in this paper have been published in the "Manual of Anthropometry" of the senior author (Blakiston, Philadelphia, 1920), in which it appears as Appendix B, pp. 171–185. Since these figures have already been published, and since especially there is great risk of typographical errors in printing so extensive a series of figures, it has been determined not to duplicate them here, but to refer readers to them.

VIII. SUMMARY OF AVERAGES AND INDICES.

HEIGHT FROM FLOOR.

	Average in man	Index to total height
Total height	1638.0 mm.	
Height of tragus	1510.0	*92.0*
Height of vertebra prominens	1399.6	*85.5*
Height of incisura sterni	1342.2	*81.7*
Height of acromium	1324.6	*81.0*
Height of nipple	1198.8	*73.3*
Height of umbilicus	986.2	*60.4*
Height of ant. spine of ilium	926.8	*56.5*
Height of trochanter	867.0	*53.0*
Height of symphysis pubis	839.0	*51.2*
Height of elbow	1035.2	*63.2*
Height of styloid process of radius	790.5	*48.3*
Height of tip of middle finger	613.8	*37.5*
Height of knee	414.6	*25.3*
Height of inner malleolus	78.7	*4.8*

SITTING HEIGHTS.

		Index to total height
Total sitting height	864.5	*52.7*
		Index to total sitting height
Height of vertebra prominens	626.9	*72.5*
Height of incisura sterni	564.2	*65.2*
Height of acromium	546.6	*33.3*

MEASUREMENTS OF THE HEAD.

Length of head	186.88 mm.
Breadth of head	147.11
Height of head	128.14
Frontal breadth	102.72
Interorbital Breadth	31.17
Zygomatic breadth	116.35
Mandibular breadth	98.27
Facial lengths	
Physionomic	175.59
Anatomical	115.35
Superior	72.07
Length of nose	53.61
Breadth of nose	32.18
Length of ear	58.89
Breadth of ear	30.99

Indices	
Cephalic	*78.96*
Height to length	*68.70*
Anatomical facial (zygomatic breadth to anatomical length)	*100.65*
Mandibulo-zygomatic (mandibular breadth to zygomatic breadth)	*84.09*
Nasal	*59.89*
Aural	*50.63*
Height of head to total height	*7.88*
Height of head to trunk length	*18.61*

MEASUREMENTS OF ARM AND LEG.

A. ARM MEASUREMENTS.

Arm stretch	1647.4 mm.
Length of arm	707.5
Length of humerus	286.7
Length of radius	245.2
Length of hand	177.6
Length of humerus + radius	531.9

Indices	To total height	To trunk length
Arm stretch	*101.0*	
Arm	*43.3*	*106.0*
Humerus	*17.5*	*42.8*
Radius	*14.9*	*36.7*

		To total height	To trunk length
Hand		*10.8*	*26.7*
Humerus + radius		*32.4*	*79.2*
Humerus to arm	*40.5*		
Radius to arm	*34.8*		
Hand to arm	*25.1*		
Radius to humerus	*85.5*		
Arm to leg	*81.5*		
Humerus to femur	*63.3*		
Radius to tibia	*73.4*		
Hand to foot	*73.6*		
Humerus + radius to femur + tibia	*67.6*		

B. LEG MEASUREMENTS.

Length of leg (height of trochanter)	867.0 mm.
Height of knee	414.6
Length of femur	453.0
Length of tibia	334.2
Length of foot	240.7
Height of inner malleolus	78.7
Length of femur and tibia	787.0

Indices		To total height	To trunk length
Leg		*53.0*	*128.6*
Knee height		*25.3*	
Femur		*27.6*	*67.7*
Tibia		*20.4*	*50.2*
Foot		*15.1*	*36.1*
Femur + tibia		*48.0*	*117.1*
Inner malleolus height		*4.8*	*11.7*
Femur to leg	*52.3*		
Tibia to leg	*38.7*		
Foot to leg	*24.1*		
Inner malleolus height to leg	*9.1*		
Tibia to femur	*73.8*		
Arm to leg	*81.5*		
Humerus to femur	*63.3*		
Radius to tibia	*73.4*		
Hand to foot	*73.6*		
Humerus + radius to femur + tibia	*67.6*		

MEASUREMENTS OF THE TRUNK.

Length of the trunk, tragus to symphysis	671.29 mm	
Length, incisura to symphysis	503.99	
Length, incisura to nipple	141.27	
Length, symphysis to nipple	362.60	
Length, incisura to umbilicus	356.69	
Length, symphysis to umbilicus	147.30	
Indices		
Trunk length to total height		*41.1*
(Length, incisura–sym.) to (total height)		*30.8*
(Length, sym.–nipple) to (length, incisura–sym.)		*72.4*
(Length, sym.–umbilicus) to (length, umbilicus–incisura)		*41.3*
(Length, sym.–umbilicus) to (length, incisura–sym.)		*28.9*
(Length, nipple–incisura) to (length, sym.–nipple)		*39.0*
Mid-point of total height	820.00 mm.	
Difference between height of symphysis and the mid-point	19.00	
Difference between the height of umbilicus and mid-point	163.99	
Difference between the height of umbilicus and spine	59.84	
Difference between height of spine and symphysis	88.04	
Difference between height of trochanter and symphysis	28.14	
Difference between height of spine and trochanter	60.23	
Difference between height of incisura and of acromium	17.99	
Difference sitting	18.89	
Difference between total height and height of vertebra prominens	238.04	
Difference sitting	237.84	
Difference between height of vertebra prominens and incisura	57.59	
Difference sitting	63.19	
Difference between height of vertebra prominens and acromium	75.94	
Difference sitting	81.19	
Difference between height of tragus and vertebra prominens	110.55	
Difference between height of tragus and incisura	168.89	
Difference between total height and height of incisura	296.09	
Difference sitting	299.74	

MEASUREMENTS OF BREADTHS.

Biacromial	352.39 mm.
Cristal	272.34
Trochanteric	236.69
Bimammillary	195.49
Spinal	256.22
Thoracic breadth	239.14
Thoracic depth	173.69

Indices		To total height	To trunk length
Biacromial		*21.5*	*52.5*
Cristal		*16.6*	*40.6*
Trochanteric		*14.4*	*35.3*
Bimammillary		*11.9*	*29.1*
Spinal		*15.6*	*38.4*
Thoracic breadth		*14.6*	*35.6*
Thoracic depth		*10.6*	*25.9*
Thoracic depth to breadth	*71.7*		
Cristal to biacromial	*77.3*		
Trochanteric to biacromial	*67.2*		
Bimammillary to biacromial	*55.5*		
Spinal to biacromial	*72.9*		
Thoracic breadth to biacromial	*68.1*		
Spinal to cristal	*94.1*		
Thoracic breadth to cristal	*87.8*		
Thoracic breadth to spinal	*93.5*		

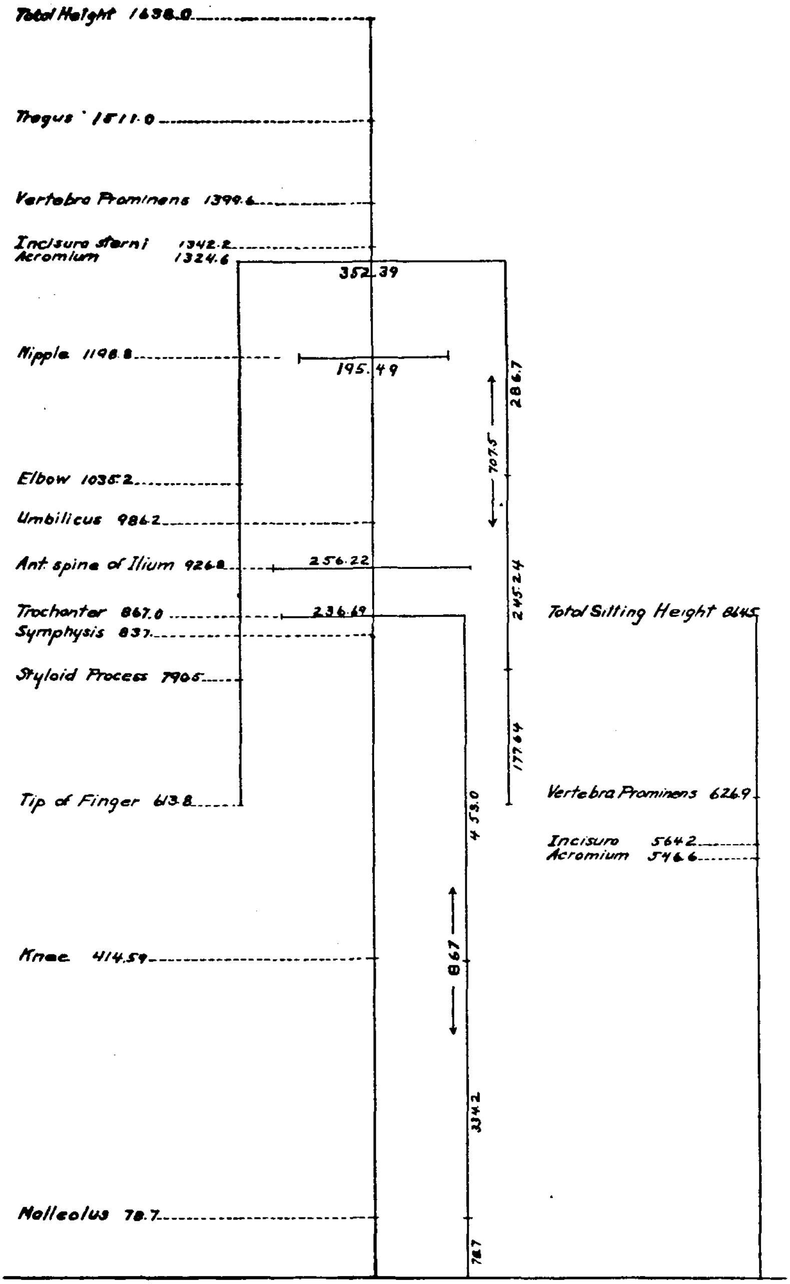
Total Height 1638.0
Tragus 1511.0
Vertebra Prominens 1399.6
Incisura sterni 1342.2
Acromium 1324.6
352.39
Nipple 1198.8
195.49
286.7
707.5
Elbow 1035.2
Umbilicus 986.2
Ant. spine of Ilium 926.8
256.22
245.24
Trochanter 867.0
Symphysis 837
236.69
Total Sitting Height 864.5
Styloid Process 790.5
177.64
Tip of Finger 613.8
453.0
Vertebra Prominens 626.9
Incisura 564.2
Acromium 546.6
Knee 414.59
867
334.2
Malleolus 78.7
78.7

FIGURE 41.

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Vol. 59. No. 17.—January, 1925.

RECORDS OF MEETINGS, 1923–24.

BIOGRAPHICAL NOTICES.

OFFICERS AND COMMITTEES FOR 1924–25.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

One thousand one hundred and twenty-fifth Meeting.

OCTOBER 10, 1923.— STATED MEETING.

The Academy met at its House at 8 P.M.

The PRESIDENT in the Chair.

There were forty-three Fellows and nine guests present.

In the absence of the Recording Secretary, the Corresponding Secretary was requested to assume his duties.

The Records of the Annual Meeting, of May 9, were read and approved.

The following letters were presented by the Corresponding Secretary: — from J. C. Arthur, C. L. Becker, Manton Copeland, L. C. Cornish, T. L. Davis, William Hovgaard, J. R. Jack, W. H. Lawrence, R. I. Lee, D. A. MacInnes, S. A. Mitchell, M. I. Rostovtzeff, E. C. Stakman, S. W. Stratton, Oswald Veblen, E. P. Warner, and Hans Zinsser, accepting Fellowship; and from Henri Guy, accepting Foreign Honorary Membership.

The Corresponding Secretary announced that an invitation for representation at the Joseph Leidy Centenary had been referred to the President with power.

The President announced the death of seven Fellows: — Louis Bell (Class I, Section 2); Louis Derr (Class I, Section 2); Rudolph Hering (Class I, Section 4); Raphael Pumpelly (Class II, Section 1); Stephen Paschall Sharples (Class I, Section 3); William Roscoe Thayer (Class III, Section 3); Arthur Gordon Webster (Class I, Section 2); and of one Foreign Honorary Member, John Morley, Viscount Morley of Blackburn (Class III, Section 3).

The following amendment to the Statutes, recommended by a special committee to which it had been referred, was adopted by ballot, twenty-eight votes being cast in the affirmative, and one in the negative:

Addition to Chapter II, Article 5 (at the end of the second paragraph): "Any Fellow shall also be exempt from annual dues who has paid such dues for forty years, or, having attained the age of seventy-five, has paid dues for twenty-five years."

New members present were introduced by Vice-President Kennelly of Class I, and by Professor Parker of Class II.

The following communication was presented:

Mr. William S. Franklin: "Determinism in Biology and Indeterminism in Physics."

Seven papers were presented by title:

"A Revision of the Atomic Weight of Boron. The Analysis of Boron Trichloride and Boron Tribromide," by Gregory P. Baxter and Arthur F. Scott, presented by G. P. Baxter.

"The Compressibility of Five Gases to High Pressures," by P. W. Bridgman.

"The Compressibility and Pressure Coefficient of Resistance of Rhodium and Iridium," by P. W. Bridgman.

"The Effect of Tension on the Thermal and Electrical Conductivity of Metals," by P. W. Bridgman.

"The Thermal Conductivity of Liquids under Pressure," by P. W. Bridgman.

"The Joule-Thomson Effect in Air: First Paper," by John R. Roebuck, presented by P. W. Bridgman.

"The Occurrence of Cordilleran and other Western Plants in the Unglaciated Areas of Northeastern America," contribution from the Gray Herbarium of Harvard University, presented by M. L. Fernald.

The Meeting was then dissolved.

One thousand one hundred and twenty-sixth Meeting.

NOVEMBER 14, 1923.— STATED MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were thirty-eight Fellows and one guest present.

The Records of the Meeting of October 10 were read and approved.

The Corresponding Secretary announced the receipt of a letter of acceptance of Foreign Honorary Membership from George Albert Boulenger; and of biographical notices of Edward Emerson Barnard, by E. B. Frost, and of George Lincoln Goodale, by Oakes Ames.

The Corresponding Secretary reported that the Council had appointed Edwin B. Wilson to be a member of the Committee of Publication, to fill the unexpired term of the late Louis Derr; and William M. Wheeler to represent the Academy in the Division of Foreign Relations of the National Research Council in place of the late A. G. Webster.

He also announced as the Committee on Methods of Electing Associates, Messrs. E. B. Wilson, W. C. Wait and Thomas Barbour.

The Corresponding Secretary reported that a meeting in memory of Professor Webster was to be held in Worcester on December 7, to which the members of the Academy were invited.

The President announced the death of two Fellows: — Samuel Walker McCall (Class III, Section 1); and Charles Proteus Steinmetz (Class I, Section 4).

The following communication was presented:

Mr. Samuel R. Detwiler (introduced by Mr. George H. Parker): "The Transplantation of Limbs in Relation to the Development of the Nervous System," with lantern illustrations.

Two papers were presented by title:

"Marine Borers in Bermuda," by E. L. Mark.

"New Species of Shipworms in Bermuda," by William F. Clapp, presented by E. L. Mark.

The Meeting was dissolved at 9.30 P.M.

One thousand one hundred and twenty-seventh Meeting.

DECEMBER 12, 1923.— STATED MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were forty Fellows and two guests present.

The Records of the Meeting of November 14 were read and approved.

The Corresponding Secretary announced the receipt of biographical notices of Samuel Leonard Abbot, by A. C. Lane; James Mills Peirce, by W. E. Byerly; Charles Proteus Steinmetz, by A. E. Kennelly; and Robert Wheeler Willson, by H. T. Stetson.

On the recommendation of the Council, the following appropriations were voted:

$200 toward the Annual Tables of Constants and Numerical Data, published under the auspices of the National Research Council.[1]

$56.23 for expense in connection with the restoration of the Rumford Monument at Auteuil.[1]

$500 for the expense of certain repairs to the House.

$54.40 for the binding of publications of the Academy, to be sent to the University of Tokyo.

The President read a minute on the life and services of Professor Arthur Gordon Webster, as adopted by the Council.

The President announced the death of Charles Francis Jenney, Fellow in Class III, Section 1.

The following communications were presented:

Mr. Richard P. Strong: "A Flagellate Infecting Plants and its Animal Hosts," with lantern illustrations.

Mr. Emory L. Chaffee: "The Electrical Response of the Retina to Stimulation by Light," with lantern illustrations.

The Meeting was dissolved at 9.45 P.M.

One thousand one hundred and twenty-eighth Meeting.

January 9, 1924.— Stated Meeting.

The Academy met at its House at 8.10 P.M.

The President in the Chair.

There were forty-one Fellows and six guests present.

The Records of the Meeting of December 12 were read and approved.

The President announced the death of James Madison Morton, Fellow in Class III, Section 1.

The President announced that Volume 27 of the Proceedings of the Academy was desired in order to complete the set of publications to be presented to the University of Tokyo.

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[1] This charge was later assumed by the Rumford Committee.

The following communications were presented:

Mr. Allyn A. Young: "The Reparations Problem."

Mr. Charles Macfie Campbell: "Some Forms of Psychotherapy."

The Meeting was dissolved at 10.10 P.M.

One thousand one hundred and twenty-ninth Meeting.

FEBRUARY 13, 1924.— STATED MEETING.

The Academy met at its House at 8.05 P.M.

The PRESIDENT in the Chair.

There were forty-five Fellows and three guests present.

The Records of the Meeting of January 9 were read and approved.

The Corresponding Secretary announced the receipt of biographical notices of William Brewster, by H. V. Neal; and Silas Weir Mitchell, by W. W. Keen.

The Corresponding Secretary also announced that an invitation had been received from Presidents Angell of Yale, Butler of Columbia, and Hibben of Princeton, to join in the preparation of a volume of congratulatory letters to Ex-President Eliot on his ninetieth birthday, and that this invitation had been referred by the Council to the President with power.

The President announced the death of four Fellows: — Basil Lanneau Gildersleeve (Class III, Section 2); Joseph Lipka (Class I, Section 1); Jacques Loeb (Class II, Section 3); Woodrow Wilson (Class III, Section 1).

The President announced the appointment of Professor Norton A. Kent as Acting Corresponding Secretary for the remainder of the year.

The President presented an invitation from the University of Naples to send a representative to the celebration of the six hundredth anniversary of its founding, to be held on May 7.

It was

Voted, That the representative be chosen by the Chair; also that the President appoint a Fellow of the Academy to write an appropriate reply in Latin.

The following communications were presented:

Mr. Charles H. Haskins: "Some Phases of Mediæval Science," with lantern illustrations.

Mr. R. Clipston Sturgis: "Forty Years of Architecture," with lantern illustrations.

The Meeting was dissolved at 9.25 P.M.

One thousand one hundred and thirtieth Meeting.

MARCH 12, 1924.— STATED MEETING.

The Academy met at its House at 8.15 P.M.

The PRESIDENT in the Chair.

There were thirty-one Fellows present.

The Records of the Meeting of February 13 were read and approved.

The President announced the death of Charles Warren Clifford, Fellow in Class III, Section 1.

On recommendation of the Council, the following appropriations were made for the ensuing year:

From the income of the General Fund, $7,775, to be used as follows:

for General and Meeting expenses	$600.00
for Library expenses	1,800.00
for Books, Periodicals and Binding	1,200.00
for House expenses	2,800.00
for Assistant Librarian Emerita	500.00
for Treasurer's expenses	875.00

From the income of the Publication Fund, $3,416.20, to be used for publication.

From the income of the Rumford Fund, $3,333.55, to be used as follows:

for Research	$1,000.00
for Books, Periodicals and Binding	200.00
for Publication	600.00
for use at the discretion of the Committee	1,533.55

From the income of the C. M. Warren Fund, $1,070.41, to be used at the discretion of the Committee.

The President appointed the Nominating Committee as follows:

Forris J. Moore, of Class I.

Reid Hunt, of Class II.

Ephraim Emerton, of Class III.

The following communications were presented:

Mr. Frederick A. Saunders: "Recent Advances in Spectroscopy."

Mr. Charles A. Coolidge: "Some Remarks on Chinese Architecture, and a Trip to the Ming Tombs," with lantern illustrations.

Two papers were presented by title:

"A Revision of the Atomic Weight of Germanium. I. The Analysis of Germanium Tetrachloride," by Gregory Paul Baxter and William Charles Cooper, Jr.

"Summary of a Study of Stellar Distribution," by Harlow Shapley and Annie J. Cannon.

The Meeting was dissolved at ten o'clock.

One thousand one hundred and thirty-first Meeting.

APRIL 9, 1924.— STATED MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were forty-three Fellows and one guest present.

The Records of the Meeting of March 12 were read and approved.

The Acting Corresponding Secretary announced the receipt of biographical notices of Louis Bell, by A. E. Kennelly; and of Hermann, Graf zu Solms-Laubach, by B. L. Robinson.

On the recommendation of the Council, it was

Voted, That power be granted to the Library Committee to give certain medical pamphlets to the Boston Medical Library, and to dispose, by sale or gift, of certain duplicate books and broken sets of periodicals.

On the recommendation of the Council, it was

Voted, That the income of $5,000, the amount of the Mills bequest, be appropriated for the use of the Committee of Publication.

The President announced the death of Thomas Corwin Mendenhall, Fellow in Class I, Section 2.

The following communications were presented:

Mr. George H. Parker: "The Carbon Dioxide Produced by Nerve."

Mr. W. J. V. Osterhout: "The Selective Action of Living Cells," with lantern illustrations.

Two papers were presented by title:

"Observations on Animal Life in the Thermal Waters of Yellowstone Park, with a Consideration of the Thermal Environment," by Charles T. Brues.

"Some Properties of Third-Terminal Electric Conducting Networks," by Arthur E. Kennelly.

At 9.50 the Meeting was dissolved.

One thousand one hundred and thirty-second Meeting.

MAY 14, 1924.— ANNUAL MEETING.

The Academy met at its House at 8.10 P.M.

The PRESIDENT in the Chair.

There were thirty-nine Fellows and two guests present.

The Records of the Meeting of April 9 were read and approved.

The President announced the death of two Fellows: — Ernest Fox Nichols (Class I, Section 2), and Theophil Mitchell Prudden (Class II, Section 4); and of one Foreign Honorary Member, Johannes Eugen Bülow Warming (Class II, Section 2).

The following report of the Council was presented:

Since the last report of the Council, there have been reported the deaths of nineteen Fellows:— Louis Bell, Charles Warren Clifford, Louis Derr, Basil Lanneau Gildersleeve, Rudolph Hering, Charles Francis Jenney, Joseph Lipka, Jacques Loeb, Samuel Walker McCall, Thomas Corwin Mendenhall, James Madison Morton, Ernest Fox Nichols, Theophil Mitchell Prudden, Raphael Pumpelly, Stephen Paschall Sharples, Charles Proteus Steinmetz, William Roscoe Thayer, Arthur Gordon Webster, Woodrow Wilson; and two Foreign Honorary Members: — John Morley, Viscount Morley of Blackburn, and Johannes Eugen Bülow Warming.

Seventeen Fellows and two Foreign Honorary Members were elected by the Council and announced to the Academy in May 1923.

The roll now includes 570 Fellows and 64 Foreign Honorary Members (not including those elected in May 1924).

The annual report of the Treasurer, Harold Murdock, was read, of which the following is an abstract:

General Fund.

Receipts.

Income on hand April 1, 1923		$3,718.12
From Investments	$4,961.31	
" Assessments	3,240.00	
" Admissions	90.00	
" Sundries	195.95	8,487.26
		$12,205.38

Expenditures.

Assistant Librarian Emerita	$500.00	
Expenses of Library	2,338.10	
Assistant Treasurer	250.00	
Treasurer's Expenses	965.24	
Books and Binding	1,171.09	
General Expenses	871.08	
House Expenses	2,868.50	
President's Expenses	40.00	$9,004.01

Interest on Bonds, bought	$44.25	
Adjustment of dividend on 10 shrs. A. T. & T. Co. rec'd in conversion	10.50	
Income transferred to principal	374.67	429.42
		$9,433.43
Balance, April 1, 1924		2,771.95
		$12,205.38

Rumford Fund.

Receipts.

Income on hand April 1, 1923	$6,453.72	
From Investments	3,844.73	$10,298.45

Expenditures.

Research	$1,200.00	
Purchase and Binding of Books and Periodicals	214.98	
Publications	1,914.97	
Tables of Constants	200.00	
Account of Restoration of Rumford Monument	56.23	$3,586.18
Income transferred to principal		191.49
		$3,777.67
Balance, April 1, 1924		6,520.78
		$10,298.45

C. M. Warren Fund.

Receipts.

Income on hand April 1, 1923	$1,982.20	
From Investments	1,142.08	$3,124.28

Expenditures.

Research	$1,000.00	
Vault Rent — part	3.00	$1,003.00
Income transferred to principal		1,056.45
		$2,059.45
Balance, April 1, 1924		1,064.83
		$3,124.28

PUBLICATION FUND.

Receipts.

Income on hand April 1, 1923		$6,383.95
From Income Appleton Fund	$1,261.75	
" Income Centennial Fund	2,499.03	
" Authors' Reprints	381.22	
" Sale of Publications	606.24	4,748.24
		$11,132.19

Expenditures.

Publications	$6,084.31	
Vault Rent — part	10.00	$6,094.31
Adjustment of dividend on 7 shrs. A. T. & T. Co. rec'd in conversion	$7.35	
Income transferred to principal	177.30	184.65
		$6,278.96
Balance, April 1, 1924		4,853.23
		$11,132.19

FRANCIS AMORY FUND.

Receipts.

From Investments	$1,720.00

Expenditures.

Publishing Statements	$75.15	
Interest on Bonds bought	19.75	$94.90
Income transferred to principal		1,625.10
		$1,720.00

The following reports were also presented:

REPORT OF THE LIBRARY COMMITTEE.

The Librarian begs to report for the year 1923–24, as follows:

During the year, 70 volumes and 21 unbound numbers of periodicals have been borrowed by 15 Fellows and 5 libraries, the Library of the Massachusetts Institute of Technology alone having borrowed 41 volumes and numbers. Many books have been consulted and used at the Academy. All books taken out have been returned or satisfactorily accounted for.

The number of books on the shelves at the time of the last report was 39,083. During the year, 542 volumes have been added, making the number now on the shelves 39,625. This includes 68 purchased from the General Fund, 21 from the Rumford Fund, and 444 received by gift or exchange. The number of pamphlets added was 75.

The expenses charged to the Library during the financial year ending April 1, 1924, are:

Salaries	$2,196.00
Binding:	
General Fund	589.65
Rumford Fund	33.40
Purchase of periodicals and books:	
General Fund	583.09
Rumford Fund	181.58
Miscellaneous	142.10

In the death of Dr. Arthur Gordon Webster on May 15, 1923, the Academy lost a Librarian who had devoted himself conscientiously and effectively to the interests of the Academy Library for a period of eight years. An appreciation of his long and faithful service in this office should be recorded in this report.

Since Mrs. Smith gave up her position in July 1923, the Assistant Librarian has carried on the work without an assistant, except during December, January and February, when a special assistant was employed on part time to get into shape a list of the duplicate books, society publications and periodicals in the Library, in the expectation of disposing of them shortly by sale or exchange.

The cataloguing of the pamphlet collection was contemplated, but the lack of funds has necessitated postponing this work to some future date. The collection is now arranged alphabetically by authors in metal filing cases. By vote of the Academy on April 9, 1924, the medical pamphlets of this collection were offered to the Boston Medical Library, which has accepted the gift with grateful appreciation.

Respectfully submitted,

H. M. Goodwin, *Librarian.*

May 14, 1924.

Report of the Rumford Committee.

The Committee organized on October 10, 1923, by electing Theodore Lyman as Chairman and P. W. Bridgman as Secretary.

The following grants in aid of researches in Light and Heat have been made during the year 1923–24:

October 10, 1923. To Professor Harlow Shapley of Harvard University for the purchase of a thermo-electric microphotometer	$500
October 10, 1923. To Professor R. W. Wood of Johns Hopkins Univeristy for the purchase of a Lummer Plate for the study of Magneto-Optic Polarization Phenomena	250
February 13, 1924. To Professor R. W. Wood of Johns Hopkins University in continuation of the grant of October 10	200
February 13, 1924. To Professor Norton A. Kent of Boston University for the continuation of his research on the Constitution of Spectral Lines	250

March 12, 1924. To Professor P. W. Bridgman to aid in the continuation of his researches on the Thermo-Dynamic Properties of Matter $500

The following papers in the Proceedings have been published with aid from the Rumford Committee since the presentation of the last Report of this Committee:

The Effect of Pressure upon Optical Absorption, by Frances G. Wick, Vol. 58, No. 16.

The Effect of Tension on the Thermal and Electrical Conductivity of Metals, by P. W. Bridgman, Vol. 59, No. 6.

The Thermal Conductivity of Liquids under Pressure, by P. W. Bridgman, Vol. 59, No. 7.

The Compressibility of Five Gases to High Pressures, by P. W. Bridgman, Vol. 59, No. 8.

The Committee has authorized the publication of a paper by Professor John R. Roebuck on the Joule-Thomson Effect in Air.

The sum of $200 has also been appropriated to assist in the publication of a paper by Dr. Harlow Shapley and Miss Cannon on Stellar Distribution.

The Chairman has approved the expenditure of $112.45 to assist in the restoration of the Tablet to Count Rumford at Auteuil, France. He has also approved the payment of $200 as the annual subscription towards the Tables of Physical Constants of the National Research Council.

This year no request for reports of progress has been sent out to those working under grants from the Committee.

For many years Professor Arthur Gordon Webster and Dr. Louis Bell served as members of this Committee, Professor Webster acting as Secretary. The members of the committee wish to record their appreciation of the services of these gentlemen and to express their deep regret at their loss.

THEODORE LYMAN, *Chairman.*

May 14, 1924.

REPORT OF THE C. M. WARREN COMMITTEE.

The Committee had at its disposal at the beginning of the fiscal year 1923–24, $3,190.47. During the year ending March 31, 1924, grants to the amount of $1,000.00 were made, $1,000.00 was transferred to the principal of the Fund and $136.27 in premiums on bonds was charged to the income. The balance on March 31, 1924, was, accordingly, $1,054.47. The appropriation of the Academy for the coming year was $1,070.41. The amount at the disposal of the Committee for the year 1924–25 is $2,124.61.

Since the last annual report awards have been made as follows:

April 9, 1923. To Prof. David E. Worrall, Tufts College, $250 to purchase apparatus and chemicals in connection with his work on the chemistry of mustard oils.

April 9, 1923. To Prof. R. R. Renshaw, New York University, $200 for special apparatus needed in the study of the causes of the physiological activity of choline and its derivatives. He was also loaned the potentiometer returned by Prof. Willard, of the University of Michigan, at the completion of his researches.

November 7, 1923. To Prof. James B. Conant, Harvard University, $150 to aid in his researches on Oxidation and Reduction.

November 7, 1923. To Prof. Harold Hibbert, Yale University, $300 to aid in his researches on the Constitution and Properties of Polysaccharides.

December 5, 1923. To Prof. Wilder D. Bancroft, Cornell University, $100 to assist in his work on Contact Catalysis.

Reports of progress have been received from the following: Profs. Worrall, Renshaw, Kraus and Fay.

During the past year researches carried out with the aid of grants from this Committee have been published by the following:

Dr. R. L. Datta on "The Temperature of Explosion for Endothermic Substances. II. Trinitro-Meta-Cresolates and Their Explosion Temperatures," Jr. Am. Chem. Soc., Oct. 1923.

Prof. Charles A. Kraus on "Reactions of Strongly Electro-Positive Metals with Organic Substances in Liquid Ammonia Solution. I. Preliminary Investigations," Jr. Am Chem. Soc., March 1923.

Prof. Henry Fay on "The Determination of Nitrogen in Steel," Chem. and Met. Eng., Feb. 1, 1922.

Prof. James B. Conant on "Electrochemical Study of Reversible Reduction of Organic Compounds," Jr. Am. Chem. Soc., June 1922; "Free and Total Energy Changes in the Reduction of Quinones," Jr. Am. Chem. Soc., Nov. 1922; "Addition Reactions of Phosphorus Halides. VI. The 1,2 and 1,4 Addition of Diphenyl-Chlorophosphine," Jr. Am. Chem. Soc., Jan. 1923; "An Electrochemical Method of Studying Irreversible Organic Reductions," Jr. Am. Chem. Soc., April 1923; and "Reduction Potentials of Quinones. I. The Effect of the Solvent on the Potentials of Certain Benzoquinones," Jr. Am Chem. Soc., Sept. 1923.

The Chairman of the Committee has been collecting reprints of papers published in the past by men who have received grants. Up to the present reprints of 32 articles have been received.

James F. Norris, *Chairman.*

May 14, 1924.

Report of the Committee of Publication.

The Committee of Publication reports as follows for the year April 1, 1923, to March 31, 1924:

The Committee and the Academy lost the valuable services of Mr. Derr, representing Class I, and holding the chairmanship of the Committee, through his death about a year ago. This event caused some very slight disturbance in the conduct of the work of your Committee and would have caused much more had it not been for the readiness with which Mr. Howard, representative of Class III, took up the work and carried it on until December, when Mr. Wilson consented to serve temporarily as representative of Class I, to conduct the routine editorial work for the Committee.

During the past twelve months since the presentation of the last report, there have been printed No. 4, of Vol. 14 of the Memoirs; Nos. 9–17 inclusive of Vol. 58 of the Proceedings, and Nos. 1–9 of Vol. 59.

The financial report is as follows:

Receipts.

Balance, April 1, 1923	$6,146.41	
Appropriation for 1923–24	3,368.32	
Sale of Publications	606.24	
Received for authors' reprints	381.22	**$10,502.19**

Expenses.

Engraving, printing and binding	$5,836.09	
Cartage and mailing	242.80	
Committee's expenses	5.42	$6,084.31
Balance on hand, March 31, 1924		$4,417.88

From this report it appears that the balance has been reduced by $1,728.53 during the year, despite the payments on the part of the Rumford Committee for the publication of articles on light and heat of $1,914.97. This means that the publications of the Academy have cost during the fiscal year just closed about $3,850.00 more than the income of the Publication Fund, including income from sale of publications, extra author's reprints, etc. Even with a continuance of contributions by the Rumford Committee toward the expenses of publication of articles in the field of that committee, it is safe to say that there will come a time within the next three or four years when the publications of the Academy must be considerably reduced in volume unless there is some way to reduce the cost per page now paid for printing.

Your Committee would point out that the Proceedings and Memoirs of the American Academy have long been very serviceable as a medium of publication for scientific and other material of the highest grade and that there is at present upon the Proceedings and Memoirs a large pressure of material that should be published. We have, for example, in press seven manuscripts for the Proceedings and two manuscripts for the Memoirs, material sufficient to complete Volume 14 in the Memoirs and Volume 59 of the Proceedings. Your Committee will direct every effort toward the reduction of costs of the publications but believes that it is none too soon for the Academy seriously to consider where it may look for some additional endowment for publication, because it is too much to hope that costs can be reduced to anything like the pre-war figure and it would be unfortunate, both for the Academy and for its members, if any steps shall have to be taken to reduce the space available for publishing material of the caliber that has distinguished these publications for many years.

Respectfully submitted,

E. B. Wilson, *Chairman.*

May 14, 1924.

REPORT OF THE HOUSE COMMITTEE.

The House Committee submits the following report for 1923–1924:

In July, previous to starting for Europe, Professor John O. Sumner resigned his chairmanship of the House Committee, and the duties of that office were assumed for the remainder of the Academy year by the present Chairman.

With the balance of $5.74 left from last year, an appropriation of $3,000, and $215 received from other societies for the use of the rooms, the Committee has had at its disposal the sum of $3,220.74. The total expenditure has been $3,083.50, leaving an unexpended balance on April 1, 1924, of $137.24. The expenditure has been as follows:

Janitor		$940.00
Electricity	Light	197.60
	Power	71.82
Coal		909.66
Care of elevator		106.17
Gas		60.40
Water		16.94
Telephone		71.19
Ash tickets		27.75
Upkeep		638.09
Janitor's materials and supplies		43.88
Total expenditure		$3,083.50

The amount of $215 contributed by other societies for the use of the building leaves the net expense of the House $2,868.50.

An unusual amount had to be expended this year on the upkeep of the building owing to necessary repairs to the roof, and the renewal of the copper capping on the side wall.

The main staircase well, the outside wall of the book stack, and the fire escape were painted during the summer, and it is planned to have the Lecture Hall painted this summer. Unfortunately, the income of the Academy does not warrant doing all the painting needed to put the building in perfect condition, but as much as possible should be done each year.

Meetings have been held as follows:

The Academy	
Stated meetings	8
Archæological Institute of America	2
Colonial Society	4
Geological Club of Boston	4
Harvard-Technology Chemical Club	9
New England Botanical Club	9
	36

Since July 1923, the Academy has allowed the newly organized Society of Friends of Medical Progress the free use of the Committee Room on the first floor as an office.

The Council Chamber has been used for Academy Council and Committee Meetings, and also by the Colonial Society, the Trustees of the Children's Museum, and the Thursday Evening Club.

Respectfully submitted,

William H. Lawrence, *Chairman.*

May 14, 1924.

On the recommendation of the Treasurer, it was

Voted, That the Annual Assessment be $10.00.

The annual election resulted in the choice of the following officers and committees:

Theodore Lyman, *President.*
Arthur E. Kennelly, *Vice-President for Class I.*
William M. Wheeler, *Vice-President for Class II.*
Arthur P. Rugg, *Vice-President for Class III.*
Norton A. Kent, *Corresponding Secretary.*
Charles B. Gulick, *Recording Secretary.*
Harold Murdock, *Treasurer.*
Harry M. Goodwin, *Librarian.*

Councillors for Four Years.

ARTHUR B. LAMB, *of Class I.*
FRANCIS W. PEABODY, *of Class II.*
WILLIAM B. MUNRO, *of Class III.*

Finance Committee.

ARTHUR LORD, THOMAS BARBOUR, PAUL J. SACHS.

Rumford Committee.

THEODORE LYMAN, HARRY M. GOODWIN,
ELIHU THOMSON, CHARLES L. NORTON,
PERCY W. BRIDGMAN, HARLOW SHAPLEY,
ARTHUR E. KENNELLY.

C. M. Warren Committee.

JAMES F. NORRIS, WALTER L. JENNINGS,
HENRY P. TALBOT, ARTHUR D. LITTLE,
GREGORY P. BAXTER, LAWRENCE J. HENDERSON,
FREDERICK G. KEYES.

Publication Committee.

EDWIN B. WILSON, *of Class I.*
HERBERT V. NEAL, *of Class II.*
ALBERT A. HOWARD, *of Class III.*

Library Committee.

EDWIN B. WILSON, *of Class I.*
THOMAS BARBOUR, *of Class II.*
WILLIAM C. LANE, *of Class III.*

House Committee.

WILLIAM H. LAWRENCE,
ROBERT P. BIGELOW, S. BURT WOLBACH.

Committee on Meetings.

THE PRESIDENT,
THE RECORDING SECRETARY,
GEORGE H. PARKER,
GREGORY P. BAXTER,
EDWARD K. RAND.

Auditing Committee.

GEORGE R. AGASSIZ,
JOHN E. THAYER.

The Council reported that the following gentlemen were elected members of the Academy:

Class I, Section 1 (Mathematics and Astronomy):
  William Caspar Graustein, of Cambridge, as Fellow.
  John Anthony Miller, of Swarthmore, Pa., as Fellow.
Class I, Section 2 (Physics):
  Albert Einstein, of Berlin, as Foreign Honorary Member.
  Paul Alphonse Heymans, of Brookline, as Fellow.
Class I, Section 3 (Chemistry):
  James Bryant Conant, of Cambridge, as Fellow.
Class I, Section 4 (Technology and Engineering):
  Charles Leavitt Edgar, of Brookline, as Fellow.
Class II, Section 3 (Zoölogy and Physiology):
  Edwin Garrigues Boring, of Cambridge, as Fellow.
  Edward Allen Boyden, of Newton Centre, as Fellow.
  Raymond Dodge, of Middletown, Conn., as Fellow.
  Samuel Jackson Holmes, of Berkeley, Cal., as Fellow.
Class II, Section 4 (Medicine and Surgery):
  Walter Elmore Fernald, of Waverley, as Fellow.
  Edward Wyllys Taylor, of Boston, as Fellow.
Class III, Section 1 (Philosophy and Jurisprudence):
  Charles Ambrose DeCourcy, of Boston, as Fellow.
Class III, Section 2 (Philology and Archæology):
  Ingersoll Bowditch, of Boston, as Fellow.
Class III, Section 3 (Political Economy and History):
  Claude Halstead Van Tyne, of Ann Arbor, Mich., as Fellow.

The following communications were presented:

Mr. Samuel J. Barnett: "New Researches on the Magnetization of Ferromagnetic Substances by Rotation and the Nature of the Elementary Magnet" (in collaboration with Mrs. L. J. H. Barnett), with lantern illustrations.

Mr. George L. Kittredge: "The Ballad of Lovewell's Fight."

The following papers were presented by title:

"The Bodily Proportions of Women in the United States; Based upon Measurements Taken from One Hundred Smith College Students," by Harris Hawthorne Wilder and Margaret Washington Pfeiffer.

"Granites and Pegmatites of Cape Ann, Massachusetts," by Charles H. Warren and Hugh E. McKinstry.

On motion of Professor Kennelly, the Academy signified its deep appreciation of the services of the retiring President by a rising vote.

At 10.05 the Meeting was dissolved.

BIOGRAPHICAL NOTICES.

		Page
SAMUEL LEONARD ABBOT	Alfred C. Lane	630
EDWARD EMERSON BARNARD	Edwin B. Frost	630
LOUIS BELL	Arthur E. Kennelly	633
WILLIAM BREWSTER	Herbert V. Neal	639
GEORGE LINCOLN GOODALE	Oakes Ames	640
SILAS WEIR MITCHELL	William W. Keen	644
JAMES MILLS PEIRCE	William E. Byerly	650
COUNT SOLMS	Benjamin L. Robinson	651
CHARLES PROTEUS STEINMETZ	Arthur E. Kennelly	657

SAMUEL LEONARD ABBOT (1818–1904).

Fellow in Class II, Section 4, 1850.

Samuel Leonard Abbot, Harvard A.B. 1838, M.D. 1841, died July 1, 1904, at the age of eighty-six years.

He was the last of that noteworthy group of practitioners connected with the Massachusetts General Hospital in the fore part of the nineteenth century, the first physician to out-patients, and upon the visiting staff until disqualified by age. These duties of his public professional life he performed with uniform and conscientious devotion.

He was a zealous student of ornithology, long officially connected with the Boston Society of Natural History. For many years he was associate editor of the *Boston Medical and Surgical Journal*, and at his death was the oldest living member of the Thursday Evening Club, He was long one of the administrators of the First Church of Boston.

A notice of his decease will be found in the *Boston Medical and Surgical Journal*, vol. 151, p. 55, 1904, but he had so outlived his contemporaries that at the time of his death the Academy failed to prepare a memoir.

A. C. Lane.

EDWARD EMERSON BARNARD (1857–1923).

Fellow in Class I, Section 1, 1892.

Edward Emerson Barnard was born at Nashville, Tennessee, on December 16, 1857. His father had died shortly before his birth, and his mother was in straightened circumstances, so that his early education was very meager. At the age of nine he began to support himself by acting as apprentice in a photographic establishment at Nashville, which service laid the foundation for his remarkably successful pioneer work in later years as a photographer of the heavens.

His studies in astronomy began by the chance acquisition of a lens which could serve as the foundation of a telescope for observing the moon or the stars. When he was nineteen he bought, from his meager savings, a five-inch telescope.

In May 1881, he discovered an unknown comet, but was unable to find it after the second day, so that it did not become an object of definite record. Later in the year he bagan a systematic search for comets, and was very successful in this. At Nashville he discovered ten comets.

He had a remarkably keen eye and an extraordinary memory for the objects in the sky which he found on his sweeps for comets. His earliest work showed accuracy in observation as well as in description. By studying on cloudy evenings, assisted occasionally by a tutor, he was able to fit himself to enter Vanderbilt University as a special student in 1883. He was assistant in astronomy for much of his course, which he completed in 1887.

When the Lick Observatory was organized in 1888, he was called to its staff by Director Holden, and was assigned work with the twelve-inch telescope and the comet-seeker. He discovered seven comets while at the Lick Observatory.

In 1889 he began his important work of photographing the Milky Way, using a portrait lens of the doublet type which had been employed for many years in photographers' studios. His photographs were a surprise and a delight to the scientific world.

On the first of July 1892, he was assigned one night a week with the thirty-six-inch refractor, and on September 9 he made his remarkable discovery of the fifth satellite of Jupiter, an object difficult of observation, not because of its faintness, but because of its closeness to the brilliant planet. With the large refractor, Barnard continued to observe with extraordinary skill and devotion, making a redetermination of the diameters of planets and satellites of the solar system an important part of his program.

In 1895 he was appointed professor of practical astronomy in the University of Chicago and astronomer at the Yerkes Observatory. There was delay in the completion of the Observatory, so that he did not resume micrometric work until the forty-inch telescope was ready for use, in the autumn of 1897.

Meanwhile, he was endeavoring to secure the finest possible photographic objective for further studies of the Milky Way and comets. At his solicitation, Miss Catharine W. Bruce of New York gave $7,000 to the University of Chicago for such a telescope and its mounting. Different opticians were urged to produce lenses for this special purpose, to be tested as to their suitability by Barnard, who made a trip to Europe on this account. After some years, a ten-inch lens of thirty-one inches focus, made by Brashear, was purchased as the best available, and the mounting was provided by Warner and Swasey. This instrument, known as the Bruce telescope, was completed early in 1904, and for the next eighteen years was assiduously used by Professor Barnard whenever the sky permitted, and whenever he had no assignment with the forty-inch telescope. His Atlas of the Milky Way, which will reproduce fifty areas of the galaxy in the form of photographic prints, will soon be published by the Carnegie Institution of Washington. The photographic prints have been ready for some years, and the descriptive matter was nearly completed at the time of Professor Barnard's death.

As an observer, Barnard has been well likened to William Hershel: nothing in the sky escaped his interest, and no weariness of the flesh or rigor of a Wisconsin winter could induce him to shorten his hours of painstaking labor at the forty-inch refractor or the Bruce telescope. He observed with the micrometer all the current objects which were suitable for a large refractor, including planets, satellites, faint comets, and nebulae, and he made a specialty of the triangulation of the principal stars in the globular clusters. He also took numerous photographs of these interesting objects with the forty-inch telescope, for measurement in the laboratory. It was his expectation that observations continued over a considerable period of years would reveal motions among the members of these systems, but, as a matter of fact, practically none were found. These long series of measures are still unpublished.

Barnard was elected member of the American Academy in 1892. His published papers were very numerous, including some nine hundred items. Volume XI of the Publications of the Lick Observatory was devoted to reproductions of his photographs of the Milky Way and comets. His principal papers appeared in the Astrophysical Journal, the Astronomical Journal, Astronomische Nachrichten, Monthly Notices of the Royal Astronomical Society, and Popular Astronomy.

Mr. Barnard was married in 1881 to Miss Rhoda Calvert. Their home was always a center of hospitality to a large circle of astronomical friends. They never had children. Mrs. Barnard died in 1921.

For the last nine years of his life, Professor Barnard suffered from diabetes, but after overcoming an acute attack he was able to continue his work without much limitation, until his final illness, which began about Christmas 1922. He died on February 6, 1923.

EDWIN B. FROST.

LOUIS BELL (1864–1923).

Fellow in Class I, Section 2, 1905.

The late Dr. Louis Bell was so well known to many members of the Academy that a brief account of his personality, career and accomplishments is sure to be welcome.

The family name was originally Le Bel, and is traceable back to the Norman conquest of England. The family settled in Scotland not far from the English border. Dr. Joseph Bell, a professor of surgery in Edinburgh University, was one of the line in more recent years. Members of the family emigrating to America in the seventeenth and eighteenth centuries have furnished U. S. senators, congressional representatives, a Speaker of the House, a U. S. Chief Justice, a Secretary of War, a presidential candidate and a Governor of Texas. Their main stronghold was in New Hampshire, a state to which the family supplied three governors, one of whom, Samuel Dana Bell, was said to have "carried New Hampshire in his vest pocket" for a number of years, and who represented that state, for two terms, in the U. S. Senate at Washington, D. C.

Louis Bell was the grandson of this same Governor Samuel Bell of New Hampshire. His father was General Louis Bell, who fought in Grant's army during the Civil War. General Bell, when 28 years old, commanded the attack on Fort Fisher, and was killed in action during the last few minutes of the final successful rush on the trenches, just before the close of the war. The news of his death came as a terrible shock to his young wife, in the family home at Chester, N. H., and she survived him only six months, leaving little Louis, then one year old (born Dec. 5, 1864), and his sister four years older. The two children were brought up at Chester by their grandmother, Governor Samuel Bell's widow. In remote village life, these two children were thrown much on their own resources, and mingled but little with the outer world. Louis was gifted mentally, and was given his bent in study, with full encouragement by his fond grandparent. The boy was an omnivorous reader, even at the age of eight, and was consumed with a thirst for knowledge of all kinds, but particularly for scientific knowledge. I used to wonder, in later years, how Dr. Bell became so well versed in the Bible and Bible history, until I learned that he was frequently allowed to choose, on Sunday mornings, between going down the hill to church and staying home to learn a chapter of Scripture by heart. Little Louis, with his swift memory, always chose the latter alternative.

When Louis was twelve years old, it was decided that he should be taken from his native village and sent to school at the Phillips Exeter Academy, Exeter, N. H. Thereafter, he only returned to Chester for brief and occasional visits; but he was very deeply attached to his childhood's home and he always desired to be carried back there after his death. He was rather a lonely lad at school, brilliant but living apart. His letters to his sister show how close was their attachment, and how much he depended on her guidance. When she died untimely at twenty-one, he was distracted with grief. He would never trust himself to speak of her in after-life, even to his most intimate friends.

Louis entered Dartmouth, the college of his family, in 1880, and took his A. B. degree there in 1884. In his studies, he distinguished himself in chemistry and physics, and took a year of post-graduate college work in physics. The professor of astronomy there allowed Louis to live at the College observatory, and to use the telescopes for observations, during two summers. Bell always loved telescopes and admired those who knew how to use them.

Bell was always a most loyal and devoted alumnus of Dartmouth, giving, throughout his life, unstintedly of his time and efforts to her aims. From Dartmouth he went to Johns Hopkins University, where he studied under Rowland, and where he took his Ph.D. degree in 1888.

The subject of his thesis was "The Absolute Wave Length of Light." At the Manchester British Association Meeting in September 1888, Dr. Bell presented a joint paper by Prof. Henry A. Rowland and himself on "Explanation of the Action of a Magnet on Chemical Action."

After graduating at Johns Hopkins, Dr. Bell went to join, for a year, the faculty of Purdue University, as professor of applied electricity, or what is now generally known as electrical engineering. This was a new chair at that time in Purdue. He organized the electrical engineering instruction there on a sound basis and in 1890 he went to New York as editor of the Electrical World, which has since continued to be a prominent electrical engineering weekly journal. At that time it was owned by W. J. Johnston. It was during this period that I first became acquainted with Dr. Bell as the editor of the Electrical World. Ever after that interview, I was his admirer and friend. Bell was always a very clear and forceful writer. It was fortunate for the Electrical World that its editor, in Dr. Bell, was not only a prominent electrical engineer but also a good scholar in English. He left the chief editorship in 1892 to enter the service of the General Electric Co., but he maintained literary connection with the Electrical World, editorially and otherwise, up to his death. Anyone acquainted with the personal characteristics of his literary style, could readily discern articles by him. He had the gift of writing in such a manner as to compel attention and understanding. There was frequently a background of quaint humor in his articles that tempted the reader to follow, even when the subject was dry and difficult.

It was about 1892 that the alternating-current motor began to enter commercial development. Dr. Bell, as chief engineer of the then newly organized power-transmission department of the General Electric Co., designed and installed some of the first polyphase power-transmission plants in the country.

In 1892, Dr. Bell published (with Oscar T. Crosby) The Electric Railway, which was a pioneer book on the subject. In 1896, he himself wrote Power Transmission for Electric Railroads, and in 1897, Electric Power Transmission, both of which books were soon widely known.

In 1895, he entered the profession of consulting engineering in Boston. He continued in this work for the rest of his life. At first, his work dealt mainly with electric power transmission, but afterwards he took up electric lighting more particularly, and later on he specialized in electric illuminating engineering. He published, in 1902, the textbook The Art of Illumination, which became a classic in that field. He lectured frequently on illumination at Harvard University, the Massachusetts Institute of Technology and elsewhere. In 1908, he was the third President of the Illuminating Engineering Society, and was an active member of several of its important committees. He continued to work for the Society up to the last. He was also, for about ten years, a vice-president of the Illuminating Engineering Society of Great Britain. He was consulting illuminating engineer to the Edison Illuminating Co. of Boston for a number of years, and up to his death. He did much research work with automobile head lights in the early days of automobile long-range search lighting, and contributed materially to the development of automobile head-light lenses.

Although he was always engaged in engineering work, he was keenly interested in pure science, especially in optics and optical astronomy. His book The Telescope, published in 1922, is delightful to read, both from the practical and historical, as well as the purely optical view points. He was a member of the American Astronomical Society, and of the Board of Visitors of the Harvard Observatory.

He was elected a member of the American Academy of Arts and Sciences in 1905, was on its Rumford Medal Committee, and contributed various papers to its volumes, particularly on optics. A list is appended of his papers in the Proceedings of the Academy, and in those of the Illuminating Engineering Society, only. No attempt will be made here to give a list of his contributions to technical literature generally, because their name is legion. The fertility of his inventiveness is also shown by the fact that he took out more than forty patents, mainly in the applications of optics and illumination. He gave much time and attention to the preparation of rules for good lighting, serving on a State Commission in Massachusetts in the preparation of a Lighting Code, and also, for some ten years, on a committee of the Illuminating Engineering Society, on Lighting Legislation.

Two weeks after his death, the Council of the Illuminating Engineering Society, in recognition of his many valued services, adopted a fine resolution of esteem and tribute to his memory. This appears in the Society's Transactions for July 1923.

During the Spanish American War, Dr. Bell was Technical Officer of the Volunteer Electrical Corps, dealing with electrical harbor and Atlantic coast defenses from the Chesapeake Bay to the Canadian Border.

In the Great War, he was a member of the Advisory Committee on the Council of National Defense. He was specially occupied with telescopic sights for naval guns. He also developed a very practicable system of invisible signalling by ultra-violet light.

Shortly before his death he was busied, under the stimulus of the investigator's instinct, in making researches on the spectrum of the powerful Sperry arc-lamp projector.

Dr. Bell married Sarah G. Hemenway of Somerville, Mass. in 1893, who with his son Louis and two grandchildren survives him. Bell was a home-making and home-loving man, shining most brightly when acting as host. He was a witty and gracious conversationalist, loving, for its own sake, a humor which never degenerated into satire or sarcasm.

In his leisure hours, which were rather few and far between, Bell enjoyed rifle and revolver shooting. He was always a good marksman, and indeed maintained, in that way, a family tradition of several generations. The Bells were always good shots. He was a member, and had been the President, of the Massachusetts Rifle Association. He also held at one time (New York, 1892) the American amateur revolver-shooting championship. In 1903, he was one of a team of the American Revolver Association which won an international shooting match with a similar French team. He would often spend his Saturday afternoons on the rifle range at Walnut Hill, Mass., where he was very popular and had many friends. He looked upon the rifle rather as an instrument of scientific precision in target hitting, than as a deadly weapon. He was, on the other hand, however, an eager student of military history.

Some two years ago, his usually robust health broke down with an attack of pneumonia and after that it failed rapidly; but his courage was invincible. At the last, he had great hopes, although in reality quite illusory hopes, of recovery. He passed away, happily, during sleep, on June 14, 1923. His final resting place is among the tombs of his kindred, at the quiet village cemetery of Chester, in the farming country of New Hampshire, where many of the existing old homesteads were celebrated in colonial times.

Five days after his death, the posthumous honorary degree of Doctor of Science was awarded to him by his Alma Mater, Dartmouth College. Dr. E. M. Hopkins, the President, delivered on that occasion the following allocution, which Dr. Bell's numerous friends will heartily endorse:

"Louis Bell, teacher, writer, investigator; pioneer in the development of electrical transmission; authority in the fields of illumination and optics; fruitful in the work of making the accumulations of the laboratory of service to mankind; whose scientific achievements have been combined with wide interest in literature and art, and who, as a writer and lecturer, has brought literary finish and quiet humor to the exposition of sound common sense; loyal alumnus of the college, and ever interested in her welfare."

Publications by Dr. Louis Bell in the Proceedings of the American Academy of Arts and Sciences.

"The Physiological Basis of Illumination." Vol. 43, No. 4, 1907.

"Note on some Meteorological Uses of the Polariscope." Vol. 43, No. 15, 1907.

"On the Opacity of Certain Glasses for the Ultra-violet." Vol. 46, No. 24, 1910.

"On the Ultra-violet Component of Artificial Light." Vol. 48, No. 1, 1912.

"Types of Abnormal Color Vision." Vol. 50, No. 1, 1914.

"The Pathological Effects of Radiant Energy upon the Eye" (with F. H. Verhoeff and C. B. Walker). Vol. 51, No. 13, 1915.

"Ghosts and Oculars." Vol. 56, No. 2, 1920.

"Notes on the Early Evolution of the Reflector." Vol. 57, No. 4, 1921.

Publications by Dr. Louis Bell in the Transactions of the Illuminating Engineering Society.

"The Illumination of the Building of the Edison Electric Illuminating Company of Boston" (with L. B. Marks and W. D'A. Ryan). Vol. II, No. 7, 1907.

"Coefficients of Diffuse Reflections." Vol. II, No. 7, 1907.

"Response to the Address of Welcome to Convention." Vol. IV, No. 7, 1909.

"The Principles of Shades and Reflectors." Vol. IV, No. 8, 1909.

"Street Photometry." Vol. V, No. 5, 1910.

"Photometry at Low Intensities." Vol. VI, No. 7, 1911.

"Report of Committee on Progress" as Chairman of Committee. Vol. VI, No. 7, 1911.

"The Pathological Effects of Radiation on the Eye" (with F. H. Verhoeff). Vol. XVI, No. 9, 1921.

"Report of Sub-Committee on Glare" as Chairman. Vol. XVII, No. 10, 1922.

A. E. KENNELLY.

WILLIAM BREWSTER (1851–1919).

Fellow in Class II, Section 3, 1891.

William Brewster, a Fellow in the American Academy of Arts and Sciences, was born at Wakefield, Massachusetts, July 5, 1851, and died in Cambridge, Massachusetts, July 11, 1919. His father was John Brewster, a well-known Boston banker. His mother was Rebecca Parker (Noyes). William was the youngest of four children, his brothers and sisters all dying in early childhood. Educated in the public schools of Cambridge, he was forced by defective eyesight to abandon plans for a college course. While a young lad he learned to stuff birds and during his lifetime acquired one of the largest private collections of birds in the country, a collection which he donated to the Agassiz Museum of Harvard University. A year's experience in his father's bank satisfied him that he was not suited to a business career. As a result he devoted his life to scientific work, especially in the field of ornithology. Brewster married Caroline F. Kettell of Boston on February 9, 1878. His publications include over three hundred titles, including his Birds of Lower California and Birds of the Cambridge Basin. Property acquired by him in Cambridge and Concord was made a sanctuary for birds. For many years of his life he was Curator of Birds and Mammals in the Agassiz Museum. He was one of the founders of the American Ornithologists Union and of the Nuttall Club, of both of which he became President, as well as of the Massachusetts Audubon Society. Amherst gave him the honorary degree of Master of Arts in 1880, and he was given the same degree by Harvard in 1889.

William Brewster was an untiring and accurate observer of bird life, absolutely truthful, judicial in temperament. Physically a handsome man, he had a genius for friendship. He has left a permanent impress upon the science of ornithology.

HERBERT V NEAL.

GEORGE LINCOLN GOODALE (1839-1923).

Fellow in Class II, Section 2, 1874.

George Lincoln Goodale, Fisher Professor Emeritus of Natural History and Honorary Curator of the Botanical Museum of Harvard University, died in Cambridge on April 12, 1923, in the eighty-fourth year of his age. He was born in Saco, Maine, on August 3, 1839, the son of Stephen Lincoln Goodale and Prudence Aiken (Nourse) Goodale.

From Amherst College he received the A.B. degree in 1860 and six years later the honorary degree of Master of Arts. From 1863 to 1866 he practised medicine in Portland, Maine, having received, in 1863, the M.D. degree from Harvard.

At the age of thirty-three he began his active connection with Harvard University, having come there from Bowdoin College, where he had held a professorship in the natural sciences from 1868 to 1872. At Harvard he began his term of service as lecturer on vegetable physiology and instructor in botany. In 1873 he was appointed assistant professor and five years later was made full professor.

From October 6, 1879 to July 31, 1909 he was Director of the Botanic Garden of Harvard University, a position which brought him in close contact with influential men of Boston and afforded an opportunity for the development of his administrative powers. The Botanic Garden in the early days of Goodale's directorship was the botanical center of the University. Until the classes became too large for the lecture room, which had a capacity of about eighty students, it was at the Botanic Garden that laboratory work was done and instruction in botany was given. It was at the Garden, also, that the collections of botanical specimens, which were later transferred to the University Museum, were assembled, treasured and exhibited. Shortly after his appointment to the directorship of the Garden, Goodale found that the growth and progress of his department depended on substantial additions to the sources of income. He immediately set about the task of enlisting the sympathies of men who were interested in horticulture and who would help him with advice or money. Soon delapidated structures were giving place to new buildings and the invested funds of the Garden were being increased by gifts. So that in addition to the eminent services rendered as an inspiring teacher through thirty-seven years, Goodale was stimulating that development of botany which will always make his term of office memorable in the annals of Harvard University.

During these years, Goodale built up a strong department in the botanical sciences. Early in his career he foresaw the necessity for a well-equipped botanical museum which would arouse the interest of the public and serve as a valuable aid in the courses offered to students. With untiring energy he labored toward the fulfillment of an ideal. His reports to the President of the University emphasized again and again the need of additional space for the rapidly growing classes and for the collections of specimens that were crowding the limited accommodations at the Garden. With the museum in his mind and before its erection was assured, he visited Leopold and Rudolph Blaschka at their home in Dresden for the purpose of inducing them to make models in glass of plants and parts of flowers. Only after repeated refusals did the Blaschkas consent to construct the desired models and enter into the contract which resulted in the now famous collection of glass flowers that is one of the most popular and instructive attractions of the museums maintained by Harvard University.

In 1890–91 he went round the world, visiting the leading botanical establishments of Ceylon, Java, Straits Settlements, Australia and Japan, forming valuable contracts with scientific men and enriching his store of scientific knowledge. The influence of this trip to the tropics was far-reaching. As a result of his observations of the vast importance of botany in the commercial enterprises of the tropics, he determined to make the exhibitions of the new museum reveal the part that plants have played in the service of man.

Of the two great interests of Goodale's life, the museum which he built and a proposed work on economic botany for which he had assembled many critical specimens, the second was interrupted by administrative duties and by failing health. His interest in economic botany was very deep. He was convinced that this branch of his subject was one of the most important for modern times. In his report to the president of the University, for the years 1889–90, he set forth his plan as follows: "It seems no longer wise to withhold a declaration of the policy which I advocate in regard to the Museum of

Economic Botany. It is my wish to make the collection an authoritative cabinet of type specimens to be described in an illustrated series of studies." To those of us who knew the scope of the project and who recognized Goodale's exceptional capacity for its fulfillment, it must ever be a source of disappointment that he did not live to carry it through and fill a gap in botanical literature that still exists.

Although Goodale was the author of many papers on plant physiology and economic botany, and brought out in 1882 his Wild Flowers of America for which Isaac Sprague prepared the colored plates, it is his Physiological Botany, published in 1885, which stands as his most significant contribution to his subject. This work was prepared for the series of university textbooks which Asa Gray had planned as a complete course of botanical study. Goodale's textbook of Physiological Botany was a painstaking production which abounds in helpful citations and is characterized by straightforward exposition. This textbook was the result of profound knowledge and crystallized, as it were, the experiences he had gained in opening up for American students the subject of plant physiology as understood at the time by leaders in botanical thought. L. H. Bailey, in a biographical sketch of Goodale published in the twenty-fifth volume of Rhodora, writes of this work as follows: "It is a solid substantial book that students of the present day would do well to understand." It was only in the last two decades of the nineteenth century that physiological botany began to take its place in the development of science in America. Goodale was among the leaders in this field and his textbook stands as a creditable monument to the part he took in the introduction of physiological botany to American colleges. He hoped to issue a second edition of this textbook. With this aim in view he accumulated a vast amount of material, but he became so much occupied with other work that a second edition never appeared.

His desire to increase the influence of Harvard University in the realm of science led to the establishment of a tropical garden and experiment station in Cuba, near the city of Cienfuegos. He succeeded in interesting Mr. E. F. Atkins of Boston in this enterprise, which is now one of the important branches of Harvard's establishments for research in applied biology and a repository of an extensive collection of economic plants. In this garden originated the first seedlings of the sugar cane to be raised in Cuba. Toward the end of his life he had the satisfaction of knowing that a liberal endowment had been secured to perpetuate the work he had started.

It was my good fortune to accompany Goodale to Cuba when in 1899 he made his first visit to the proposed site of the new garden. It was on this trip that I began to know the sterling character of the man and to form that deep affection for him which increased with the passing of time. As I think back through the years in which it was my privilege, both as student and colleague, to know Goodale, the outstanding characteristics of his personality were delightful courtesy toward all who had dealings with him; consummate tact in handling the delicate situations which confronted him as head of a university department, and punctilious integrity in his business transactions with the many good friends who helped him to bear the burdens of building up the Botanic Garden and the Botanical Museum.

It was as a charming and finished lecturer, I think, that Goodale stands out most prominently in the memory of those who attended his courses in Harvard College. As a teacher he was forceful and concise. His demonstrations were fascinating and skillful. Combined with unique gifts of clear thinking and careful exposition were a commanding presence and a convincing manner. If, putting aside Goodale's achievements which stand as concrete evidence of his capacity, we judge him simply through his influential contacts with students during a long life as an inspiring teacher, we must arrive at the conclusion that his position among scientific men is a commanding one and that he made an indelible impression on the history of botany and botanical thought in America.

From 1888 to 1909 Goodale was Fisher Professor of Natural History. In 1909, when he retired from active teaching, he was made Fisher Professor Emeritus and Honorary Curator of the Botanical Museum. After his retirement he lived quietly at his home at 5 Berkeley Street, Cambridge. Here he converted a room into a laboratory and carried on his researches in museum technique. His visits to the Museum grew less and less frequent as his bodily strength waned, but to the end the Museum was uppermost in his daily thoughts and what might be its future was the only disturbing question in his mind.

Beyond the University his work was well known. In 1889 he was vice-president of the Biological Section of the Association for the

Advancement of Science. In 1890–91 he was president of the Association. In 1890 he received the LL.D. degree from Amherst College. In 1894 Bowdoin and in 1896 Princeton University honored him with similar degrees. He was an honorary fellow of the New York Academy of Sciences and of the Royal Society of New Zealand, and member of the Philosophical Society and National Academy of Sciences. He held membership in many botanical societies and was in attendance at the International Botanical Congress held in Brussels in 1910.

In 1866 he married Henrietta Juel Hobson who survives him, as do his two sons, Dr. Joseph Lincoln Goodale of Boston and Francis G. Goodale of Weston, Mass.

OAKES AMES.

SILAS WEIR MITCHELL (1829–1914).

Fellow in Class II, Section 3, 1865.

PERSONAL RECOLLECTIONS BY W. W. KEEN, M. D.

There is one serious difficulty in my writing my personal recollections of Dr. Weir Mitchell, namely, the impossibility of compressing within a reasonable compass his many and varied activities covering sixty years, and also my own friendship with him during almost as long a period.

He was one of what, some years ago, I described as "The Philadelphia Triumvirate"— Henry Charles Lea, Horace Howard Furness and Silas Weir Mitchell. No other three citizens shed such luster upon their native city during many years past.

Weir Mitchell was born on February 15, 1829; graduated from the Jefferson Medical College, when twenty-one years of age, in 1850; read his first paper at the age of twenty-three, and wrote his last novel at the age of eighty-three. He sank to rest after a brief illness, in undimmed splendor, on January 4, 1914, when almost eighty-five years of age.

He was just eight years my senior, but he graduated in medicine early and I graduated late. He had so much the start of me that I always held him rather in reverence as a father than as an elder brother.

As Dr. Charles W. Burr noted, "As a boy he had that best education, hearing his elders converse on things worth talking about"— not a bad hint for young parents to-day.

In the sixty years of his active scientific and literary life, he published over two hundred and fifty books, papers, reports, etc. He personally taught a generation of eager pupils. He prepared himself for the Chair of Physiology but, sad to say, he was twice defeated for such a Chair, both in the Jefferson Medical College, his Alma Mater, and also at the University of Pennsylvania. How utterly blind these two Boards of Trustees were to an extraordinary genius!

His election to the Board of Trustees of the University of Pennsylvania, a number of years later, marked the final victory of the younger progressives of the Medical Department in that university, led by Harrison Allen, William Pepper, James Tyson, H. C. Wood, and others. The most suggestive, and at the same time witty congratulation on his election was uttered by Harrison Allen —"Thou shalt not be King but thou shalt beget Kings."

My personal acquaintance with Dr. Mitchell began three days after I had started on my medical studies, in September 1860, when he asked me to help him in some experiments on snakes. He was already a distinguished physiologist, known in Europe as well as at home. Then began the most intimate medical friendship of my life, closed only by his death over fifty-three years later, with never a cloud so large as a man's hand between us.

The best term by which to describe him was that which I often applied to him, "a yeasty man." His own mind was always fermenting with new ideas, new researches, new papers, new poems, new novels. He gave points to the neurologists, the orthopedists, the gynecologists, the ophthalmologists, the anatomists, the surgeons, the botanists.

The last evening I spent with him, only a few days before his final illness, when he was almost eighty-five years old, his yeasty mind was still fermenting. His mind, in its ninth decade, was still hospitable to new ideas. "Why do not the botanists," he suddenly exclaimed to me, "try experiments such as the following." Then he proceeded to develop a scheme for certain experiments on plants, not knowing that, at that very time, half-way round the globe, Sir J. C. Bose, a wonderfully talented Hindu — who was knighted by the king for his scientific work and received a higher accolade from the Royal Society, being made an F. R. S.— was actually carrying out *in extenso* the idea that Mitchell briefly suggested to me in that very conversation.

He was the first to describe erythromelalgia; investigated with William Thomson eye strain as the cause of persistent headache; and with Thomson and myself the final distribution of all the optic nerves in the retina beyond the chiasm; discovered the chiasm of the respiratory nerves in the turtle; studied the diseases of stumps, including the curiously persisting phantom limbs after amputation; discovered, with Morris J. Lewis, more than one muscular reflex; devised the Mitchell "rest cure;" studied with Morehouse and myself the dreadful, persistent pain after wounds of nerve trunks (causalgia), and introduced the use of atropin with morphin, for its relief, an investigation which produced the little tablets which we now have for use in various proportions of these remedies by means of the hypodermic needle.

He was a large factor in introducing the wide use of the thermometer, the electric battery and the hypodermic syringe during the Civil War. I do not think that, outside of our hospital at Turner's Lane, there were, in the whole Army of the Potomac, a half dozen of these three devices for diagnosis and treatment.

His remarkable presidential address at the second meeting of the Triennial Congress of American Physicians and Surgeons in 1891, on the "Early History of Instruments of Precision," is still a classic and a storehouse of knowledge for the profession.

His thorough historical studies of the dress, manners and peculiarities of conversation of the eighteenth century, in order to write his "Youth of Washington," for a reproduction of the very atmosphere of the times in which Washington lived, may well serve as a model to other historians. His studies of the relation of weather to neuralgia, covering some years of coöperation by a most intelligent army officer of the line, is also a model for other similar investigations.

Sir James Paget, when President of the great International Congress of Medicine in London in 1881, was well within the mark, when as I sat opposite him at table as his guest during the Congress he said to me, "Dr. Mitchell is one of the most distinguished medical men in your country," adding, after a brief pause, "or in any country."

The venoms of various snakes — the rattle snake, the moccasin, the cobra — and of the related South American poisons, corroval and vao, also interested him for over half a century. His own researches, in which I assisted him for several years, were of great importance for he first discovered that these venoms were toxic albumins. He also stimulated and, in various ways, aided Noguchi and later Reichert in the investigations of the effects of these venoms upon the blood and especially on the crystallography of the blood, the subject of his very first paper in 1852. He was also one of the original and most active trustees of the Carnegie Institution, which gave him opportunities to aid and stimulate others in their researches.

The Civil War, which broke out on April 15th, 1861, gave him an unrivalled opportunity to study injuries and diseases of the nervous system, upon which his alert mind quickly seized. Among the special hospitals which his friend, Surgeon General Hammond established at Mitchell's instance, was one devoted entirely to these neurological diseases and injuries, first located in the Christian Street Hospital and later chiefly at the Turner's Lane Hospital, both in Philadelphia. He and Dr. George R. Morehouse were appointed Acting Assisting Surgeons of the U. S. Army, in charge of these wards. It was the most fortunate event of all my professional life that Mitchell asked Hammond to transfer me to the Turner's Lane Hospital as their junior in the service. I was ordered there early in 1863 about a year after my graduation. The reason he asked for my transfer, he told me years afterwards, was, that he had early found "that he could not kill me with hard work"— a cherished encomium.

I have never wearied in telling the story of his broad-minded generosity. He was the guiding spirit of our "firm," if I may use such a term, at a time when it was extremely unusual for papers, books, etc., to be written by more than a single author. Soon after I joined them, Mitchell plotted our work. He assigned to himself a paper on "Reflex Paralysis" (Circular No. 6 Surgeon General's Office, U. S. Army, 1864), a book on "Gun Shot Wounds and Other Injuries of Nerves" (J. B. Lippincott Co., 1864), with several other papers. Morehouse was to write on certain other subjects, such as "Epilepsy," "Choreal Affections," etc., and I was to do the experimental work on the "Antagonism between Morphin and Atropin," under their direction, on which later Mitchell's text was founded. I, myself, was also to write a detailed paper of nearly thirty pages, published in the American

Journal of the Medical Sciences, for October 1864, on "Malingering." We were the first, I believe, to introduce ether anesthesia as a means for detecting these shirkers. Not a few amusing stories arising in the course of these investigations are recorded in that paper.

How well I recall the hours and hours, not seldom long after midnight, when I wrote literally thousands and thousands of pages of histories (for dictation and the typewriter were still below the horizon). Many of these were the foundation of the paper on "Reflex Paralysis," and the book on "Gun Shot Wounds and Other Injuries of Nerves," on "Malingering," and other later papers. Most of the material for Dr. Morehouse's papers was destroyed by a fire in his office and "carbon copies," alas, were utterly undreamed of.

I am not sure but that this fire was a great relief to Dr. Morehouse. He was a most accomplished and delightful talker, with a head full of ideas. His knowledge was extensive. He could converse by the hour most entertainingly and set forth his ideas logically and philosophically. But put a pen in his hand, and it became for the nonce paralyzed. He wrote very few papers. Medicine lost much of value by his silence.

Note, now, the large-minded, big-hearted generosity of Dr. Mitchell. Ninety-nine men out of a hundred would have published their books and papers, "By S. Weir Mitchell, M.D. and George R. Morehouse, M.D." as authors, and in the preface would have expressed their "obligation to Acting Assistant Surgeon, Dr. W. W. Keen, for his faithful and intelligent work in recording the histories and carrying out the experiments as planned by the authors," or some similar phraseology. Not so Mitchell. The paper on "Reflex Paralysis" and the book on "Gun Shot Wounds and Other Injuries of Nerves" were by "S. Weir Mitchell, M.D., George R. Morehouse, M.D., and W. W. Keen, M.D.," and the long paper on "Malingering" was "By W. W. Keen, M.D., S. Weir Mitchell, M.D., and George R. Morehouse, M.D." I, a mere medical kid, barely two years out of the medical school, had my name precede that of Mitchell, already recognized in Europe as well as in America as a master mind in research, as well as that of Morehouse. This, too, is a suggestion for imitation today, for recognition limited to a preface, and sometimes not even appearing there, is not altogether unknown.

But even better than this remarkably generous treatment was the profound influence that Mitchell exerted upon my whole professional life. He gave me the scientific bent, the ardor in the search for scientific truth, the eager and constant desire for "Work," that "Master Word" in medicine, as Osler has well called it. He taught me how to observe and how to elicit, often literally to "dig up," histories from unobservant patients. As the eye of an expert geologist almost instantly sees the meaning of a barely visible outcropping of a vein of precious ore, Mitchell taught me to recognize and to seize at once upon some small and apparently insignificant fact which, when fully investigated, proved to be of great value in practice, or even the key to the secret of the patient's chief malady.

Even in the sixties, Mitchell showed a certain tremor of his hands, yet, in spite of this unsteadiness, I never saw him injure or divide a nerve or a blood vessel or a tissue which he desired not to cut.

His recreations were largely in the woods and in fishing. He is well remembered on Mt. Desert by his laying out the Cadillac trail. For many years he camped out with Phillips Brooks, Bishop McVickar and the Rev. Dr. Charles D. Cooper. On Sunday they always held a service and Mitchell told me with great glee how, on one occasion, Bishop Brooks' tongue got derailed in the middle of the Lord's Prayer and he had to begin over again. This accident is not limited to bishops.

I have no time to consider his extensive list of novels, essays, poems, etc. His biography, soon to be published, will detail these and many other interesting facts.

He was always and everywhere a *primum mobile*, starting the wheels going and always to accomplish some good piece of work. Osler was the only rival I have ever known in this stimulating influence over others. William Pepper was a near rival. Fortunate, indeed, were the many who have benefited by the influence which Mitchell shed all around him. I count myself the most fortunate of them all.

W. W. KEEN.

JAMES MILLS PEIRCE (1834–1906).

Fellow in Class I, Section 1, 1862.

James Mills Peirce, son of Professor Benjamin Peirce, was born in Cambridge on May 1, 1834. He was graduated at Harvard in 1853 and immediately entered the Harvard Divinity School, and after taking his degree of Bachelor of Divinity he preached for a short time in a church in Somerville.

A ministerial life soon palled on him and he accepted a tutorship at Harvard in 1856. He became Assistant Professor of Mathematics in 1859, University Professor of Mathematics in 1869, and Perkins Professor of Mathematics and Astronomy in 1885.

He was for many years secretary of the Academic Council, and on the formal organization of the Graduate School of Arts and Sciences in 1890 he became its Dean.

He died in harness on the 21st of March 1906, in the seventy-second year of his age, having nearly completed his fiftieth year of service in the Harvard Faculty.

His life was devoted to the liberalizing of Harvard College and the development on the academic side into Harvard University. He was one of the pioneers in introducing and expanding the elective system in the College; and during the long administration of his classmate, President Eliot, he worked shoulder to shoulder with him in fostering graduate study in the University; and when finally the Graduate School of Arts and Sciences was established he served devotedly and untiringly for many years as its Dean.

For nearly fifty years he was a picturesque figure in the College, living the larger portion of that time in the College Yard, peculiarly accessible to the students, many of whom became his intimate friends. Always genial, kindly, and helpful, he was a man who could unbend without seeming to realize that he was unbending.

He was a deep and thorough scholar in his special subject, rather a student than an investigator; an excellent teacher, and an unusually clear and polished lecturer. His intellectual activities were by no means confined to his chosen science; he was an omnivorous reader, rather fonder of the drama than of other departments of literature, a fine Shakespearean scholar, and a remarkable dramatic reader.

The stage and whist were his two great passions, and toward the end of his life they occupied most of his evenings, for he was a bachelor and a clubman.

During his college course James Mills Peirce was a member of the Alpha (Harvard) Chapter of the Psi Upsilon fraternity. Among his fellow-members were W. W. Goodwin, Ephraim W. Gurney, Benjamin J. Jeffries, Alexander Agassiz, Theodore Lyman, and Robert Treat Paine. Although the chapter later became inactive Professor Peirce always retained his interest in the fraternity and attended frequently to the end of his life the graduate banquets which were held in Boston.

W. E. Byerly.

COUNT SOLMS (1842–1915).

Foreign Honorary Member in Class II, Section 2, 1896.

In estimating a great scholar, inspiring teacher, or productive scientist, statistical matter takes secondary rank. The bulk of his published output, size of his classes, length of his service, extent of his organizing and administrative activities — all these quantitative matters, which seem to have made the sum of his achievement, are much less significant, certainly far less interesting, than those human traits which made him what he was. His energy, sincerity, breadth of mind, his clarity of diction, skill of hand, quickness of apprehension, fairness in controversy, kindly interest, even his sense of humor — just these things which cannot be measured are what make a man a vital force in his epoch. It is through them that he catches the attention of his contemporaries, influences his colleagues, guides and stimulates his students, and thus leaves his impress upon a branch of knowledge.

Hermann, Count of Solms-Laubach, was born December 23, 1842, near the small village of Laubach in Upper Hesse, and died at Strasburg in Alsace November 14, 1915. His family was of old nobility and had maintained its independence even to the Napoleonic period. His father was Count Otto of Solms-Laubach and his mother, born a princess of Wied, was an aunt of Carmen Sylva, the poetess queen of Roumania. She was also a niece of that Prince Maximilian of Wied-Neuwied whose name is familiar in botany from his effective explorations in Brazil early in the nineteenth century.

Count Solms, after early instruction by family tutors and in an elementary school, passed through the gymnasium at Giessen and entered the well-known university at the same place. It is said that his early inclination toward history and taste for natural science were nearly balanced, but that he was influenced toward the latter as a life work by Leuckhart's stimulating lectures on animal anatomy. From boyhood he seems to have had a passion for observing and collecting plants and it was toward botany rather than zoölogy that his impulse turned. Prompt in his decision, he left the University of Giessen after a single semester and went to Berlin to study with Alexander Braun. There he devoted himself — as he afterward was disposed to maintain, all too exclusively — to botany. He achieved his doctorate at Berlin in 1865, his dissertation dealing with the systematic position of the Genus Lathraea.

After several months of travel, during which he formed some lifelong friendships with notable botanists and gained acquaintance with the important botanical collections in Geneva and elsewhere, he went to the University of Halle whither he was attracted by de Bary and where he soon became instructor in botany. In 1872, during the reorganization of the University of Strasburg, de Bary was selected to head the botanical establishments of that institution. Doubtless through his influence Solms was called there too as Professor extraordinarius. In this capacity he served from 1872 to 1878. The epoch and place were remarkable in the history of botany. The great fame of de Bary brought to his laboratory a cosmopolitan group of students of exceptional talent, destined to make good and attain high distinction in divers fields and many lands. The evolutionary theory had brought new points of view. Improvements in microscope, microtome and staining technique had opened wide vistas for exploration. Here Count Solms passed the still formative years between instructorship and independent appointment. But little older than many of the talented band he was helping to guide he left an enduring impression upon them and retained through life their friendship and admiration.

In 1879 he was chosen to succeed Grisebach at Göttingen. Four years later he made a journey to the tropics which greatly broadened his experience and always remained to him a source of satisfaction. The Dutch had established an admirable laboratory at the Botanical Garden of Buitenzorg in Java. Under the able direction of Treub it became a place of international hospitality. Solms was one of the first to avail himself of the unusual privileges thus offered. There he found a wealth of opportunity, particularly in plant anatomy, and there he was able to assemble much material both for his personal investigation and to furnish fascinating problems for his future students.

In 1888 Count Solms received an honor of the first magnitude, namely a call to the University of Berlin. This position, however, he accepted only reluctantly and almost immediately resigned in order to take one much less conspicuous, the botanical professorship at Strasburg, which had become vacant through the death of de Bary. It is believed that several motives led Solms to this course. Though social by nature he had a horror of large or formal gatherings and probably dreaded social contacts and especially court functions which his scientific distinction combined with his noble birth would have rendered unavoidable in a great city and imperial capital. On the other hand it is certain that he felt in the highest degree the honor of succeeding at Strasburg his revered master, whose work it became his ambition to continue. It is not improbable that he hesitated to assume the large administrative duties at Berlin through some doubt of his ability in this phase of scientific work and from preference for less interrupted personal research. Finally, he was of the Rhinelands by birth and traditions.

Whatever may have been the grounds for his decision, it was promptly made and so far as is known was never regretted. It is interesting to speculate on what might have occurred if Solms had held his appointment to Berlin University instead of making way for the later incumbent there who, copious in his personal output and endowed with exceptional talent for organization, took the direction of the Berlin Imperial Botanical Garden and Museum just when the colonial expansion of the German Empire opened correspondingly wide opportunities in descriptive science.

As he appeared to his students at Strasburg in the late eighties and early nineties Solms was a man of extraordinary energy and incessant activity. He was tall, erect, broad-shouldered but otherwise rather slender. With high forehead, fine nose, alert and frienoly blue eyes, and copious reddish-brown hair and beard he would have been rather handsome had his carriage been more sedate, but he was loose-hung and apt to assume in the eagerness of discussion awkward postures. His gait was vigorous but had nervous uncertainty. His clothes, excellent in quality, were conservatively chosen for comfort and with frank disregard of fashion. They were worn carelessly and often impatiently adjusted during conversation as when in the midst of an eager account of some recent theory or discovery he would suddenly interrupt himself to tie a shoe lace or give new centering to a cravat that had wandered far to the right or left.

On the lecture platform he was always clear and consecutive, often brilliant. His manner of delivery, however, had marked peculiarities. His voice was raised almost to a shout. He rapidly paced back and forth on the rostrum and his gesticulation was almost incessant and frequently violent. He handled illustrative specimens somewhat recklessly and occasionally to their damage. But all this was quickly forgotten by his listeners who were soon attentive to the compelling logic of his argument. Though facts were subjected to critical and ruthless examination, they were never the end in themselves. Their interest lay in what they proved. This was the secret by which he so perfectly held the attention of his hearers. He often told them what they had previously read or heard, but always imparted to such matter new interest by his explanations and interpretation.

He devoured and digested scientific literature in vast quantities, and had the rare ability of selective assimilation. Matter diffusely recorded by colleagues in works of tedious detail he acquired with great rapidity and presented in condensed form, clarified and arranged with admirable sense of values.

Such were his gifts as a lecturer. It is unfortunate that in his writings he did not attain the same lucidity. An almost morbid regard for precision, combined with anxiety to accord all possible credit to previous exponents of his subject, led him to overload his own composition with quotations. As a result, his writings, instead of having the consistency of his speech, present an often bewildering variety of contrasting views. They require close reading.

As a director of laboratory research he was friendly in his interest and uncommonly generous in supplying material of value often from his personal stores, but he was not very suggestive. Indeed, he purposely left his students to devise their own methods and work out their own salvation. When they gave evidence of progress, his gratification was immediate and doubtless greater because their discoveries were their own rather than the reflected result of promptings on his part.

His personal investigations were at all times enthusiastically pursued. So great was his eagerness in them that he quickly surrounded himself by a tremendous array of books, papers, specimens and apparatus, which soon left him only some small corner of a desk or laboratory table on which to write. Though his manual cleverness seemed subject to handicap from nervous uncertainty in his movements, he was not lacking in skill. He had, furthermore, boundless courage in attacking problems of the most baffling difficulty, as in studying the anatomy of the rarest fossil plants or revising extant genera like Pandanus or Rafflesia.

Compared with his compatriots, most of whom were already tending strongly toward specialization, Solms had great breadth of interest. Many of his early studies had to do with the anatomical and developmental traits of algae and fungi. He wrote on certain geographic aspects of the bryophytes. He published repeatedly on the screw-pines. His most notable taxonomic work concerned the phanerogamic parasites on which he became the world's greatest authority and concerning which he collaborated in systematic works of the first importance, such as the Flora Brasiliensis, the Natürliche Pflanzenfamilien, and Pflanzenreich. Much of his most remarkable work was in the realm of paleophytology and there he was one of the first to bring to bear upon the structure and classification of vegetable fossils a comprehensive knowledge of the anatomy and taxonomy of extant groups. He took more than an incidental interest in problems of variation and those presented by plants of cultivation.

In assuming the duties of de Bary he fell heir to the editorship of the Botanische Zeitung, which he patiently directed through many years with the minimum of change in its already rather archaic form.

Solms's death, occurring at a time when war was a grim actuality and men's thoughts were distracted from all else, met with far less notice than would otherwise have been the case. Biographical accounts of him have been few, but one of them has been of exceptional excellence, namely that by his associate, intimate friend, and official successor, Prof. Ludwig Jost[1] — a sketch replete with scientific insight, photographic portraiture, and affectionate touches of humor. From England also there have been worthy tributes to his memory by Prof. D. H. Scott[1] and Dr. Otto Stapf.[2]

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[1] Berichte der Deutschen Botan. Gesellsch., vol. xxxiii, pp. (85)–(112).

Fond of travel and facile in acquiring foreign languages Count Solms made frequent visits to Switzerland and Italy and in his active years went not rarely to England where he was specially interested by the superb collections of plants — living, dried and fossil. He became very fond of many of his English colleagues and spoke of them in terms of high regard. In matters of race, national customs, religious preferences or political views he was uniformly tolerant, though often facetious in comment. Toward a very few of his contemporary scientists he was clearly antipathetic but seemed less irritated by the difference of their views than by the tediousness with which these were presented. A bore he could not abide.

Count Solms never married. His household was long directed by his sister and after her death by two nieces, the Countesses Sophie and Anna of Solms-Rödelheim. When he had directed the Botanical Institute at Strasburg for twenty years he retired from active service and acquired an excellent house overlooking the Botanical Garden. Here he passed his remaining years in great comfort, often visiting the scenes of his former activities and, so far as his health permitted, continuing his investigations. He remained cheerful, was interested in and sympathetic with the work of his successor, and was jocose regarding the timorous beginnings of coeducation just then taking place in the scientific laboratories of Strasburg.

At his home, which was one of simplicity and great charm, he and his nieces practised a quiet and delightful hospitality. A notable member of the household was a parrot, an accomplished and gentlemanly bird, that came to the table on the Count's forefinger and there, supplied with napkin and appropriate small dishes, did full justice to a *table d'hôte*, partaking of all courses from soup to dessert, though showing particular enthusiasm for the salad.

Seldom has a man of Solms's rank pursued science with such fine devotion. Even more rarely has a born scientist possessed interests of such breadth or a personality so striking.

B. L. Robinson.

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[1] Nature, xcvi, p. 541 (1916), reprinted in Bot. Gaz., xli, p. 433 (1916).
[2] Proc. Linn. Soc., 128th Session, pp. 71–73.

CHARLES PROTEUS STEINMETZ (1865–1923).

Fellow in Class I, Section 4, 1911.

The sudden death of Dr. C. P. Steinmetz on October 26, 1923, at Schenectady, N. Y., has removed one of the most remarkable men in electrical engineering that the world has ever known. His life is a triumphant history of genius and energy, hampered by obstacles and physical disabilities.

He was born in Breslau, Germany, on April 9, 1865. His father was an official of that city and gave the boy a good education. Charles entered the University of Breslau in 1882, specializing in mathematical and physical sciences. Later, he transferred his studies to the University of Berlin; but his stay there was short. He manifested the strong individuality of his character and warmly espoused the cause of socialism. This brought him under the ban of the German government authorities, to whom socialistic propaganda were iniquities of the most glaring type. It was necessary for young Steinmetz to leave Germany promptly, for political reasons, and in 1888 he went to the Zurich Polytechnic School, where he studied mechanical engineering. In the following year, he decided to try his fortune in America. He was needy and had to borrow his passage money from the friend who suggested the venture. Furthermore, he was handicapped physically because he was humpbacked; but he was young, energetic and full of hope.

Steinmetz reached New York in 1889, with a letter of introduction from the Elektrotechnische Zeitung, a leading electrical journal of Berlin. He found employment as a draughtsman with the Eickemeyer Manufacturing Co. of Yonkers, N. Y. He was soon promoted by them to the office of electrical designer, and put in charge of electrical work in their factory. He had never had any practical electric experience in Germany; but he had had, as we have seen, a good fundamental training in physics and mathematics.

In the year following his arrival in New York, he made his first contribution to electrotechnical literature, "Notes on the Law of Hysteresis" in the Electrical Engineer. In that year he also became an Associate of the American Institute of Electrical Engineers. He contributed his first papers to that Institute in 1892, on the laws of hysteresis. These papers showed remarkable grasp and originality. They were the forerunners in the Institute Proceedings of a long series of technical papers, many of which have become classically famous. His writings flowed into a number of channels in electrical literature and soon aroused world-wide attention. Commencing with a paper in the Proceedings of the International Electrical Congress of Chicago (August 1893), on "Complex Quantities and Their Use in Electrical Engineering," a subject then relatively young, he proceeded to develop and expand the applications of complex numbers to electrical machinery and circuits in a manner that was very remarkable. He was soon a familiar figure at the meetings and discussions of the American Institute of Electrical Engineers. He would stand with his knee on a chair, and his arm on the back of the same, his large head low on his shoulders, and a cigar in the left corner of his mouth, his eyes flashing with vivacity and intelligence. He would thus speak on the topic of the evening for perhaps twenty minutes, with the audience spell-bound. His remarks were always keen, clear and to the point. He held the rare combination of a mathematical mind, a practical sense of realities, and a power of clear exposition. There was no one who could address a meeting of electrical engineers with such magnetic effect as Steinmetz. He was elected President of the American Institute of Electrical Engineers for the year 1901–02. He was also President of the Illuminating Engineering Society in 1915–16.

When the Eickemeyer electrical interests were acquired by the General Electric Co. in 1893, Steinmetz joined that company, and was attached to the computing department of the Lynn works. Here he rapidly distinguished himself, and in a few months became the leading spirit in the design and computation of alternating-current machinery. In 1894 the headquarters of the General Electric Co. was transferred to Schenectady, N. Y., and Steinmetz organized there the designing department. He also developed a research department. For nearly thirty years he was the chief consulting engineer of the General Electric Co., and his responsibilities were often very large. During this long period of work, he took out nearly two hundred United States patents for electrical inventions. They were not, in general, of a sensational nature, but they were sound and serviceable. Although his papers in the technical journals were most remarkable on the theoretical side, his practical skill in design and invention was very noteworthy.

In 1902, Steinmetz was appointed professor of electro-physics at Union College, Schenectady, and he held this chair until his death. He received the honorary degree of M.A. from Harvard University in 1902, and that of Ph.D. from Union College in 1903. He was elected a non-resident member of the American Academy of Arts and Sciences October 11, 1911.

Dr. Steinmetz was the author of some ten well-known books on electrotechnics, including electric illumination. Some of these have been translated into other languages. They are mathematical and nearly all utilize and develop complex arithmetic. While occasionally employing the calculus and differential equations, they are mainly written in simple algebra. His mathematics was like his reasoning, direct and without circumlocution. His literary style was quite characteristic, terse and explicit. Although he acquired a thorough conversational knowledge of the English language, his writings contain certain small traces of foreign idiomatic construction, imported from the German. So widespread has become the study of his books, that a few of these Steinmetzian phrases have become current in modern American electrotechnical literature. His most recent book Relativity and Space, published in 1923, is perhaps the best extant exposition of Einsteinian philosophy in terms that the engineer is able to grasp.

The pervasive energy of his mentality vented itself in other channels than engineering. He successfully entered the fields of civics and politics in his city of residence. He had been President of the Schenectady Board of Education since 1912, and also President of the Schenectady Common Council since 1916. He was always ready to devote time and energy to lectures and addresses in various American cities, on technical, scientific and sociological topics. He expounded the beliefs that the progress of civilization depends greatly on the development of electric power distribution, and also on the elimination of waste in material distribution.

As a personality, he was noteworthy and impelling. His brilliant eyes, short bushy hair and the characteristic attitudes associated with his physical deformity, fascinated attention. No one who had once seen Steinmetz speak could ever forget him. He was an inveterate smoker, and the inevitable cigar is a prominent object in most of his portraits. He was also very fond of strong coffee. In all other respects, however, he was most temperate in habits. He lived a bachelor's life, almost outside the pale of feminine society. He was cheerful, generous and kindly, fond of recreation in the open air, but cut off from sports except boating and canoeing. He was entirely free from disdain or affectation, and was approachable to every questioner, no matter how crude the question. He held strong opinions on many subjects and he enjoyed temperate debate. His strong individuality, however, resented rules and restrictions. In the General Electric Company's works, where the printed rules of non-smoking were abundantly displayed, the only exception, permitted by universal consent, was Dr. Steinmetz's cigar.

His bachelor home was an open house to many of his fellow-engineers, and he liked to see them enjoy it. He had special tastes and recreations. At one time, he kept a number of alligators as pets. He had a fine collection of cacti in his greenhouses and was well versed in their names, habitats and modes of living. He had a large aquarium containing plants and fish. A trap door and ladder gave access to a grotto below the water level, where, beside a large glass window, he could sit and smoke, while watching the slow-moving fishes and working out his problems.

His death occurred from heart failure, shortly after his return from a trip to a convention of the American Institute of Electrical Engineers on the Pacific Coast, where he had read a paper on "High-Voltage Insulation," the last of so long and notable a series of new contributions to engineering literature. He was still actively working and producing.

His death removes a great and unique personality from the ranks of electrical engineering, a genius, one out of many millions of men, hampered by a deformed body but endowed with a most energetic mind. A most highly individualistic man, he devoted his life to the theory and practice of electrical engineering. All electrical engineers realize how great was his triumph.

A. E. Kennelly.

American Academy of Arts and Sciences

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Theodore William Richards	Cambridge
Martin André Rosanoff	Pittsburgh, Pa.
Miles Standish Sherrill	Winchester
Harry Monmouth Smith	Brookline
Julius Oscar Stieglitz	Chicago, Ill.
Henry Paul Talbot	Newton
Richard Chase Tolman	Washington, D. C.
William Hultz Walker	Bridgton, Me.
Willis Rodney Whitney	Schenectady, N. Y.
Robert Seaton Williams	Cambridge
Alpheus Grant Woodman	Watertown

Class I., Section IV.— *Technology and Engineering.*— 46.

Henry Larcom Abbot	Cambridge
Comfort Avery Adams	Cambridge
Bernard Arthur Behrend	Boston
William Herbert Bixby	Chicago, Ill.
Francis Tiffany Bowles	Boston
Charles Francis Brush	Cleveland, O.
William Hubert Burr	New Canaan, Conn.
John Joseph Carty	New York, N. Y.
Harry Ellsworth Clifford	Newton
Arthur Powell Davis	Washington, D. C.
Theodore Harwood Dillon	Cambridge
Gano Dunn	New York, N. Y.
William Frederick Durand	Palo Alto, Cal.
Charles Leavitt Edgar	Brookline
Frederic Harold Fay	Boston

Desmond FitzGerald	Brookline
John Ripley Freeman	Providence, R. I.
George Washington Goethals	New York, N. Y.
John Hays Hammond	New York, N. Y.
Ira Nelson Hollis	Worcester
William Hovgaard	Brookline
Hector James Hughes	Cambridge
Alexander Crombie Humphreys	New York, N. Y.
James Robertson Jack	Cambridge
Dugald Caleb Jackson	Cambridge
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
Gaetano Lanza	Philadelphia, Pa.
William Henry Lawrence	Boston
Charles Thomas Main	Winchester
Lionel Simeon Marks	Cambridge
Edward Furber Miller	Newton
Frederick Law Olmsted	Brookline
Charles Francis Park	Taunton
William Barclay Parsons	New York, N. Y.
Harold Pender	Philadelphia, Pa.
Albert Sauveur	Cambridge
Peter Schwamb	Arlington
Henry Lloyd Smyth	Watertown
Charles Milton Spofford	Brookline
Samuel Wesley Stratton	Cambridge
George Fillmore Swain	Brookline
Edward Pearson Warner	Cambridge
George Chandler Whipple	Cambridge
Robert Simpson Woodward	Washington, D. C.
Joseph Ruggles Worcester	Boston

CLASS II.— *Natural and Physiological Sciences.*— 190.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 53.

Wallace Walter Atwood	Worcester
George Hunt Barton	Cambridge
Norman Levi Bowen	Washington, D. C.

Isaiah Bowman	New York, N. Y.
Thomas Chrowder Chamberlin	Chicago, Ill.
John Mason Clarke	Albany, N. Y.
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	New Haven, Conn.
William Jackson Humphreys	Washington, D. C.
Ellsworth Huntington	New Haven, Conn.
Oliver Whipple Huntington	Newport, R. I.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, H. I.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
James Furman Kemp	New York, N. Y.
Alfred Church Lane	Cambridge
Andrew Cowper Lawson	Berkeley, Cal.
Charles Kenneth Leith	Madison, Wis.
Waldemar Lindgren	Brookline
Frederic Brewster Loomis	Amherst
Alexander George McAdie	Readville
John Campbell Merriam	Washington, D. C.
William John Miller	Northampton
Charles Palache	Cambridge
Percy Edward Raymond	Lexington
William North Rice	Middletown, Conn.
Austin Flint Rogers	Palo Alto, Cal.
Robert Wilcox Sayles	Chestnut Hill
Waldemar Theodore Schaller	Washington, D. C.
Charles Schuchert	New Haven, Conn.
William Berryman Scott	Princeton, N. J.

Hervey Woodburn Shimer	Hingham
Thomas Wayland Vaughan	La Jolla, Cal.
Charles Doolittle Walcott	Washington, D. C.
Robert DeCourcy Ward	Cambridge
Charles Hyde Warren	New Haven, Conn.
David White	Washington, D. C.
Herbert Percy Whitlock	New York, N. Y.
Bailey Willis	Palo Alto, Cal.
Arthur Winslow	Boston
John Eliot Wolff	Pasadena, Cal.
Jay Backus Woodworth	Cambridge
Frederick Eugene Wright	Washington, D. C.

Class II., Section II.—*Botany*.—35.

Oakes Ames	North Easton
Joseph Charles Arthur	Lafayette, Ind.
Irving Widmer Bailey	Cambridge
Liberty Hyde Bailey	Ithaca, N. Y.
Edward Wilber Berry	Baltimore, Md.
Douglas Houghton Campbell	Palo Alto, Cal.
George Perkins Clinton	New Haven, Conn.
John Merle Coulter	Chicago, Ill.
Bradley Moore Davis	Ann Arbor, Mich.
Edward Murray East	Jamaica Plain
Rollins Adams Emerson	Ithaca, N. Y.
Alexander William Evans	New Haven, Conn.
Merritt Lyndon Fernald	Cambridge
Robert Almer Harper	New York, N. Y.
John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge
Fred Dayton Lambert	Tufts College
Jacob Goodale Lipman	New Brunswick, N. J.
Burton Edward Livingston	Baltimore, Md.
George Richard Lyman	Morgantown, W. Va.
Elmer Drew Merrill	Berkeley, Cal.
Winthrop John Vanleuven Osterhout	Cambridge
Charles Vancouver Piper	Washington, D. C.

Alfred Rehder	Jamaica Plain
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Erwin Frink Smith	Washington, D. C.
John Donnell Smith	Baltimore, Md.
Elvin Charles Stakman	St. Paul, Minn.
William Codman Sturgis	New York, N. Y.
Roland Thaxter	Cambridge
William Trelease	Urbana, Ill.
William Henry Weston, Jr.	Cambridge

Class II., Section III.—*Zoölogy and Physiology.*—61.

Nathan Banks	Cambridge
Thomas Barbour	Boston
Francis Gano Benedict	Boston
Henry Bryant Bigelow	Concord
Robert Payne Bigelow	Brookline
Edwin Garrigues Boring	Cambridge
William T. Bovie	Milton
Edward Allen Boyden	Newton Center
John Lewis Bremer	Boston
Charles Thomas Brues	Boston
Hermon Cary Bumpus	Duxbury
Walter Bradford Cannon	Cambridge
Thorne Martin Carpenter	Boston
William Ernest Castle	Belmont
Charles Value Chapin	Providence, R. I.
Benjamin Preston Clark	Boston
Samuel Fessenden Clarke	Williamstown
Edwin Grant Conklin	Princeton, N. J.
Manton Copeland	Brunswick, Me.
Joseph Augustine Cushman	Sharon
William Healey Dall	Washington, D. C.
Charles Benedict Davenport	Cold Spring Harbor, N. Y.
Raymond Dodge	Middletown, Conn.

Gilman Arthur Drew	Woods Hole
Cecil Kent Drinker	Boston
Alexander Forbes	Milton
Samuel Henshaw	Cambridge
Samuel Jackson Holmes	Berkeley, Cal.
Leland Ossian Howard	Washington, D. C.
Herbert Spencer Jennings	Baltimore, Md.
Charles Willison Johnson	Brookline
Charles Atwood Kofoid	Berkeley, Cal.
Frederic Thomas Lewis	Waban
Ralph Stayner Lillie	Cleveland, O.
Richard Swann Lull	New Haven, Conn.
Edward Laurens Mark	Cambridge
Ernest Gale Martin	Palo Alto, Cal.
Albert Davis Mead	Providence, R. I.
Gerrit Smith Miller	Washington, D. C.
Edward Sylvester Morse	Salem
Herbert Vincent Neal	Tufts College
Henry Fairfield Osborn	New York, N. Y.
George Howard Parker	Cambridge
William Patten	Hanover, N. H.
Raymond Pearl	Baltimore, Md.
John Charles Phillips	Wenham
Henry Augustus Pilsbry	Philadelphia, Pa.
Herbert Wilbur Rand	Cambridge
Arthur Clarence Redfield	Boston
William Emerson Ritter	La Jolla, Cal.
Percy Goldthwait Stiles	Newtonville
John Eliot Thayer	Lancaster
William Lyman Underwood	Belmont
Addison Emory Verrill	Westville, Conn.
John Broadus Watson	New York, N. Y.
Arthur Wisswald Weysse	Boston
William Morton Wheeler	Jamaica Plain
Harris Hawthorne Wilder	Northampton
Edmund Beecher Wilson	New York, N. Y.
Frederick Adams Woods	New York, N. Y.
Robert Mearns Yerkes	Washington, D. C.

Class II., Section IV.— *Medicine and Surgery.*— 41.

Edward Hickling Bradford	Boston
Charles Macfie Campbell	Cambridge
Alexis Carrel	New York, N. Y.
Henry Asbury Christian	Boston
Stanley Cobb	Ponkapoag
Rufus Cole	New York, N. Y.
Harvey Cushing	Boston
David Linn Edsall	Cambridge
Walter Elmore Fernald	Waverley
Simon Flexner	New York, N. Y.
Joseph Lincoln Goodale	Boston
Robert Battey Greenough	Boston
Ross Granville Harrison	New Haven, Conn.
William Henry Howell	Baltimore, Md.
Reid Hunt	Boston
Henry Jackson	Boston
Elliott Proctor Joslin	Boston
William Williams Keen	Philadelphia, Pa.
Roger Irving Lee	Cambridge
Frank Burr Mallory	Brookline
William James Mayo	Rochester, Minn.
Samuel Jason Mixter	Boston
Francis Weld Peabody	Boston
William Lambert Richardson	Boston
Milton Joseph Rosenau	Brookline
Frederick Cheever Shattuck	Boston
Theobald Smith	Princeton, N. J.
Charles Wardell Stiles	Washington, D. C.
Richard Pearson Strong	Boston
Edward Wyllys Taylor	Boston
William Sydney Thayer	Baltimore, Md.
Ernest Edward Tyzzer	Wakefield
Frederick Herman Verhoeff	Boston
Henry Pickering Walcott	Cambridge
John Warren	Boston
John Collins Warren	Boston

William Henry Welch	Baltimore, Md.
Francis Henry Williams	Boston
Simeon Burt Wolbach	Boston
Horatio Curtis Wood	Philadelphia, Pa.
Hans Zinsser	Boston

CLASS III.— *Moral and Political Sciences.*— 197.

SECTION I.— *Theology, Philosophy and Jurisprudence.*— 49.

Thomas Willing Balch	Philadelphia, Pa.
Simeon Eben Baldwin	New Haven, Conn.
Willard Bartlett	Brooklyn, N. Y.
Joseph Henry Beale	Cambridge
Charles Henry Brent	Buffalo, N. Y.
Howard Nicholson Brown	Boston
Leslie Colby Cornish	Augusta, Me.
Charles Ambrose De Courcy	Boston
Edmund Burke Delabarre	Providence, R. I.
James De Normandie	Lincoln
Frederic Dodge	Belmont
Edward Staples Drown	Cambridge
William Harrison Dunbar	Cambridge
William Herbert Perry Faunce	Providence, R. I.
William Wallace Fenn	Cambridge
Frederick Perry Fish	Brookline
Paul Revere Frothingham	Boston
George Angier Gordon	Boston
Alfred Hemenway	Boston
William Ernest Hocking	Cambridge
Charles Evans Hughes	Washington, D. C.
Frederick John Foakes Jackson	New York, N. Y.
William Lawrence	Boston
Frederick Lawton	Boston
Arthur Lord	Plymouth
William Caleb Loring	Boston
Nathan Matthews	Boston
William McDougall	Cambridge

Edward Caldwell Moore	Cambridge
John Bassett Moore	New York, N. Y.
George Herbert Palmer	Cambridge
Charles Edwards Park	Boston
Leighton Parks	New York, N. Y.
Francis Greenwood Peabody	Cambridge
George Wharton Pepper	Philadelphia, Pa.
Roscoe Pound	Belmont
Elihu Root	New York, N. Y.
James Hardy Ropes	Cambridge
Arthur Prentice Rugg	Worcester
Austin Wakeman Scott	Cambridge
Henry Newton Sheldon	Boston
Moorfield Storey	Boston
William Howard Taft	Washington, D. C.
William Jewett Tucker	Hanover, N. H.
William Cushing Wait	Medford
Eugene Wambaugh	Cambridge
Edward Henry Warren	Boston
Winslow Warren	Dedham
Samuel Williston	Belmont

Class III., Section II.— *Philology and Archæology.*— 56.

Francis Greenleaf Allinson	Providence, R. I.
William Rosenzweig Arnold	Cambridge
Maurice Bloomfield	Baltimore, Md.
Franz Boas	New York, N. Y.
Ingersoll Bowditch	Jamaica Plain
Carl Darling Buck	Chicago, Ill.
Eugene Watson Burlingame	New Haven, Conn.
Edward Capps	Princeton, N. J.
George Henry Chase	Cambridge
Walter Eugene Clark	Chicago, Ill.
Roland Burrage Dixon	Cambridge
Franklin Edgerton	Philadelphia, Pa.
William Curtis Farabee	Philadelphia, Pa.
Jesse Walter Fewkes	Washington, D. C.

Jeremiah Denis Mathias Ford	Cambridge
Pliny Earle Goddard	New York, N. Y.
Charles Hall Grandgent	Cambridge
Louis Herbert Gray	Lincoln, Neb.
Charles Burton Gulick	Cambridge
Roy Kenneth Hack	Cincinnati, O.
William Arthur Heidel	Middletown, Conn.
George Lincoln Hendrickson	New Haven, Conn.
Bert Hodge Hill	Athens, Greece
Elijah Clarence Hills	Berkeley, Cal.
William Henry Holmes	Washington, D. C.
Edward Washburn Hopkins	New Haven, Conn.
Joseph Clark Hoppin	Pomfret, Conn.
Albert Andrew Howard	Cambridge
William Guild Howard	Cambridge
Aleš Hrdlička	Washington, D. C.
Eugene Xavier Louis Henry Hyvernat	Washington, D. C.
Carl Newell Jackson	Cambridge
Hans Carl Gunther von Jagemann	Cambridge
James Richard Jewett	Cambridge
Alfred Louis Kroeber	Berkeley, Cal.
Kirsopp Lake	Cambridge
Henry Roseman Lang	New Haven, Conn.
Charles Rockwell Lanman	Cambridge
John Livingston Lowes	Cambridge
David Gordon Lyon	Cambridge
Clifford Herschel Moore	Cambridge
George Foot Moore	Cambridge
Hanns Oertel	Munich, Germany
Chandler Rathfon Post	Cambridge
Edward Kennard Rand	Cambridge
George Andrew Reisner	Cambridge
Edward Robinson	New York, N. Y.
Fred Norris Robinson	Cambridge
Rudolph Schevill	Berkeley, Cal.
Edward Stevens Sheldon	Cambridge
Herbert Weir Smyth	Cambridge
Franklin Bache Stephenson	Washington, D. C.

Charles Cutler Torrey	New Haven, Conn.
Alfred Marston Tozzer	Cambridge
Clark Wissler	New York, N. Y.
James Haughton Woods	Cambridge

Class III., Section III.— *Political Economy and History*.— 47.

Wilbur Cortez Abbott	Cambridge
Brooks Adams	Quincy
George Burton Adams	New Haven, Conn.
Charles McLean Andrews	New Haven, Conn.
John Spencer Bassett	Northampton
Carl Lotus Becker	Ithaca, N. Y.
Charles Jesse Bullock	Cambridge
Thomas Nixon Carver	Cambridge
Edward Channing	Cambridge
John Bates Clark	New York, N. Y.
Archibald Cary Coolidge	Boston
Richard Henry Dana	Cambridge
Clive Day	New Haven, Conn.
Davis Rich Dewey	Cambridge
Ephraim Emerton	Cambridge
Henry Walcott Farnam	New Haven, Conn.
Max Farrand	New Haven, Conn.
William Scott Ferguson	Cambridge
Irving Fisher	New Haven, Conn.
Worthington Chauncey Ford	Cambridge
Edwin Francis Gay	New York, N. Y.
Frank Johnson Goodnow	Baltimore, Md.
Evarts Boutell Greene	Champaign, Ill.
Arthur Twining Hadley	New Haven, Conn.
Albert Bushnell Hart	Cambridge
Charles Homer Haskins	Cambridge
Isaac Minis Hays	Philadelphia, Pa.
Charles Downer Hazen	New York, N. Y.
George La Piana	Cambridge
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge

William MacDonald	New York, N. Y.
Charles Howard McIlwain	Cambridge
Roger Bigelow Merriman	Cambridge
Samuel Eliot Morison	Concord
William Bennett Munro	Boston
Charles Lemuel Nichols	Worcester
James Ford Rhodes	Boston
Michael I. Rostovtzeff	Madison, Wis.
William Milligan Sloane	New York, N. Y.
John Osborne Sumner	Boston
Frank William Taussig	Cambridge
Frederick Jackson Turner	Madison, Wis.
Claude Halstead Van Tyne	Ann Arbor, Mich.
George Grafton Wilson	Cambridge
George Parker Winship	Charles River
Allyn Abbott Young	Cambridge

Class III., Section IV.— *Literature and the Fine Arts.*— 45.

Irving Babbitt	Cambridge
George Pierce Baker	Cambridge
William Sturgis Bigelow	Boston
Le Baron Russell Briggs	Cambridge
Charles Allerton Coolidge	Boston
Frederick Shepherd Converse	Boston
Samuel McChord Crothers	Cambridge
Wilberforce Eames	New York, N. Y.
Edward Waldo Emerson	Concord
William Emerson	Cambridge
Arthur Fairbanks	Cambridge
Frank Edgar Farley	Middletown, Conn.
Arthur Foote	Brookline
Edward Waldo Forbes	Cambridge
Kuno Francke	Gilbertsville, N. Y.
Daniel Chester French	New York, N. Y.
Horace Howard Furness	Philadelphia, Pa.
Robert Grant	Boston
Morris Gray	Boston

Chester Noyes Greenough	Cambridge
James Kendall Hosmer	Minneapolis, Minn.
Mark Antony DeWolfe Howe	Boston
Archer Milton Huntington	New York, N. Y.
George Lyman Kittredge	Cambridge
William Coolidge Lane	Cambridge
John Ellerton Lodge	Boston
Charles Martin Tornov Lœffler	Medfield
Charles Donagh Maginnis	Brookline
Allan Marquand	Princeton, N. J.
Albert Matthews	Boston
Harold Murdock	Brookline
William Allan Neilson	Northampton
William Lyon Phelps	New Haven, Conn.
Arthur Kingsley Porter	Cambridge
Herbert Putnam	Washington, D. C.
Denman Waldo Ross	Cambridge
Paul Joseph Sachs	Cambridge
John Singer Sargent	London, Eng.
Ellery Sedgwick	Boston
Henry Dwight Sedgwick	Cambridge
Richard Clipston Sturgis	Boston
Charles Howard Walker	Boston
Owen Wister	Philadelphia, Pa.
George Edward Woodberry	Beverly
Charles Henry Conrad Wright	Cambridge

FOREIGN HONORARY MEMBERS.—65.

(Number limited to seventy-five.)

Class I.— *Mathematical and Physical Sciences.*— 25.

Section I.— *Mathematics and Astronomy.*— 8.

Johann Oskar Backlund	Petrograd
Arthur Stanley Eddington	Cambridge
Jacques Salomon Hadamard	Paris
Godfrey Harold Hardy	Oxford
Felix Klein	Göttingen
Tullio Levi-Civita	Rome
Charles Emile Picard	Paris
Charles Jean de la Vallée Poussin	Louvain

Class I., Section II.— *Physics.*— 8.

Svante August Arrhenius	Stockholm
Albert Einstein	Berlin
Oliver Heaviside	Torquay
Sir Joseph Larmor	Cambridge
Hendrik Antoon Lorentz	Haarlem
Max Planck	Berlin
Sir Ernest Rutherford	Cambridge
Sir Joseph John Thomson	Cambridge

Class I., Section III.— *Chemistry.*— 4.

Fritz Haber	Berlin
Henri Louis Le Chatelier	Paris
Wilhelm Ostwald	Leipsic
William Henry Perkin	Oxford

Class I., Section IV.— *Technology and Engineering.*— 5.

Heinrich Müller Breslau	Berlin
Ferdinand Foch	Paris

Joseph Jacques Césaire Joffre	Paris
Vsevolod Evgenievich Timonoff	Petrograd
William Cawthorne Unwin	London

CLASS II.— *Natural and Physiological Sciences.*— 22.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 10.

Frank Dawson Adams	Montreal
Charles Barrois	Lille
Waldemar Christofer Brögger	Christiania
Sir Archibald Geikie	Haslemere, Surrey
Viktor Goldschmidt	Heidelberg
Albert Heim	Zürich
Emmanuel de Margerie	Paris
Gustaf Adolf Frederik Molengraaff	Delft
Sir William Napier Shaw	London
Johan Herman Lie Vogt	Trondhjem

CLASS II., SECTION II.— *Botany.*— 4.

John Briquet	Geneva
Adolf Engler	Berlin
Ignatz Urban	Berlin
Hugo de Vries	Lunteren

CLASS II., SECTION III.— *Zoölogy and Physiology.*— 4.

George Albert Boulenger	Brussels
Maurice Caullery	Paris
Sir Edwin Ray Lankester	London
George Henry Falkiner Nuttall	Cambridge

CLASS II., SECTION IV.— *Medicine and Surgery.*— 4.

Rt. Hon. Sir Thomas Clifford Allbutt	Cambridge
Sir Thomas Barlow, Bart.	London
Francis John Shepherd	Montreal
Sir Charles Scott Sherrington	Oxford

CLASS III.— *Moral and Political Sciences.*— 18.

SECTION I.— *Theology, Philosophy and Jurisprudence.*— 3.

Rt. Hon. Arthur James Balfour, Earl of Balfour	Prestonkirk
Raymond Poincaré	Paris
Rt. Hon. Sir Frederick Pollock, Bart.	London

CLASS III., SECTION II.— *Philology and Archæology.*— 6.

Wilhelm Dörpfeld	Athens
Henri Guy	Grenoble
Hermann Georg Jacobi	Bonn
Arthur Anthony Macdonell	Oxford
Alfred Percival Maudslay	Hereford
Ramon Menendez Pidal	Madrid

CLASS III., SECTION III.— *Political Economy and History.*— 4.

Adolf Harnack	Berlin
Alfred Marshall	Cambridge
Henri Pirenne	Ghent
Rt. Hon. Sir George Otto Trevelyan, Bart.	London

CLASS III., SECTION IV.— *Literature and the Fine Arts.*— 5.

Georg Brandes	Copenhagen
Thomas Hardy	Dorchester
Jean Adrien Antoine Jules Jusserand	Washington, D. C.
Rudyard Kipling	Burwash
Sir Sidney Lee	London

STATUTES AND STANDING VOTES

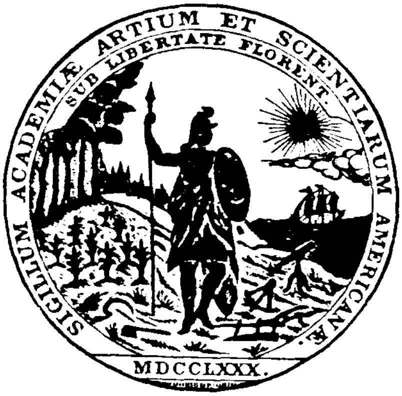
STATUTES

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *April* 12, *and December* 13, 1922, *February* 14, *March* 14, *and October* 10, 1923.

CHAPTER I

THE CORPORATE SEAL

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ.
SUB LIBERTATE FLORENT.
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art. 2.

CHAPTER II

Fellows and Foreign Honorary Members and Dues

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*
  Section 1. Mathematics and Astronomy
  Section 2. Physics
  Section 3. Chemistry
  Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*
  Section 1. Geology, Mineralogy, and Physics of the Globe
  Section 2. Botany
  Section 3. Zoölogy and Physiology
  Section 4. Medicine and Surgery

Class III. *The Moral and Political Sciences*
  Section 1. Theology, Philosophy, and Jurisprudence
  Section 2. Philology and Archæology
  Section 3. Political Economy and History
  Section 4. Literature and the Fine Arts

Article 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more that Two hundred and ten in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

Article 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii. art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments. Any Fellow shall also be exempt from annual dues who has paid such dues for forty years, or, having attained the age of seventy-five, has paid dues for twenty-five years.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x, art. 2.

ARTICLE 7. *Diplomas signed* by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix. art. 1, 7; chap. x. art. 2.

CHAPTER III

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS

The procedure in the election of Fellows and Foreign Honorary Members shall be as follows:

Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by two Fellows of that Section or by three Fellows of any Sections, and sent to the Corresponding Secretary accompanied by a statement of the qualifications of the nominee and brief biographical data.

Notice shall be sent to every Fellow not later than the fifteenth of January in each year, reminding him that all nominations must be in the hands of the Corresponding Secretary before the fifteenth of February following.

A list of the nominees, giving a brief account of each, with the names of the nominators, shall be sent to every Fellow with a request that he return the list with such confidential comments and indications of preference as he may choose to make.

All the nominations, with any comments thereon and with expressions of preference on the part of the Fellows, shall be referred to the appropriate Class Committees, which shall canvass them, and report their recommendations in writing to the Council before the Stated Meeting of the Academy in April.

Elections of Fellows and Foreign Honorary Members shall be made by the Council before the Annual Meeting in May, and announced at that meeting.

Persons nominated in any year, but not elected, may be carried over to the list of nominees for the next year at the discretion of the Council, but shall not be further continued unless renominated.

See Chap. ii.; chap. vi. art. 1; chap. ix. art. 1.

CHAPTER IV

OFFICERS

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x. art. 1.

ARTICLE 2. If any officer be unable, through death, absence, or disability, to fulfil the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

ARTICLE 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi. art. 2.

ARTICLE 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi. art. 2.

ARTICLE 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V

THE PRESIDENT

ARTICLE 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1.

CHAPTER VI

The Secretaries

Article 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

Article 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv. art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

ARTICLE 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII

THE TREASURER AND THE TREASURY

ARTICLE 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap. ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art. 1.

CHAPTER VIII

THE LIBRARIAN AND THE LIBRARY

ARTICLE 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

ARTICLE 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

ARTICLE 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

ARTICLE 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

ARTICLE 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

ARTICLE 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

ARTICLE 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6; chap. x. art. 1, 2.

CHAPTER IX

THE COUNCIL

ARTICLE 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x. art. 1.

ARTICLE 2. Seven members shall constitute a quorum.

ARTICLE 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

ARTICLE 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

ARTICLE 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

ARTICLE 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii. art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X

STANDING COMMITTEES

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance*, to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap. vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee*, to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iii) *The Cyrus Moors Warren Committee*, to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publication*, to consist of three Fellows, one from each Class, to whom all communications submitted to the

Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2; chap. ix. art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI

Meetings, Communications, and Amendments

Article 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

Article 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

Article 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

Article 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

Article 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

Article 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

Article 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

Article 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii; chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

STANDING VOTES

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. The Council, under such rules respecting nominations as it may prescribe, may elect as Associates of the Academy a limited number of men of mark in affairs or of distinguished service in the community.

Associates shall be entitled to the same privileges as Fellows, but shall not have the right to vote.

The admission fee and annual dues of Associates shall be the same as those of Fellows residing within fifty miles of Boston.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

INDEX.

Abbot, S. L., Notice of, 630.
Ames, Oakes, Biographical Notice of G. L. Goodale, 640.
Amory (Francis) Fund, 617.
Animal Life in the Thermal Waters of Yellowstone Park, Observations on, 369, 614.
Annual Tables of Constants, Appropriation for, 610.
Architecture, Forty Years of, 612.
Arthur, J. C., accepts Fellowship, 607.
Assessments, Annual, 625.
Associates, Committee on Methods of Electing, 609.

Ballad of Lovewell's Fight, 628.
Barnard, E. E., Notice of, 630.
Barnett, S. J., New Researches on the Magnetization of Ferromagnetic Substances by Rotation and the Nature of the Elementary Magnet, 628.
Baxter, G. P., and Cooper, W. C., Jr., A Revision of the Atomic Weight of Germanium. I. The Analysis of Germanium Tetrachloride, 233, 613.
Baxter, G. P., and Scott, A. F., A Revision of the Atomic Weight of Boron. The Analysis of Boron Trichloride and Boron Tribromide, 19, 608.
Becker, C. L., accepts Fellowship, 607.
Bell, Louis, death of, 607; Notice of, 633.
Biographical Notices, 629.
Books, Periodicals and Binding, Appropriation for, 612.
Boring, E. G., elected Fellow, 627.
Boron, A Revision of the Atomic Weight of, 19, 608.
Boron Trichloride and Boron Tribromide, The Analysis of, 19, 608.
Boulenger, G. A., accepts Foreign Honorary Membership, 609.
Bowditch, Ingersoll, elected Fellow, 627.
Boyden, E. A., elected Fellow, 627.
Brewster, William, Notice of, 639.
Bridgman, P. W., The Compressibility and Pressure Coefficient of Resistance of Rhodium and Iridium, 107, 608; The Compressibility of Five Gases to High Pressures, 171, 608; The Effect of Tension on the Thermal and Electrical Conductivity of Metals, 117, 608; The Thermal Conductivity of Liquids under Pressure, 139, 608.
Brues, C. T., Observations on Animal Life in the Thermal Waters of Yellowstone Park, with a Consideration of the Thermal Environment, 369, 614.
Byerly, W. E., Biographical Notice of J. M. Peirce, 650.

Campbell, C. M., Some Forms of Psychotherapy, 611.
Cannon, A. J. *See* Shapley, Harlow, and Cannon, A. J.
Carbon Dioxide Produced by Nerve, 614.
Chaffee, E. L., The Electrical Response of the Retina to Stimulation by Light, 610.
Chinese Architecture, Some Remarks on, 613.
Clapp, W. F., New Species of Shipworms in Bermuda, 277, 609.
Clifford, C. W., death of, 612.
Committee on Methods of Electing Associates, 609.
Compressibility and Pressure Coefficient of Resistance of Rhodium and Iridium, 107, 608.
Compressibility of Five Gases to High Pressures, 171, 608.
Conant, J. B., elected Fellow, 627.
Conductivity of Metals, The Effect of Tension on the Thermal and Electrical, 117, 608.
Coolidge, C. A., Some Remarks on Chinese Architecture, 613.
Cooper, W. C., Jr. *See* Baxter, G. P., and Cooper, W. C., Jr.

Copeland, Manton, accepts Fellowship, 607.
Cordilleran and Other Western Plants in the Unglaciated Areas of Northeastern America, The Occurrence of, 608.
Cornish, L. C., accepts Fellowship, 607.
Council, Report of, 614.

Davis, T. L., accepts Fellowship, 607.
DeCourcy, C. A., elected Fellow, 627.
Derr, Louis, death of, 607.
Determinism in Biology and Indeterminism in Physics, 608.
Detwiler, S. R., The Transplantation of Limbs in Relation to the Development of the Nervous System, 609.
Dodge, Raymond, elected Fellow, 627.
Duplicates, Library Committee given power to dispose of, 613.
Durand, E. J., The Genera Midotis, Ionomidotis and Cordierites, 1.

Edgar, C. L., elected Fellow, 627.
Einstein, Albert, elected Foreign Honorary Member, 627.
Electrical Response of the Retina to Stimulation by Light, 610.

Fellows deceased (19) —
  Louis Bell, 607.
  C. W. Clifford, 612.
  Louis Derr, 607.
  B. L. Gildersleeve, 611.
  Rudolph Hering, 607.
  C. F. Jenney, 610.
  Joseph Lipka, 611.
  Jacques Loeb, 611.
  S. W. McCall, 609.
  T. C. Mendenhall, 614.
  J. M. Morton, 610.
  E. F. Nichols, 614.
  T. M. Prudden, 614.
  Raphael Pumpelly, 607.
  S. P. Sharples, 607.
  C. P. Steinmetz, 609.
  W. R. Thayer, 607.
  A. G. Webster, 607.
  Woodrow Wilson, 611.
Fellows elected (14) —
  E. G. Boring, 627.
  Ingersoll Bowditch, 627.
  E. A. Boyden, 627.
  J. B. Conant, 627.
  C. A. DeCourcy, 627.
  Raymond Dodge, 627.
  C. L. Edgar, 627.
  W. E. Fernald, 627.
  W. C. Graustein, 627.
  P. A. Heymans, 627.
  S. J. Holmes, 627.
  J. A. Miller, 627.
  E. W. Taylor, 627.
  C. H. Van Tyne, 627.
Fellows, List of, 663.
Fernald, M. L., The Occurrence of Cordilleran and Other Western Plants in the Unglaciated Areas of Northeastern America, 608.
Fernald, W. E., elected Fellow, 627.
Flagellate Infecting Plants and Its Animal Hosts, 610.
Foreign Honorary Members deceased (2) —
  John Morley, Viscount Morley of Blackburn, 607.
  J. E. B. Warming, 614.
Foreign Honorary Member elected —
  Albert Einstein, 627.
Foreign Honorary Members, List of, 680.
Franklin, W. S., Determinism in Biology and Indeterminism in Physics, 608.
Frost, E. B., Biographical Notice of E. E. Barnard, 630.

Genera Midotis, Ionomidotis and Cordierites, The, 1.
General Fund, 615; Appropriations from the Income of, 610, 612.
Germanium, A Revision of the Atomic Weight of, 233, 613.
Germanium Tetrachloride, The Analysis of, 233, 613.
Gildersleeve, B. L., death of, 611.
Goodale, G. L., Notice of, 640.
Goodwin, H. M., Report of the Library Committee, 618.
Granites and Pegmatites of Cape Ann, Massachusetts, 313, 628.
Graustein, W. C., elected Fellow, 627.
Guy, Henri, accepts Foreign Honorary Membership, 607.

Haskins, C. H., Some Phases of Mediæval Science, 612.
Hering, Rudolph, death of, 607.
Heymans, P. A., elected Fellow, 627.
Holmes, S. J., elected Fellow, 627.

House Committee, Report of, 625.
House Expenses, Appropriation for, 610, 612.

Jack, J. R., accepts Fellowship, 607.
Jenney, C. F., death of, 610.
Joule-Thomson Effect in Air: First Paper, The, 608.

Keen, W. W., Biographical Notice of S. W. Mitchell, 644.
Kennelly, A. E., Biographical Notice of Louis Bell, 633; of C. P. Steinmetz, 657; Some Properties of Three-Terminal Electrical Conducting Networks, 295, 614.
Kent, N. A., appointed Acting Corresponding Secretary, 611.
Kittredge, G. L., The Ballad of Lovewell's Fight, 628.

Lane, A. C., Biographical Notice of S. L. Abbot, 630.
Lawrence, W. H., accepts Fellowship, 607; Report of the House Committee, 625.
Lee, R. L., accepts Fellowship, 607.
Library, Appropriation for, 612.
Library Committee, Report of, 618; given power to dispose of medical pamphlets and of duplicates, 613.
Lipka, Joseph, death of, 611; Trajectory Surfaces and a Generalization of the Principal Directions in Any Space, 49.
Loeb, Jacques, death of, 611.
Lyman, Theodore, Report of the Rumford Committee, 619.

McCall, S. W., death of, 609.
MacInnes, D. A., accepts Fellowship, 607.
McKinstry, H. E. *See* Warren, C. H., and McKinstry, H. E.
Magnetization of Ferromagnetic Substances by Rotation and the Nature of the Elementary Magnet, New Researches on, 628.
Marine Borers in Bermuda, 257, 609.
Mark, E. L., Marine Borers in Bermuda, 257, 609.
Mendenhall, T. C., death of, 614.
Miller, J. A., elected Fellows, 627.
Mills Bequest, Appropriation of income for Committee of Publication, 614.
Mitchell, S. A., accepts Fellowship, 607.
Mitchell, S. W., Notice of, 644.
Morley, John, death of, 607.
Morton, J. M., death of, 610.
Murdock, Harold, Report of the Treasurer, 615.

Naples University invites Academy to send representative to Six Hundredth Anniversary, 611.
National Research Council, W. M. Wheeler appointed to Division of Foreign Relations, 609.
Neal, H. V., Biographical Notice of William Brewster, 639.
Nichols, E. F., death of, 614.
Nominating Committee, 613.
Norris, J. F., Report of the C. M. Warren Committee, 620.

Officers elected, 625; List of, 661.
Osterhout, W. J. V., The Selective Action of Living Cells, 614.

Parker, G. H., The Carbon Dioxide Produced by Nerve, 614.
Peirce, J. M., Notice of, 650.
Pfeiffer, M. W. *See* Wilder, H. H., and Pfeiffer, M. W.
Pierce, G. W., Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters, 79.
Piezoelectric Crystal Resonators and Crystal Oscillators Applied to the Precision Calibration of Wavemeters, 79.
Prudden, T. M., death of, 614.
Psychotherapy, Some Forms of, 611.
Publication Committee, Report of, 622; Income from Mills Bequest appropriated for use of, 614.
Publication Fund, 617; Appropriation from the Income of, 612.
Pumpelly, Raphael, death of, 607.

Records of Meetings, 605.
Reparations Problems, The, 611.
Robinson, B. L., Biographical Notice of Count Solms, 651.
Roebuck, J. R., The Joule-Thomson Effect in Air: First Paper, 608.
Rostovtzeff, M. I., accepts Fellowship, 607.
Rumford Committee, Report of, 619.

Rumford Fund, 616; Appropriations from the Income of, 612; Papers published by aid of, 620.
Rumford Monument, Appropriation for Restoration of, 610.
Rumford Premium, 699.

Saunders, F. A., Recent Advances in Spectroscopy, 613.
Science, Some Phases of Mediaeval, 612.
Scott, A. F. *See* Baxter, G. P., and Scott, A. F.
Selective Action of Living Cells, 614.
Shapley, Harlow, and Cannon, A. J., Summary of a Study of Stellar Distribution, 213, 613.
Sharples, S. P., death of, 607.
Shipworms in Bermuda, New Species of, 277, 609.
Solms, Count, Notice of, 651.
Spectroscopy, Recent Advances in, 613.
Stakman, E. C., accepts Fellowship, 607.
Standing Committees elected, 626; List of, 661.
Standing Votes, 698.
Statutes, 683; amendment to, 607.
Steinmetz, C. P., death of, 609; Notice of, 657.
Stellar Distribution, Summary of a Study of, 213, 613.
Stratton, S. W., accepts Fellowship, 607.
Strong, R. P., A Flagellate Infecting Plants and its Animal Hosts, 610.
Sturgis, R. C., Forty Years of Architecture, 612.

Taylor, E. W., elected Fellow, 627.
Thayer, W. R., death of, 607.
Thermal Conductivity of Liquids under Pressure, 139, 608.
Three-Terminal Electrical Conducting Networks, Some Properties of, 295, 614.
Tokyo University, Appropriation for Binding Set of Academy Publications for, 610.
Trajectory Surfaces and a Generalization of the Principal Directions in any Space, 49.
Transplantation of Limbs in Relation to the Development of the Nervous System, 609.
Treasurer, Report of, 615.

Van Tyne, C. H., elected Fellow, 627.
Veblen, Oswald, accepts Fellowship, 607.

Warming, J. E. B., death of, 614.
Warner, E. P., accepts Fellowship, 607.
Warren, C. H., and McKinstry, H. E., Granites and Pegmatites of Cape Ann, Massachusetts, 313, 628.
Warren (C. M.) Committee, Report of, 620.
Warren (C. M.) Fund, 616; Appropriation from the Income of, 613.
Webster, A. G., death of, 607; Minute on Life and Services, 610.
Wheeler, W. M., appointed to National Research Council, 609.
Wilder, H. H., and Pfeiffer, M. W., The Bodily Proportions of Women in the United States; Based upon Measurements Taken from One Hundred Smith College Students, 439, 628.
Wilson, E. B., appointed to Committee of Publication, 609; Report of the Publication Committee, 622.
Wilson, Woodrow, death of, 611.
Women in the United States, The Bodily Proportions of, 439, 628.

Young, A. A., The Reparations Problem, 611.

Zinsser, Hans, accepts Fellowship, 607.